Monophonic radius distance Monophonic Number on Special Graphs

A. ANTO KINSLEY[*], M.SHANTHI[**]

[*]*Department of Mathematics,*
St.Xavier's College (Autonomous), Palayamkottai-627002.
Affiliated to Manonmaniam Sundaranar University, Abishekapatti,
Tirunelveli-627012, India.

[**]*Research Scholar(Reg No. 11822) ,*
Department of Mathematics, St.Xavier's College (Autonomous), Palayamkottai-627002.
Affiliated to Manonmaniam Sundaranar University, Abishekapatti,
Tirunelveli-627012, India.

Abstract- Let G be a connected graph of order at least two. We study about the monophonic sets and define a new set called the monophonic radius- distance monophonic set of a graph G. The monophonic radius was found for some corona related , path and cycle related graphs are found. Also Monophonic radius distance monophonic number are found.

Keywords: Monophonic distance, $r_m -$ distance monophonic set, $r_m -$ distance monophonic number

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I. INTRODUCTION

By a graph $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. For basis graph terminology, we refer to Harary [1,4]. A chord of a path is an edge joining two non-adjacent vertices of P. A path P is called monophonic if it is a chordless path. For any two vertices u and v in a connected graph G, the monophonic distance $d_m(u, v)$ from u to v is defined as the length of the longest $u - v$ monophonic path in G. The monophonic eccentricity $e_m(v)$ of a vertex v in G is $e_m(v) = max\{d_m(u, v): u \in V(G)\}$. The monophonic radius $rad_m(G)$ of G is $rad_m(G) = min\{e_m(v): v \in V(G)\}$. The monophonic diameter $diam_m(G)$ of G is $diam_m(G) = max\{e_m(v): v \in V(G)\}$. A vertex v is a simplicial vertex of a graph G if $< N(v) >$ is complete. A vertex v is an universal vertex of a graph G, if it is a full degree vertex of G. In this paper we study the distance monophonic sets and numbers for various graphs with respect to the monophonic radius distance. Throughout this paper we refer r_m as monophonic radius.

II. THE $r_m -$ DISTANCE MONOPHONIC SET

Definition 2.1. For a connected graph $G = (V, E)$ of order at least two, the set M of vertices of G is a $r_m -$ distance monophonic set of G. If each vertex of G lies on an $x - y$ monophonic path of length r_m for some vertices x and y in M where r_m is the monophonic radius. The minimum cardinality of a $r_m -$ distance monophonic set G is the $r_m -$ distance monophonic number of G, denoted by $m_{r_m}(G)$.

Theorem 2.2. For the corona graph $C_n \odot K_m$, the monophonic radius is $n - 1$.

Proof. For the corona graph $C_n \odot K_m$, there are n(m+1) vertices. Among those $n(m + 1)$ vertices, the nm vertices have the monophonic eccentricity n and all the other n vertices of $C_n \odot K_m$ have the monophonic eccentricity $n - 1$. Thus the minimum monophonic eccentricity which is monophonic eccentricity radius is $n - 1$.

Theorem 2.3. For the corona graph $C_n \odot K_m$, r_m-distance monophonic number is nm.

Proof. For the corona graph $C_n \odot K_m$, by the theorem 2.2 , the monophonic radius is $n - 1$ all of the m vertices of $C_n \odot K_m$ form the minimum r_m- distance monophonic distance. Hence $m_{r_m}(G) = nm$.

Theorem 2.4. For the corona $K_n \odot K_m$ $n \geq 4, m \geq 1$, the monophonic radius, r_m = 2.

Proof. In the corona graph $K_n \odot K_m$, there are nm vertices. Among the nm vertices all the n copies of K_m vertices has the eccentricity 3 and all the n vertices of K_n has the eccentricity 2. Hence minimum eccentricity is 2 for the corona graph $K_n \odot K_m$. Hence the monophonic radius of $K_n \odot K_m n \geq 4, m \geq 1$ is 2.

Theorem 2.5. For the corona $K_n \odot K_m n \geq 4, m \geq 1$, $m_{r_m}(G)$ $= mn + r_m$. Where r_m is the monophonic radius of $K_n \odot K_m$.

Proof. Let the corona graph $K_n \odot K_m$= G we have the monophonic radius is 2. Also we know that every simplicial vertex belongs to every r_m-distance monophonic set. Here all the n copies of K_m vertices are the simplicial vertices of $K_n \odot K_m$. Hence all these nm vertices belongs to every r_m – distance monophonic set of $K_n \odot K_m$. But these vertices alone does not form the r_m – distance monophonic set. Hence include any two vertices of K_n with nm vertices to form a minimum r_m–distance monophonic set of $K_n \odot K_m$. Hence $m_{r_m}(G) = mn + 2$. Here $r_m = 2$ for every $K_n \odot K_m$. Hence we also can written as $m_{r_m}(G) = mn + r_m$.

Theorem 2.6. For the corona graph, $P_n \odot \overline{K_m}$, $n \geq 2$, the monophonic radius, $r_m = \begin{cases} \frac{n+2}{2} & if\ n \equiv 0(mod\ 2) \\ \frac{n+1}{2} & if\ n \equiv 1(mod\ 2) \end{cases}$

Proof. Let G be the corona graph $P_n \odot \overline{K_m}$, $n \geq 2$.

Case (i) $n \equiv 1(mod\ 2)$. All the vertices of P_n has the eccentricity as follow. The vertices v_i and v_{n+1-i} where $1 \leq i \leq \lceil n/2 \rceil$ has the eccentricity $n+1-i$. Also for the n copies of $\overline{K_m}$ vertices every i[th] copy and $(n+1-i)^{th}$ copy of $\bar{k}_m$ vertices has the eccentricity $n+2-i$ where $1 \leq i \leq \lceil n/2 \rceil$. This implies the $v_{\lceil n/2 \rceil}$vertex has the minimum eccentricity as $n+1-\lceil n/2 \rceil$ = n+1-$\left(\frac{n+1}{2}\right) = \frac{n+1}{2}$. Thus the monophonic radius of $P_n \odot \overline{K_m}$ where $n \equiv 1(mod\ 2)$ is $\frac{n+1}{2}$

Case (ii) $n \equiv 0(mod\ 2)$

The vertices v_i and v_{n+1-i} has the same eccentricity as $n+1-i$ where $1 \leq i \leq n/2$. Also each vertex i^{th} copy and $(n+1-i)^{th}$ copy has the same eccentricity as $n+2-i$. Here the vertices $v_{n/2}$ and $v_{n+1-n/2}$ has the minimum eccentricity as $n+1-n/2 = \frac{2n+2-n}{2} = \frac{n+2}{2}$. Hence the monophonic radius of $P_n \odot \overline{K_m}$ where $n \equiv 0(mod\ 2)$ is $\frac{n+2}{2}$.

Theorem 2.7. For the corona graph $P_n \odot \overline{K_m}$, $n \geq 2$, $m_{r_m}(G) = mn$.

Proof. Let G be the corona graph $P_n \odot \overline{K_m}$, $n \geq 2$ we know that $r_m = \begin{cases} \frac{n+2}{2} & if\ n \equiv 0(mod\ 2) \\ \frac{n+1}{2} & if\ n \equiv 1(mod\ 2) \end{cases}$. Here all the vertices in then copies of $\overline{K_m}$ are the simplicial vertices. Hence these nm vertices belong to every $r_m -$ distance monophonic set and this nm vertices covers all the vertices of $P_n \odot \overline{K_m}$ in the $r_m -$ distance monophonic path and its minimum. Hence $m_{r_m}(G) = mn$.

Theorem 2.8. For the graph ${C_n}^2, n \geq 9$

$$r_m = \begin{cases} \frac{2n-6}{3} & n \equiv 0(mod\ 3) \\ \frac{2n-5}{3} & n \equiv 1(mod\ 3) \\ \frac{2n-7}{3} & n \equiv 2(mod\ 3) \end{cases}$$

Proof. Let us consider the graph ${C_n}^2$, n≥9. We will see n arise in three cases.

Case (i) $n \equiv 0(mod\ 3)$

In ${C_n}^2$ with $\equiv 0(mod\ 3)$, all the vertices of ${C_n}^2$ have the same eccentricity of $\frac{2n-6}{3}$. Hence the monophonic radius is $\frac{2n-6}{3}$. i.e) $r_m = \frac{2n-6}{3}$

Case (ii) $n \equiv 1(mod\ 3)$

In ${C_n}^2$ with $n \equiv 1(mod\ 3)$, all the vertices of ${C_n}^2$ have the same eccentricity of $\frac{2n-5}{3}$. Hence the monophonic radius is $\frac{2n-5}{3}$. i.e) $r_m = \frac{2n-5}{3}$

Case (iii) $n \equiv 2(mod\ 3)$

In ${C_n}^2$ with $n \equiv 2(mod\ 3)$, all the vertices of ${C_n}^2$ have the same eccentricity of $\frac{2n-7}{3}$. Hence the monophonic radius is $\frac{2n-7}{3}$ i.e) $r_m = \frac{2n-7}{3}$

Theorem 2.9. For the graph ${C_n}^2$, $n \geq 9$

$$m_{r_m}(G) = \begin{cases} \frac{n-r}{2q} & r = 0,2,3,5 \\ \frac{n-r}{2q} + 1 & r = 1,4 \end{cases} \text{ where } n = 6q + r, 0 \leq r < 6$$

Proof. For the graph ${C_n}^2$, $n \geq 9$ we will have six cases with respect to the remainder of modulo 6.

Case (i) $r = 0$

For the case $r = 0$, we have to divide n vertices into $n/6$ sets and label them to S[1],S[2], . . . $S_{n/6}$. In each sets S[1],. …$S_{n/6}$ we have to label each vertices with $v_{i1}, v_{i2}, v_{i3}, v_{i4}, v_{i5}, v_{i6}$ where $1 \leq i \leq n/6$. Choose the vertices v_{11},v_{14} in S[1] and $v_{n/6}$ vertex in $S_{n/6}$ to form the minimum r_m- distance monophonic set of ${C_n}^2$. $m_{r_m}(G) = \frac{n}{2q}$ where $n = 6q$.

Case (ii) $r = 1$

For the case $r = 1$, we have to divide n vertices into $n/6$ sets. We will have S[1],. …$S_{n/6}$ and one more vertex which we label as x_1. In each sets S[1],. …$S_{n/6}$ we have to label each vertices with $v_{i1}, v_{i2}, v_{i3}, \ldots v_{i6}$ where i≤ $i \leq n/6$. Choose v_{11},v_{13} in S[1] and $v_{n/6}$ in $S_{n/6}$, x_1 to form a minimum r_m- distance monophonic set of ${C_n}^2$. Hence $m_{r_m}(G)$= 4 . This can be written as $\frac{n-1}{2q}$+1 i.e).$m_{r_m}(G) = \frac{n-r}{2q}$ +1.

Case (iii) $r = 2$.

For the case $r = 2$, we have to divide n vertices into $n/6$ sets.We will have S[1],. …$S_{n/6}$ and 2 more vertices which we label as x_1, x_2 . In each sets S[1],S[2]. …$S_{n/6}$ we have to label each vertices with $v_{i1}, v_{i2}, v_{i3}, \ldots v_{i6}$ where i≤ $i \leq n/6$. Choose v_{11},v_{14} in S[1] and $v_{n/6}{}^6$ in $s_{n/6}$ to form a minimum r_m- distance monophonic set of ${C_n}^2$. Hence $m_{r_m}(G)$= 3 . This can be written as $\frac{n-2}{2q}$ where n = 6q+2 . i.e)$m_{r_m}(G) = \frac{n-r}{2q}$.

Case (iv) $r = 3$

For the case $r = 3$, we have to divide n vertices into $n/6$ sets. We will have S[1],. …$S_{n/6}$ and 3 more vertices which we label as x_1, x_2,x_3 . In each sets S[1],S[2]. …$S_{n/6}$ we have to label each vertices with $v_{i1}, v_{i2}, v_{i3}, \ldots v_{i6}$ where i≤ $i \leq n/6$. Choose v_{11},v_{14} in S[1] and also x_3 to form a minimum r_m- distance monophonic set of ${C_n}^2$. Hence $m_{r_m}(G)$= 3 . This can be written as $\frac{n-3}{2q}$ where n = 6q+3 . i.e).$m_{r_m}(G) = \frac{n-r}{2q}$.

Case (v) $r = 4$

For the case $r = 4$, we have to divide n into $n/6$ sets. We will have S[1],S[2]. …$S_{n/6}$ and 4 vertices which we label as x_1, x_2,x_3,x_4 . In each sets S[1],. …$S_{n/6}$ we have to label each vertices with $v_{i1}, v_{i2}, \ldots v_{i6}$ where i≤ $i \leq n/6$. Choose v_{11},v_{13} in S[1] and x_2,x_4 to form a r_m- distance monophonic set of ${C_n}^2$. Hence $m_{r_m}(G)$= 4 . This can be written as $\frac{n-4}{2q}$ +1 where n = 6q +4 i.e).$m_{r_m}(G) = \frac{n-r}{2q}$+1.

Case (vi) $r = 5$

For the case $r = 5$, we have to divide n into $n/6$ sets. We will have S[1],S[2]. …$S_{n/6}$ and 5 vertices which we label as x_1,

$x_2,\ldots x_5$. In each sets $S_1,\ldots S_{n/6}$ we have to label each vertices with $v_{i1}, v_{i2}, \ldots v_{i6}$ where i≤ $i \le n/6$. Choose v_{11}, v_{14} in S_1 and x_3 to form a r_m- distance monophonic set of C_n^2 . Hence $m_{r_m}(G)=5$. This can be written as $\frac{n-5}{2q}$ where $n = 6q+5$

i.e). $m_{r_m}(G)=\frac{n-r}{2q}$.

Theorem 2.10. For the graph $P_n^{\ 2}$, $n \ge 6$

$$r_m = \begin{cases} n/3 & n \equiv 0,\ 3 (mod\ 6) \\ \frac{n-1}{3} & n \equiv 1,\ 4 (mod\ 6) \\ \frac{n+1}{3} & n \equiv 2\ (mod\ 6) \\ \frac{n-2}{3} & n \equiv 5 (mod\ 6) \end{cases}$$

Proof: *Case (i)* $n \equiv 0 (mod\ 6)$

In this case, all the vertices of $P_n^{\ 2}$ has the eccentricity as follows. We will separate n vertices into $\frac{n}{2}$ vertices by v_i and v_{n+1-i} as $1 \le i \le \frac{n}{2}$ the same eccentricity. Now we consider the eccentricity of v_i , $1 \le i \le \frac{n}{2}$. We will have the eccentricity $n - (n/3 + (2q+1))$ to v_i if $i \equiv 1,2\ (mod\ 3)$ where $i = 3q + r$ and $n - (n/3 + 2q)$ if $i \equiv 0\ (mod\ 3)$ where $1 \le i \le n/2$ and then we consider the eccentricity v_{n+1-i}, $1 \le i \le n/2$ as v_i. Now we will see the $v_{n/2}$ and $v_{n+1-n/2}$ vertices has the minimum eccentricity as and $n - (n/3 + 2q) = n - \left(\frac{n+6q}{3}\right) = n - \left(\frac{12q+r}{3}\right) = n - \frac{12q}{3}$ ($\because$ $r = 0) = 6q + r - 4q = 2q = \frac{6q}{3} = \frac{6q+r}{3} = n/3$. Hence the minimum eccentricity is $n/3$ and so the monophonic radius is $n/3$ $if\ n \equiv 0, (mod\ 6)$

Case(ii) $n \equiv 1, (mod\ 6)$

In this case, we will separate n vertices into $\frac{n-1}{2}$ vertices by v_i, v_{n+1-i} $1 \le i \le \frac{n-1}{2}$ has the same eccentricity and then we will have the remaining vertex $v_{\lceil \frac{n}{2} \rceil}$. The vertex $v_i, i \equiv 0\ (mod\ 3)$ has the eccentricity $n - \left(\frac{n-1}{3} + 2q\right)$ where $i = 3q + r$ and if $i \equiv 1, (mod\ 3)$ has eccentricity $n - \left(\frac{n-1}{3} + (2q+1)\right)$ and if $i \equiv 2, (mod\ 3)$ has the eccentricity $n - \left(\frac{n-1}{3} + (2q+2)\right)$ $1 \le i \le \left\lceil \frac{n}{2} \right\rceil$. The vertex $v_{\lceil \frac{n}{2} \rceil}$ has the minimum eccentricity as $n - \left(\frac{n-1+6q+3}{3}\right) = n - \left(\frac{6q+1-1+6q+3}{3}\right) = n - \left(\frac{12q+3}{3}\right) = n - (4q + 1) = 6q + 1 - 4q - 1 = 2q = \frac{6q}{3} = \frac{6q+1-1}{3} = \frac{n-1}{3}$. Hence the minimum eccentricity is $\frac{n-1}{3}$ andso the monophonic radius is $\frac{n-1}{3}$ and so the monophonic radius is $\frac{n-1}{3}$ if $n \equiv 1\ (mod\ 6)$.

Case(iii) $n \equiv 2\ (mod\ 6)$

In this case we will separate n vertices into $\frac{n}{2}$ vertices as v_i, v_{n+1-i} has the same eccentricity. $1 \le i \le \frac{n}{2}$. The vertex v_i has the eccentricity as $n - \left(\frac{n+1}{3} + 2q\right)$ if $i \equiv 0,1\ (mod\ 3)$ and $n - \left(\frac{n+1}{3} + (2q+1)\right)$ if $i \equiv 2\ (mod\ 3)$ where $= 3q + r$. The vertices $v_{\frac{n-2}{2}}, v_{\frac{n}{2}}, v_{n+1-\frac{n-2}{2}}, v_{n+1-\frac{n}{2}}$ has the minimum eccentricity as $n - \left(\frac{n+1}{3} + 2q\right)$.Therefore $n - \left(\frac{n+1}{3} + 2q\right) = n - \left(\frac{n+1+6q}{3}\right) = n - \left(\frac{6q+2+1+6q}{3}\right) = n - \left(\frac{12q+3}{3}\right) = n - (4q+1) = 6q + 2 - 4q - 1 = 2q + 1 = \frac{3(2q+1)}{3} = \frac{6q+3}{3} = \frac{6q+2+1}{3} = \frac{n+1}{3}$. Hence the minimum eccentricity is $\frac{n+1}{3}$ and so the monophonic radius is $\frac{n+1}{3}$ if $n \equiv 2\ (mod\ 6)$.

Case(iv) $n \equiv 3\ (mod\ 6)$.

In this case, we will separate n vertices into $\frac{n-1}{2}$ vertices as v_i, v_{n+1-i} has the same eccentricity $1 \le i \le \frac{n-1}{2}$ and the remaining vertex $v_{\lceil \frac{n}{2} \rceil}$. The vertex v_i has the eccentricity as $n - \left(\frac{n}{3} + 2q\right)$ if $i \equiv 0\ (mod\ 3)$and $n - \left(\frac{n}{3} + (2q+1)\right)$ if $i \equiv 1,2\ (mod\ 3)$. The vertices $v_{\frac{n-1}{2}}, v_{\lceil \frac{n}{2} \rceil}, v_{n+1-\frac{n-1}{2}}$, has the minimum eccentricity as $n - \left(\frac{n}{3} + (2q+1)\right)$. Therefore $n - \left(\frac{n+6q+3}{3}\right) = n - \left(\frac{6q+3+6q+3}{3}\right) = n - \left(\frac{12q+6}{3}\right) = n - (4q+2) = 6q + 3 - 4q - 2 = 2q + 1 = \frac{3(2q+1)}{3} = \frac{6q+3}{3} = \frac{n}{3}$. Hence the minimum eccentricity is $\frac{n}{3}$ and so the monophonic radius is $\frac{n}{3}$ if $n \equiv 3\ (mod\ 6)$.

Case(v)) $n \equiv 4\ (mod\ 6)$.

In this case we separate n vertices into $\frac{n}{2}$ vertices as v_i, v_{n+1-i} has the same eccentricity . The vertex v_i has the eccentricity as $n - \left(\frac{n-1}{3} + (2q+1)\right)$ if $i \equiv 1 (mod\ 3)$, $n - \left(\frac{n-1}{3} + (2q+2)\right)$ if $i \equiv 2\ (mod\ 3)$ and $n - \left(\frac{n-1}{3} + 2q\right)$ if $i \equiv 0 (mod\ 3)$, where $i = 3q + r$. The vertices $v_{\frac{n}{2}}, v_{n+1-\frac{n}{2}}$, has the minimum eccentricity as $n - \left(\frac{n-1}{3} + (2q+2)\right)$ Therefore $n - \left(\frac{n-1}{3} + (2q+2)\right) = n - \left(\frac{n-1+6q+6}{3}\right) = n - \left(\frac{6q+4-1+6q+6}{3}\right) = n - \left(\frac{12q+9}{3}\right) = n - (4q+3) = 6q + 4 - 4q + 3 = 2q + 1 = \frac{3(2q+1)}{3} = \frac{6q+3}{3} = \frac{6q+3+1-1}{3} = \frac{6q+4-1}{3} = \frac{n-1}{3}$. Hence the minimum eccentricity is $\frac{n-1}{3}$ and so monophonic radius is $\frac{n-1}{3}$ if $n \equiv 4 (mod\ 6)$

Case(vi) $n \equiv 5 (mod\ 6)$

In this case we separate n vertices into $\frac{n-1}{2}$ vertices as v_i, v_{n+1-i} has the same eccentricity and the the remaining vertex $v_{\lceil \frac{n}{2} \rceil}$. The eccentricity of v_i is $n - \left(\frac{n-2}{3} + (2q+2)\right)$if $i \equiv 0,1\ (mod\ 3)$, and $n - \left(\frac{n-2}{3} + (2q+2)\right)$ if $i \equiv 2\ (mod\ 3)$, where $i = 3q + r$. The vertex $v_{\lceil \frac{n}{2} \rceil}$ has the minimum eccentricity as $n - \left(\frac{n-2}{3} + (2q+1)\right)$ since $\left\lceil \frac{n}{2} \right\rceil \equiv 0 (mod\ 3)$.Therefore $n - \left(\frac{n-2}{3} + (2q+1)\right) = n - \left(\frac{n-2+6q+3}{3}\right) = n - \left(\frac{n+6q+1}{3}\right) = n - \left(\frac{6q+5+6q+1}{3}\right) = n - \left(\frac{12q+6}{3}\right) = n - (4q+2) = 6q + 5 - 4q - 2 = 2q + 3 = \frac{3(2q+3)}{3} = \frac{6q+9}{3} = \frac{6q+5+4+2-2}{3} = \frac{6q+5+6-2}{3} = \frac{6q+5-2}{3} = \frac{n-2}{3}$. Hence the minimum eccentricity is $\frac{n-2}{3}$ and so monophonic radius is $\frac{n-2}{3}$ if $n \equiv 5 (mod\ 6)$

Theorem 2.11. For the graph ${P_n}^2$, $n \geq 6$

$$m_{r_m}(G) = \begin{cases} \frac{n-r}{2q} + 1 & r = 0,2,3,4 \\ \frac{n-r}{2q} & r = 1,5 \end{cases} \text{ where } n = 6q + r,$$

$0 \leq r < 6$

Proof. *Case (i)* $n \equiv 0(mod\ 6)$

In this case, we know that the monophonic radius, $r_m = \frac{n}{3}$. If we divide vertex set $V(G)$ into two sets S_1, S_2 with $\frac{n}{2}$ vertices each. In the each set $S_1 \& S_2$ label the vertices with v_{1i} and v_{2i} in S_1 and $(1 \leq i \leq \frac{n}{2})$.Choose the vertices v_{1i} and $v_{1\frac{n}{2}}$, and v_{2i} and $v_{2\frac{n}{2}}$ in s_2 form the minimum r_m −distance monophonic set. Hence $m_{r_m}(G) = 4$. This can be rewritten as $\frac{n}{2q} + 1$ where $n = 6q + r$. Here $r = 0$. Therefore $m_{r_m}(G) = \frac{n-r}{2q} + 1$

Case (ii). $n \equiv 1(mod\ 6)$

In this case, we know that the monophonic radius, $r_m = \frac{n-1}{3}$. If we divide vertex set $V(G)$ into two sets S_1, S_2 with $\frac{n-1}{2}$. We have remaining one vertex label this with x_1. In each set S_1 and S_2 label the vertices with v_{1i} and $v_{2i}(1 \leq i \leq \frac{n-1}{2})$.Choose the vertices v_{1i} in S_1, and v_{2i} in S_2 and also the take the vertex x_1 to form a minimum r_m −distance monophonic set. Hence $m_{r_m}(G) = 3$. This can be rewritten as $\frac{n-1}{2q}$ where $n = 6q + r$. Here $r = 1$. Therefore $m_{r_m}(G) = \frac{n-1}{2q}$.

Case (iii). $n \equiv 2(mod\ 6)$

In this case, we know that the monophonic radius, $r_m = \frac{n+1}{3}$. We divide the vertex $V(G)$ into two sets S_1 and S_2 with $\frac{n}{2}$ vertices. In each set S_1 and S_2 label the vertices with v_{1i} and $v_{2i}(1 \leq i \leq \frac{n}{2})$.Choose the vertices v_{1i} and $v_{i\left(\frac{n}{2}-1\right)}$ in S_1 and v_{22} and $v_{2\frac{n}{2}}$ in S_2 to form a minimum r_m −distance monophonic set. Hence $m_{r_m}(G) = 4$. This can be rewritten as $\frac{n-2}{2q} + 1$ where $n = 6q + r$. Here $r = 2$. Therefore $m_{r_m}(G) = \frac{n-2}{2q} + 1$.

Case (iv). $n \equiv 3(mod\ 6)$

In this case, we know that the monophonic radius, $r_m = \frac{n}{3}$. We divide the vertex set $V(G)$ into two sets S and S_2 with $\frac{n-1}{2}$ vertices and we have remaining one vertex label this with x_1 . In each set S_1 and S_2 label the vertices with v_{1i} and $v_{2i}(1 \leq i \leq \frac{n-1}{2})$.Choose the vertices the vertices v_{1i}, $v_{i\frac{n-1}{2}}$ is S_1 and v_{22} is S_2 and also take the vertex x_1 to form a minimum r_m −distance monophonic set. Hence $m_{r_m}(G) = 4$. This can be rewritten as $\frac{n-3}{2q} + 1$ where $n = 6q + r$. Here $r = 3$. Therefore $m_{r_m}(G) = \frac{n-r}{2q} + 1$.

Case (v). $n \equiv 4\ (mod\ 6)$

In this case, we know that the monophonic radius, $r_m = \frac{n-1}{3}$. We divide the vertex set $V(G)$ into two sets S_1 and S_2 with $\frac{n}{2}$ vertices . In each set S_1 and S_2 label the vertices with v_{1i} and $v_{2i}(1 \leq i \leq \frac{n-1}{2})$. Choose the vertices the vertices v_{1i} and $v_{i\frac{n}{2}}$ is S_1 and v_{21} and $v_{2\frac{n}{2}}$ is S_2 to form a minimum r_m −distance monophonic set. Hence $m_{r_m}(G) = 4$. This can be rewritten as $\frac{n-4}{2q} + 1$ where $n = 6q + 4$. Therefore $m_{r_m}(G) = \frac{n-r}{2q} + 1$.

Case (vi). $n \equiv 5\ (mod\ 6)$

In this case, we divide the vertex set $V(G)$ into S_1 and S_2 with $\frac{n-1}{2}$ vertices and we have the remaining one vertex label it with x_1 . In each set S_1 and S_2, label the vertices with v_{1i} and $v_{2i}(1 \leq i \leq \frac{n-1}{2})$.Choose the vertices v_{1i} in S_1 and v_{21} in S_2 and also take the vertex x_1 to form a minimum r_m −distance monophonic set. Hence $m_{r_m}(G) = 3$. This can be rewritten as $m_{r_m}(G) = \frac{n-5}{2q}$ where $n = 6q + r$. Here $r = 5$. Therefore $m_{r_m}(G) = \frac{n-r}{2q}$.

References

[1] A.Anto Kinsley and M.Shanthi, Monophonic Number of a graph with respect to monophonic radius, Malaya Journal of Matematik ,Vol. S,No. 1, 168-172, (2020).

[2] F. Buckley and F. Harary, Distance in graphs, Addison- Wesley, Redwood City,CA,(1990).

[3] G. Chartrand, F. Harary and P. Zang, On the geodesic number of a graph, *Networks*.39 (2009) 1-6.

[4] G. Chartrand and P. Zang, Introduction to Graph Theory, Tata McGraw Hill, (2006).

[5] F. Harary, Graph Theory, Addison- Wesley(1956).

[6] J.John, P. Arul Paul Sudhakar, On the edge monophonic number of a graph, *Filomat* 26:6 (2012), 1081 – 1089.

[7] A. P. Santhakumaran and P.Titus, Monophonic distance in graphs, *Discrete Mathematics*, Algorithms and Applications, Vol.3, No. 2 (2011) 159 – 169.

[8] A. P. Santhakumaran, P. Titus and K. Ganesamoorthy, On the monophonic number of a graph, *J.* Appl. Math. Informatics. 32 (2014), No. 1 - 2, pp. 255 – 266.

Authors

First Author : Dr.A.Anto Kinsley,M.Sc.,M.Phil.,Ph.D.,M.Tech.,
Associate Professor,
Department of Mathematics,
St.Xavier's College (Autonomous), Palayamkottai-627002.
Affiliated to Manonmaniam Sundaranar University,
Abishekapatti,Tirunelveli-627012, India.

Second Author : Mrs. M.Shanthi, M.Sc.,M.Phil.,
Research Scholar (Reg No. 11822) ,
Department of Mathematics,
St.Xavier's College (Autonomous), Palayamkottai-627002.
Affiliated to Manonmaniam Sundaranar University,
Abishekapatti,
Tirunelveli-627012, India.