A MULTIOBJECTIVE LINEAR PROGRAMMING METHODOLOGY TO DETERMINE THE OPTIMAL TAX RATIOS

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ABSTRACT: *In this research, a multi-objective linear goal programming model is developed to assist policymakers in structuring the nation's tax system and to facilitate the country by ensuring economic prosperity, which will ultimately lead to a boom and well-being in the nation's status. Since the amount of taxes imposed impacts a nation's economic growth. Every nation's government faces constant challenges in creating and implementing an effective tax system. The scope of this analysis is restricted to specific tax rates, including general sales tax, petrol tax, food and medication tax, telecom tax, etc. Sales tax is the largest contribution and the most regressive of them all. The main objective of this study is to develop an optimal mathematical model using the weighted goal programming variation suggested by Ganesh et al. [1] in order to determine and assign the optimal tax ratios. Using this methodology, the objectives with minimum deviation can be achieved.*

KEYWORDS: *Goal Programming, Mathematical Model, Multi Criteria Decision, Tax Ratios*

1. INTRODUCTION:

To minimize undesired deviations, Charness and Copper, who first established the goal programming technique in 1950 [2], characterized three goal programming configurations. Around the corner of 1960, software was introduced to solve GP problems. In light of the contributions made by Lee (1973) [3], Ignizio (1976) [4], Lin (1980) [5], Tamiz (1996) [6], and several others have been contributed in the development of different variants of linear GP.

By using many goals at once, goal programming—a unique kind of advanced linear programming—offers a means of working towards a formal decision analysis that may examine conflicting aims. The key advantage of GP is its simplicity and ease to use. This records a massive number of applications across numerous fields. Computer packages that are widely accessible for linear programming can be used to solve it, either as a single linear programming solution or as a sequence of connected linear programming solutions. Because it is a continuation of linear mathematical programming, for which effective techniques for addressing problems exist [7] . As a result, GP can handle a wide range of variables, restrictions, and objectives.

GP models can be divided into several types, each of which has distinct versions depending on decision variables and goals, as well as variants based on distance metrics [8]. Like, Weighted goal programming, Lexicographic goal programming, Chebyshev goal programming, Integer Goal programming, Binary goal programing and Fuzzy goal programming etc.

Many researchers have applied goal programing and its different variants to address the real world problems. JB Bassey used a goal programming approach in Power Generation Expansion Planning of Nigeria [9]. Sardar and Mohsin, developed a goal programming model to analyze the outsourcing strategies for cost and capacity flexibility. The application of three different variants of goal programming provided the alternative solutions [10]. The multi-objective optimization problem using three variants of goal programming (GP) approaches: preemptive GP, non-preemptive GP and weighted max–min fuzzy GP were applied for joint decision making of inventory lot-sizing, supplier selection and carrier selection problem [11]. A goal programming-based multiple-objective integrated response and recovery model to investigate strategic supply distribution and early-stage network restoration decisions were presented by Ransikarbum K. and Mason, S. J. [12]. Perić and Bratić , applied the goal programming methodology for solving multiple objective problem of the technological variants and production plan optimization . The optimization criteria are determined and the multiple objective linear programming model for solving a problem of the technological variants and production plan optimization is formed and solved [13]. Further , Mahajan et. al developed a simplified novel goal programming method under intuitionistic fuzzy environment using both membership/non-membership functions [14].

Do et al. [15]looked into the best- worst method and linear goal programming (GP) approached in combination to create an ideal circular economy model of the wood processing chain for lowering CO_2 emissions .Three goal programming approaches—the Weighted Approach (also known as the Archimedean Goal Programming), the Preemptive Approach (also known as the Lexicographic Approach), and the Chebyshev Approach (also known as the Min–Max Goal Programming)—were utilized by Kaur et al. to address a resource allocation problem in agile-based software development [16]. On the other hand, Malik et al. investigated the use of Meta-Goal Programming in the textile production industry to achieve multiple goals at once [17]. Tekin et al. [18] investigated a trip optimization for public transportation systems using the linear goal programming method.

In literature, Raida et. al. [19], investigated the state of Indiana's tax laws and created an optimization spreadsheet model to help the state government make tax policy decisions. The local government's financial department and the Federation of Tax Administrators were two of the public sources from which the data for the 2008 fiscal year was obtained. The model was developed by considering the competing interests of various tax revenue streams, including gaming, cigarette sales, alcohol sales, individual income, property, corporate income, and sales. These are the main sources of revenue for the state of Indiana. Both sales and income taxes were utilized, partly due to the balancing effect of the regressive nature of sales taxes with the progressive nature of income taxes.

Sahiner et al. [20] investigated the tax laws of the Turkish city Isparta. The tax bases for the 2009 fiscal year were determined through the utilization of the "fgoalattain" Matlab command found in the optimization toolbox. In order to establish equitable tax regulations for the city of Isparta, the following taxes were taken into consideration: building land, entertainment, fire insurance, trading licenses, advertising, inspection and examination of slaughterhouses, toll, facilities, real estate, sanitation, construction, wholesale food markets, residential usage licenses, communication, and work permits for certain days. They looked at both progressive and regressive taxes on various goods, but the decision to consume these goods is entirely up to the individual.

Using an interactive multi-objective linear programming methodology, Chrisman el al. [21] investigated the tax structure of Peoria . The results were obtained using ADBASE computer package. They listed several goals they hoped to accomplish with tax restructuring, such as lowering property taxes, lessening household debt, and preventing companies and consumers from moving to the suburbs in order to avoid paying higher sales taxes. Additionally, they proposed sales taxes on food, medicine, durable goods, and gasoline as alternative revenue sources to the property tax.

Any nation's ability to expand economically is greatly influenced by its tax structure. A tax percentage that is too high stunts economic expansion and robs the people of their money. The nation's tax policy makers must assign the ideal tax percentage in order to preserve economic stability and give financial comfort to the citizens of any community. It is expected, the established model will determine the optimal tax ratios that assist the broader public, especially the hard-pressed common man, and will enable policymakers to alter the tax policy template by incorporating different national aims. also offers the capacity to demonstrate the compromises made between the planning's aims.

The paper is organized as follows. Section 1 is dedicated to the introduction followed by Section 2, which discussed the methodology. The problem description is given in section 3 and model formulation is presented in Section 4. The implementation of the method is presented in Section 5. However, Section 6 presents the results and finally, conclusions are given in Section 7.

2. METHODOLOGY:

In particular, the goal function is maximized or minimized using the linear programming methodology. While the goal of goal programming modelling is to reduce the positive and negative variances between the set goals and the outputs that are subsequently provided in order of priority.

The fundamental approach of the GP is to identify a specific numerical goal for every target, convert distinct, disparate destinations into a single goal, and then search for a solution that reduces the total amount of deviations between these targets' capacities and their desired levels. The main benefit of GP is that, provided the problem has a feasible region, there is always an answer; this is made possible by the addition of deviational variables [22].

$$MinG_1\left(w_{i1}^{-}d_{i1}^{-} + w_{i1}^{+}d_{i1}^{+}\right) \quad ; for i = 1\ , 2, \dots, m$$

$$MinG_2\left(w_{i2}^{-}d_{i2}^{-} + w_{i2}^{+}d_{i2}^{+}\right) \quad ; for i = 1\ , 2, \dots, m$$

:

$$MinG_k\left(w_{ik}^{-}d_{ik}^{-} + w_{ik}^{+}d_{ik}^{+}\right) \quad ; for i = 1\ , 2, \dots, m$$

Subject to $\sum_{j=1}^{n} a_{ij}x_j + d_i^{-} - d_i^{+} = b_i \quad ; for i = 1,2, \dots,$

Where $\chi_j, d_i^{-}, d_i^{+} \geq 0$

$$G_1 \ggg G_2 \ggg \dots \ggg G_k$$

Using all of the available data, goal programming will be used to reduce the number of deviations from the predetermined targets. The most important goal will be the first to be prioritized, and subsequent goals won't be considered until the primary goal has been met or has reached the point at which more advancements are not desired. The ultimate solution may not fully accomplish all of the goals, but the least amount of deviations will be made.
GP is formed of four components;

1. Decision variables
2. Deviational variables
3. System constraints
4. Goal function

The generic form of GP model is

Minimize Z = $\sum_{i=1}^{m} (d_i^{-} + d_i^{+})$

Subject to

$$\sum_{j=1}^{n} (a_{ij}\chi_j + d_i^{-} - d_i^{+}) = b_i$$
$$for\ i = 1,2, \dots, m; j = 1,2, \dots, n$$

$$\chi_j, d_i^{-}, d_i^{+} \geq 0$$

Where

χ_j= the decision variables
d_i^{-} = amount by which the ith goal is underachieved

${d_i}^+$ = amount by which the ith goal is overachieved

${d_i}^-$ *and* ${d_i}^+$ are referred to as deviational variables

3. Problem Description:

Pakistan is the fifth most populous country in the world, with an anticipated 241.9 million [23] people living there as of 2024 (Economic Survey of Pakistan). The nation's policy makers always battle to reconcile divergent objectives when it comes to tax policy in order to establish a fair regulation for the taxes it collects. The scope of this analysis is restricted to specific tax rates, including general sales tax, petrol tax, food and medication tax, telecom tax, etc. The most regressive of them all, sales tax makes up 36.2% of the overall tax income (Financial statement 2022–2023) [24]. It is also the largest contributor. The main cause of this is that the items that are taxed (food, clothing, petrol, etc.) are not an individual choice and low-income citizens pay the major share of their income.

Table I represents the estimated revenue collection from taxes

Table – I Estimated Tax Revenues

Types of taxes	Tax Revenues (PKR Billions)
POL Products	233
Food and Drug	289
Telecom	299
Sales Tax	1087.79

Based on the supposition that the nation has the following objectives, an ideal mathematical model is formed to determine the best tax ratios to satisfy national needs without burdening the general public.

The concern goals are
G1: general tax sales have to be at least PKR. 2063 billion to satisfy the economic commitments of the country
G2: popular sales tax cannot exceed sixteen% of all taxes gathered
G3: Food and drug taxes can't exceed 14 % of all taxes collected
G4: Petrol tax can not exceed PKR.10 in line with liter.

It is important to note that the Pakistani government had begun levying a fixed rupee sales tax on petroleum product instead of a percentage based on the sale price per liter.

4. MODEL FORMUALTION:

Let the variables χ_t ,χ_f , χ_s , χ_p represent tax ratios for telecom, food and drug, general sales, and petroleum tax respectively.

Mathematically, the goals are then

$1088\chi_s + 233\chi_p + 299\chi_t + 289\chi_f \geq 2063$ (Tax Revenue constraint)

$0.16(1088\chi_s + 233x_p + 299\chi_t + 289\chi_f) \geq 1088\chi_s$ (General Sales Tax constraint)

$0.14(1088\chi_s + 233\chi_p + 299\chi_t + 289\chi_f) \geq 289\chi_f$ (Food and Drug Tax constraint)

$\chi_p \leq 10$ (Petroleum Tax constraint)

$$\chi_s, \chi_p, \chi_f, \chi_t \geq 0$$

By giving the objective with the largest deviation 50% of the total weight and distributing the remaining 50% among all the other goals, we apply the WGP model to minimize the weighted sum of deviations. The GP formulation can be expressed mathematically as follows:

Objective function

$$\text{Min } Z = 0.5d_1^- + 0.167d_2^-, + 0.167d_3^- + 0.17d_4^+$$

Subject to

$$1088\chi_s + 233\chi_p + 299\chi_t + 289x_f + d_1^- - d_1^+ = 2063$$

$$-913.92\chi_s + 37.28\chi_p + 47.84\chi_t + 46.24\chi_f + d_2^- - d_2^+ = 0$$

$$152.32\chi_s + 32.62\chi_p + 41.86\chi_t - 248.54\chi_f + d_3^- - d_3^+ = 0$$

$$x_p + d_4^- - d_4^+ = 10$$

$$\chi_s, \chi_p, \chi_t, \chi_f, d_1^-, d_1^+, d_2^-, d_2^+ d_3^-, d_3^+ d_4^-, d_4^+ \geq 0$$

Where $\sum_{i=1}^{n} w_i = 1$ such that $\sum_{i=1}^{n} w_i = 0.5 + \sum_{i=1}^{n-1} w_i = 1$

Basic Variables	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	R.H.S
Name of Variables	x_s	x_p	x_t	x_f	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	d_4^-	d_4^+	
Minimize Z	0	0	0	0	0.5		0.167	0	0.167	0	0	0.17	
Goal 1	1088	233	299	289	1	-1	0	0	0	0	0	0	2063
Goal 2	-913.92	37.28	47.84	46.24	0	0	1	-1	0	0	0	0	0
Goal 3	152.32	32.62	41.86	-248.54	0	0	0	0	1	-1	0		0
Goal 4	0	0	0	1	0	0	0	-1	0	0	1	-1	10

5. SOLUTION OF THE MODEL:

Within the linear programming module, the QM for Windows computer package is used to solve the weighted goal programming model. In this work, we simply provided three iterations.

First basic feasible solution

	Cj	0	0	0	0	0.5	0	0.167	0	0.167	0	0	0.17	0	0	0	0	
	Basis	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	Artf 1	Artf 2	Artf 3	Artf 4	R.H.S
Names of variables		x_s	x_p	x_t	x_f	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	d_4^-	d_4^+					
1	Artfcl 1	1.088	233	299	289	1	-1	0	0	0	0	0	0	1	0	0	0	2063
1	Artfcl 2	-913.9	37.28	47.84	46.24	0	0	1	-1	0	0	0	0	0	1	0	0	0
1	Artfcl 3	152.3	32.62	41.86	-248.54	0	0	0	0	1	-1	0	0	0	0	1	0	0
1	Artfcl 4	0	0	0	1	0	0	0	0	0	0	1	-1	0	0	0	1	10
	Zj	-326.4	-302.9	-388.7	-87.7	-1	1	-1	1	-1	1	-1	1	1	1	1	1	2073
	cj-zj	326.4	302.9	388.7	87.7	1	-1	1	-1	1	-1	1	-1	0	0	0	0	

Third Feasible Solution

	cj	0	0	0	0	0.5	0	0.167	0	0.167	0	0	0.17	0	0	0	0	
	Basis	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	Artf 1	Artf 2	Artf 3	Artf 4	R.H.S
Name of variables		x_s	x_p	x_t	x_f	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	d_4^-	d_4^+					
1	Artfcl 1	0	0	0	2064.28	1	-1	0	0	-7.1429	7.1429	0	0	1	0	-7.14	0	2063
0	x1	1	0	0	-0.3036	0	0	-0.0009	0.0009	0.0011	-0.001	0	0	0	-0.0009	0.001	0	0
0	x2	0	1	1.2833	-6.2017	0	0	0.0043	-0.0043	0.025	-0.025	0	0	0	0.0043	0.025	0	0
1	Artfcl 4	0	0	0	1	0	0	0	0	0	0	1	-1	0	0	0	1	10
	zj	0	0	0	-2065.2	-1	1	0	0	7.149	-7.142	-1	1	1	2	9.142	1	2073
	cj-zj	0	0	0	2065.28	1	-1	0	0	-7.1429	7.142	1	-1	0	-1	-8.142	0	

Sixth feasible solution (Optimal Solution)

	cj	0	0	0	0	0.5	0	0.167	0	0.167	0	0	0.17	0	0	0	0	
	Basis	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	Artf 1	Artf 2	Artf 3	Artf 4	R.H.S
Number of variables		x_s	x_p	x_t	x_f	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	d_4^-	d_4^+					
0		0	0	0	1	0.0005	-0.005	0	0	-0.0035	0.0035	0	0	0.0005	0	-0.0035	0	0.999
0	x1	1	0	0	0	0.0001	-0.0001	-0.0009	0.0009	0	0	0	0	0.0001	-0.0009	0	0	0.3034
0	Artfcl 3	0	1	1.2833	0	0.003	-0.003	0.0043	-0.0043	0.0043	-0.0043	0	0	0.003	0.0043	0.0043	0	6.1979
0	Artfcl 4	0	0	0	0	-0.0005	0.0005	0	0	0.0035	0.0035	1	-1	-0.0005	0	00.0035	1	9.0006
	zj	0	0	0	0	1	0	0.334	0	-0.334	0	0	0.34	0	0	0	0	0
	cj-zj	0	0	0	0	-0.5	0	-0.167	0	-0.167	0	0	-0.17	0	0	0	0	

After six iterations, the optimal solution is obtained as

$$X_s = 0.30$$

$$\chi_p = 6.20$$

$$\chi_f = 1$$

$$d^-{}_4 = 9$$

$$\chi_t = d_1^- = d_1^+ = d_2^- = d_2^+ = d_3^- = d_3^+ = d_4^+ = 0$$

6. RESULTS AND DISCUSSION:

All the objectives have been satisfied by the solution; the only one left unfulfilled is the petrol tax, which is missed by PKR 9 per liter. The country's policymakers will be able to select the best tax ratios for the upcoming years. This study is a valuable resource for the nation as policymakers work to adapt the tax system to the demands and circumstances of the state. The strategy could also assist other emerging nations in aligning their tax policies with the needs of their citizens.

7. CONCLUSION:

Using the goal programming methodology which is a popular approach to interactive multi-objective linear programming, in order to allocate and determine the optimal tax ratios for constructing a tax structure that maximizes growth and increases revenue while minimizing excess burden and preserving public morale, we developed an optimized mathematical model.

The government will be able to analyze Pakistani taxation and set tax ratios for the upcoming years based on strategies and policies by evaluating the results over the long and short terms and establishing goals to the insights gained from this.

Since the nation's budgetary planning and the process of economic changes both hinge on the correctness of ideal tax ratios. This study is conducted in an effort to provide a framework for better understanding the trade-offs and available options for enhancing the nation's tax system.

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