# ON FUZZY NEUTROSOPHIC LOCAL COVERING DIMENSIONS

**ABDULGAWAD. A. Q. AL-QUBATI[1,2]**

[1] **Department of Mathematics, College of Sciences and Arts, Najran University, Najran, SaudiArabia.**

[2] **Department of Mathematics, University of Hodeidah, Hodeidah, Yemen.**

***Abstract*: ** The covering dimension of topological spaces is considered one of the most important dimensions functions in dimension theory, and the locally covering dimension is no less important than it. Also, it closely related to covering dimension and other topological dimensions, so, in this paper, we study some interesting properties of zero and local covering dimension of fuzzy neutrosophic topological spaces, first, we establish a zero dimensionality and covering dimension of fuzzy neutrosophic spaces then we find the locally covering dimension of fuzzy neutrosophic regular spaces. Furthermore, a quantity of appealing properties and characterizations of subset theorem, fuzzy neutrosophic local homeomorphism and other related properties are studied.

**Keywords:**. Fuzzy neutrosophic topology, fuzzy neutrosophic covering dimension, fuzzy neutrosophic locally covering dimension, zero-dimensionality

[2000] 54A05, 54A08, 54D10..

## 1. INTRODUCTION

The fuzzy set was introduced by Zadeh [1] in 1965, where each element had a degree of membership. The intuitionstic fuzzy set (Ifs for short) on a universe X was introduced by K. Atanassov [2] in 1986 as a generalization of fuzzy set, where besides the degree of membership and the degree of non- membership of each element. Smarandache [3,4] proposed and developed the concept of a neutrosophic set as an improvement of a fuzzy set. The neutrosophic sets become popular over fuzzy sets due to their indeterminacy component which handles the hesitancy efficiently and in a better way than even the highest level fuzzy set i.e. IVIFS. The neutrosophic set contains three independent components namely, the truth membership T , the Indeterminacy membership I, and the Falsity membership F. The neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics, as an extension of the concept of the fuzzy set theory introduced by Zadeh. In [5] Salama and AL-Blowi introduced the concept of neutrosophic topological space (NTS) and in [6]Y.Veereswari introduced the concept of fuzzy neutrosophic topological spaces. The basic concepts of topological spaces are extended to fuzzy neutrosophic topological spaces by many authors. The covering dimension and zero dimensionality in fuzzy topological spaces were studied by some authors see [7,8], [9-11].In [12] we have introduced and investigated the concepts of intuitionistic fuzzy coveing dimension and zero dimensionality of intuitionistic fuzzy topological space, in view of the definition of Chang [13] also, we studied some of them properties.

In this paper a new approach to use of fuzzy neutrosophic set in dimension theory for that we introduce the concept of fuzzy neutrosophic locally covering dimension, we prove subset theorem and topological property for fuzzy neutrosophic regular space.

In Section 2, we reviewed the basic concepts in fuzzy neutrosophic topology which are important prerequisites that are needed in this paper.

In Section 3, we define the notion of the fuzzy neutrosophic covering dimension further, we develop the concept of ordinary covering dimension theory and intuitionistic fuzzy covering dimension using the concept of fuzzy neutrosophic topology. In addition, we prove some results which are similar to the classical ones.

In Section 4, we define zero dimensionality in fuzzy neutrosophic topological spaces, we prove some theorems about this concept.

Finally, in Section 5, we define the concept of fuzzy neutrosophic locally covering dimension and we introduce and discuss important theorems in this consept.

## 2.Preliminaries

The concept of fuzzy neutrosophic set with operations used in this study can be found in [3,4,14,15].

**Definition 2.1**[6] A fuzzy neutrosophic topology [ FNT for short ] on a nonempty set T is a family $\psi$ of neutrosophic fuzzy sets in T satisfy the following axioms:

(T1) $0_T^{\sim}, 1_T^{\sim} \in \psi$ .

(T2) If $A_1, A_2 \in \psi$, then $A_1 \cap A_2 \in \psi$.

(T3) If $A_\lambda \in \psi$ for each $\lambda \in \Lambda$, then $\bigcup_{\lambda\in\Lambda} A_\lambda \in \psi$ .

In this case the pair $(T, \psi)$ is called a fuzzy neutrosophic topological space [ FNTS for short ], The elements of $\psi$ are called fuzzy neutrosophic open sets [ NOS for short ]. A fuzzy neutrosophic set F is closed if and only if $C(F)$ is fuzzy neutrosophic open T .

**Example 2.2**[5] Let $T = \{t\}$ and $R_1, R_2$ , $R_3$ and $R_4$ are FN sets on T defined as follows:
$R_1 = \{t, 0.5, 0.5, 0.4, t \in T\}$.

$R_2 = \{t, 0.4, 0.6, 0.8, t \in T\}$.

$R_3 = \{t, 0.5, 0.6, 0.4, t \in T\}$.

$R_4 = \{t, 0.4, 0.5, 0.8, t \in T\}$.

Then the family $\psi = \{0_{\tilde{T}}, 1_{\tilde{T}}, R_1, R_2, R_3, R_4\}$ is a NT on T.

**Example 2.3** Let T = {a, b,c} and $M, N, C$ and D be IF sets on T defined as follows:

$$M = \left\langle t, \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.2}, \frac{c}{0.5}\right). \left(\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.2}\right) \right\rangle,$$
$$N = \left\langle t, \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.2}\right). \left(\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.4}\right) \right\rangle,$$
$$C = \left\langle t, \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.3}\right). \left(\frac{a}{0.2}, \frac{b}{0.2}, \frac{c}{0.2}\right) \right\rangle,$$
$$D = \left\langle t, \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right), \left(\frac{a}{0.4}, \frac{b}{0.2}, \frac{c}{0.5}\right). \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.4}\right) \right\rangle$$

Then the family $\psi = \{0_{\tilde{T}}, 1_{\tilde{T}}, M\, N, C, D\}$ is a FNT on T.

**Definition 2.4** Let A be a fuzzy neutrosophic set in FNTS (T, ψ), the set $\psi_Y = \{ Y \cap O : O \in O\ \psi \}$is a fuzzy neutrosophic topology on Y called the induced fuzzy neutrosophic topology on Y andthe pair $(Y, \psi_Y)$ is called the fuzzy neutrosophic subspace of (T, ψ).

**Definition 2.5** A fuzzy neutrosophic set A of FNTS (T, ψ) is said to be fuzzy neutrosophic neighbourhood of a fuzzy neutrosophic point P if there exists a fuzzy neutrosophic open set U such that $P \subseteq U \subseteq A$ (dented by FN-nbd ).

**Definition 2.6** A family $\mathbb{U} = \{U_\lambda : \lambda \in \Lambda\}$ is said to be fuzzy neutrosophic open cover of T if and only if $\bigcup_{\lambda \in \Lambda} U_\lambda = 1_{\tilde{T}}$ and a collection $\mathbb{V} = \{V_\alpha : \alpha \in \Delta\}$ is said to be FNOS refinement of $\mathbb{U}$ if $\bigcup_{\alpha \in \Delta} U_\alpha = 1_{\tilde{T}}$ and each $V_\alpha$ is contained in some members $U_\lambda$ of $\mathbb{U}$.

**Definition 2.7**[6] Let A be a fuzzy neutrosophic set of FNTS (T, ψ). Then

FNint(A) = $\bigcup\{G : G \in \psi, G \subseteq A\}$ is called the fuzzy neutrosophic interior of A, denoted by FN-int(A) or $\overset{\circ}{A}$ .

FNint(A) = $\bigcap\{F : F \in \psi^c, A \subseteq F\}$ is called the fuzzy neutrosophic closure of A, denoted byFN-cl(A) or $\overline{A}$ .

**Definition 2.8** A FNTS $(T, \psi)$ is said to be fuzzy neutrosophic regular spaces (FNRS.for short) if for every fuzzy neutrosophic point P in T and every fuzzy neutrosophic open set U such that $P \subseteq U$, there exists a fuzzy neutrosophic fuzzy set V such that $P \subseteq V \subseteq \overline{V} \subseteq U$.

**Definition 2.9 A** FNTS $(T, \psi)$ is said to be fuzzy neutrosophic normal spaces (FNNS.for short) if for every fuzzy neutrosophic closed set F in T and every fuzzy neutrosophic open set U such that $F \subseteq U$, there exists a fuzzy neutrosophic fuzzy set V such that $F \subseteq V \subseteq \overline{V} \subseteq U$

## 3. Fuzzy neutrosophic covering dimension.

**Definition 3.1** Let T be a nonempty set. A family $\mathbb{U} = \{U_\lambda : \lambda \in \Lambda\}$ of FNOSs in T is said to be of order $n(n > 1)$ written $ord_{FNO}\ \mathbb{U} = n$, if n is the greatest integer such that there exists a quisi-coinsident subfamily of $\mathbb{U}$ having n + 1 elements

**Remark 3.2** From the above definition if $ord_{FNO}\{\mathbb{U}\} = n$, then for each $n+2$

distinct indexes

$\lambda_1, \lambda_2, \ldots, \lambda_n \in \Lambda$ we have $U_{\lambda_1} \cap U_{\lambda_2} \cap \ldots \cap U_{\lambda_{n+2}} = \emptyset$ then it is quisi-coinsident, in particular if $ord_{FNO}\{\mathbb{U}\} = -1$, then $\mathbb{U}$ consists of the empty fuzzy neutrosophic sets and $ord_{FNO}\{\mathbb{U}\} = 0$, then $\mathbb{U}$ consist of pairwise disjoint fuzzy neutrosophic sets which are not all empty.

**Definition 3.3** The covering dimension of a FNTS $(T, \psi)$ denoted $dim_{FNT}(T)$ is the least integer n such that every finite fuzzy neutrosophic open cover of $1_T^{\sim}$ has a finite open refinement of order not exceeding n or $+\infty$ if there exists no such integer. Thus $dim_{FNT}(T) = -1$ if and onlyif $T = \emptyset$ and $dim_{FNT}(T) \leq n$ if every finite fuzzy neutrosophic open cover cover of $1_T^{\sim}$ has a finite open refinement of order $\leq n$. We have $dim_{FNT}(T) = n$ if it is true that $dim_{FNT}(T) \leq n$, but it is false that $dim_{FNT}(T) \leq n-1$. Finally $dim_{FNT}(T) = +\infty$ if for every positive integer n it is false that $dim_{FNT}(T) \leq n$.

**Theorem 3.4** The following statements are equivalent for FNTS $(T, \psi)$.

(1) $dim_{FNT}(T) \leq n$.

(2) For every finite fuzzy neutrosophic open cover $\mathbb{U} = \{U_1, U_2, \ldots, U_k\}$ of $1_T^{\sim}$ there exists a finite fuzzy neutrosophic open cover $\{V_1, V_2, \ldots, V_k\}$ of $1_T^{\sim}$ of order less than or equal to n, and $V_i \leq U_i$, for i = 1, 2,..., k.

(3) If $U_1, U_2, \ldots, U_{n+2}$ is an intuitionistic fuzzy open cover of $1_T^{\sim}$ then there exists a non-fuzzy neutrosophic quasi-coinsident open cover $V_1, V_2, \ldots, V_{n+2}$ of $1_T^{\sim}$ such that $V_i \leq U_i$, for i = 1, 2,. , n+2.

**Proof** (1) ⇒ (2) Suppose that $\dim_{FNT}(T) \leq n$ and let $\mathbb{U} = \{U_1, U_2, \ldots, U_k\}$ of $1_T^{\sim}$. Let $\mathbb{V}$ be a finite fuzzy neutrosophic open refinement of $\mathbb{U}$ such that $ord_{FNO}\{\mathbb{U}\} \leq n$, if $V \in \mathbb{V}$, then $V \subseteq U_i$, for some i, let each $V \in \mathbb{V}$ be associated with one FNSs $U_i$ containing it. Let $\eta_i$ be the union of allthose members of V thus associated with $U_i$. Then each $\eta_i$ is an FNOS and $\eta_i \subseteq U_i$.

Let $M = \eta_1, \eta_2, \ldots \eta_k$, to show that $ord_{FNO}\{M\} \leq n$, that is, every fuzzy neutrosophic quasi-coinsident subfamily of $\mathbb{N}$ contains at most n + 1 members. Suppose if possible, there exists a fuzzy neutrosophic quisi-coinsident subfamily $M_i$ of M containing (n+2) members. Then there exists $t \in T$ such that , for each $\eta_\alpha, \eta_\beta \in M_i$, we have

$\mu_{\eta_\alpha}(x) + \mu_{\eta_\beta}(x) > 1$, that is $\mu_{\eta\alpha}(x) > \mu_{\eta\beta}(x)$. Now, since $\eta_\sigma = \bigcup\{v_{i_\sigma} \in V : v_{i_\sigma} \subseteq U_i$, as associated in the construction of $\eta_\sigma\}$

$(\sigma = \alpha, \beta)$, and since $\mathbb{V}$ is a fuzzy neutrosophic finite cover of $t \in T$.and

$\mu_{\eta_\beta}(t) = max\{\mu_{v1_\beta}(t), \ldots, \mu_{vs_\beta}(t)\}$.

If we choose $V_{k_\beta}$ and $V_{l_\beta}$ such that $\mu_{\eta_\alpha}(t) = \mu_{Vk_\alpha}(t)$ and $\mu_{\eta_\beta}(t) = \mu_{V_{l_\beta}}(t)$.

Clearly, We get $V_{k\alpha}$ and$V_{k\beta}$ fuzzy neutrosophic quasi-coincident at t. In this way we obtain corresponding to every fuzzy neutrosophic quasi-coincident pair $\eta_x\alpha$ and $\eta_x\beta$ at t, a pair $V_{i\alpha}$ and$V_{i\beta}$ of$V$'s which are distinct in themselves as well as distinct from others and fuzzy neutrosophic quasi- coincident at t.

The collection of all these members of $\mathbb{V}$ chosen above constitute a fuzzy neutrosophic quasi-coincident subfamily of V having n + 2 members. This is contradiction to the fact that $ord_{F\,NO}\{U\} \le n$. Thus $ord_{F\,NO}\{\mathbb{U}\} \le n$

The proof of (2) ⇒ (3) , (3) ⇒ (2) and (3) ⇒ (1) follows immediately from Theorem (3.5) of [12].■

**Proposition 3.5** Let $(Y, \psi_Y)$ be a fuzzy neutrosophic closed subspace of FNTS $(T, \psi)$. Then $\dim_{FNT}(Y) \le \dim_{FNT}(T)$.

**Proof** Since Y is a neutrosophic fuzzy closed subspace of T, $\mu_Y$ is a neutrosophic fuzzy closed set in T. We must show that if $\dim_{FNT}(T) \le n$, then $\dim_{F\,NT}(Y) \le n$,. Clearly if $\dim_{F\,NT}(T) = -1$, then $T = 0_T^{\sim}$ and hence $Y = 0_T^{\sim}$, then $\dim_{F\,NT}(Y) = -1$ , and if $\dim_{F\,NT}(T) = \infty$ , then the theorem is obvious.

Suppose that $\dim_{F\,NT}(T) \le n < \infty$ and let $\mathbb{U}^Y = \{U_1^Y, U_2^Y, ..., U_n^Y\}$ be an open cover of $1_Y^{\sim}$ . Then $\mathbb{U} = \{U_1, U_2, ..., U_k - \mu_Y\}$ is neutrosophic fuzzy open cover of $1_T^{\sim}$ so $\mathbb{U}$ has a neutrosophic fuzzy open refinement $\mathbb{V}$ such that $ord_{F\,NO}\{\mathbb{V}\} \le n$, let $\mathbb{V}^Y = \{V|Y : V \in \mathbb{V}\}$ we claim that $ord_{F\,NO}\mathbb{V}^Y \le n$, let $\{V_{i_1}^Y, V_{i_2}^Y, ..., V_{i_{n+2}}^Y\}$ be a subfamily of $\mathbb{V}^Y$, since $ord_{F\,NO}\{\mathbb{V}\} \le n$, and since $\{V_{i_1}, V_{i_2}, ..., V_{i_{n+2}}\}$ is a subfamily of $\mathbb{V}$ having n + 2 members which is non-overlapping. That is for each $t \in T$ andin particular for each $t \in Y$ there exists subscripts $i_q$, $i_r$ such that $\mu_{V_{i_q}}(t) + \mu_{V_{i_r}}(t) \le 1$ ,i.e

$$\mu_{V_{i_q}}(t) \not> \mu_{V^c_{i_r}}(t) \text{ or } \rho_{V_{i_q}}(t) \not< \rho_{V^c_{i_r}}(t) \text{ or } \gamma_{V_{i_q}}(t) \not< \gamma_{V^c_{i_r}}(t)$$ [3,4]

This in turn implies that every subfamily of $\mathbb{V}^Y$ having n + 2 members is non-quasi-coincident andhence $ord_{F\,NO}\mathbb{V}^Y \le n$. Thus $\dim_{FNT}(Y) \le \dim_{F\,NT}(T)$.■

## 4.Zero dimensional in fuzzy neutrosophic topological spaces.

**Proposition 4.1** Let $(T, \psi)$ be a FNTS, then $\dim_{FNT}(T) = 0$ if and only if every finite fuzzy neutrosophic opn cover of $1_T^{\sim}$ has has a refinement consisting of disjoint crisp clopen neutrosophic fuzzy sets.

**Proof** By Remark (3.2) if $\mathbb{U} = \{U_\lambda : \lambda \in \Lambda\}$ is a disjoint crisp clopen cover of $1_T^{\sim}$ then $ord_{F\,NO}\mathbb{U} = 0$ and hence $\dim_{F\,NT}(T) = 0$.

Now, let $= \{U_\lambda : \lambda \in \Lambda\}$ be a finite fuzzy neutrosophic open cover of $1_T^{\sim}$ , since $\dim_{FNT}(T) = 0$, there exists a finite open refinement $\mathbb{V} = \{V_1, V_2, ..., V_k\}$ of $\mathbb{U}$ such that $ord_{F\,NO}\mathbb{U} = 0$, it followsthat every pair of elements of V are non-overlapping. Now, we show that each member of $\mathbb{V}$ is crisp clopen fuzzy neutrosophic set, let $V_i \in \mathbb{V}$ then $V_i$ is non-overlapping with the union of remaining members of $\mathbb{V}$ which is an open fuzzy set, since V is also cover of $1_T^{\sim}$ that is $v_i \bigcap (\bigcup_{i \ne j} v_j) = 1_T^{\sim}$

and since for each i, $V_i$ is non-overlapping with, $\bigcup_{i \ne j} V_j$ we have $v_i \bigcap (\bigcup_{i \ne j} v_j) = 0$, fo each i.

Hence $V_i + \bigcup_{i \ne j} V_j = 1_T^{\sim}$ by definition each $V_i + \bigcup_{i \ne j} V_j = 1_T^{\sim}$ is crisp and clopen fuzzy neutrosophic set in T and by Remark (3.2) the members of $\mathbb{V}$ are pairwise disjoint.■

**Proposition 4.2** If T = {t} is singleton space and (T, ψ) is an FNTS. Then $\dim_{FNT}(T) = 0$.
**Proof** Let $\mathbb{U} = \{U\}$ be a singleton family of fuzzy neutrosophic open sets which is a fuzzy neutrosophic open cover of $1_T^{\sim}$, there is a fuzzy neutrosophic open refinement of $\mathbb{U}$ which is $\mathbb{U}$, then $\mathbb{U} = 1_T^{\sim}$ but the $ord_{FNO}\{\mathbb{U}\} = 0$ it follows that $\dim_{FNT}(T) = 0$ .■

**Theorem 4.3** A closed subspace $(Y, \psi_Y)$ of zero dimensional FNTS $(T, \psi)$ is also zero-dimensional.

**Proof** Let $\mathbb{U}^Y = \{U^Y, U^Y, ..., U^Y\}$ be an fuzzy neutrosophic open cover of $1_T^{\sim}$, then $\mathbb{U} = \{U_1, U_2, ..., 1_T^{\sim} - \psi_Y\}$ is a fuzzy neutrosophic open cover of $1_Y^{\sim}$, by Proposition (4.1) $\mathbb{U}$ has an open refinement $\mathbb{V}$ consisting of disjoint crisp clopen fuzzy neutrosophic sets such that $ord_{FNO}\{\mathbb{U}\} = 0$ , let $V^Y = \{V|Y, V \in V\}$ we claim that $ord_{FNO}\mathbb{V}^Y = 0$, since V consisting of disjoint crisp clopenfuzzy neutrosophic sets, this implies that $V_Y$ consisting of disjoint crisp clopen fuzzy neutrosophic set, since $ord_{FNO}$ $\mathbb{U} = 0$ and $dim_{FNT}(T) = 0$, then by Proposition (3.5) we get $ord_{FNO}V^Y = 0$and $dim_{FNT}(Y) = 0$.■

## 5. Fuzzy neutrosophic Local dimension

In this section the concept of locally covering dimension and Local home of FNTS are introduced, same theorem about them are extended and proved.

**Definition 5.1** The local covering dimension of non-empty FNTS $(T, \psi)$ denoted by $Locdim_{FNT}(T)$is the least integer n such that for every FNP P in T there exists some open fuzzy neutrosophic set U with $P \subseteq U$ such that $\dim_{FNT}(\overline{U}) \leq n$ or ∞ if there exists no such integer.

If $T = 1_T^{\sim}$ i.e. T is empty, then $Locdim_{FNT}(T) = -1$ .

Thus $Locdim_{FNT}(T)$ is the least integer n such that there exists a fuzzy neutrosophic open cov- ering $\mathbb{U} = \{U_\lambda : \lambda \in \Lambda\}$ of $1_T^{\sim}$ such that $\dim_{FNT}(\overline{U}) \leq n$, for each $\lambda \in \Lambda$ and every nonempty FNTS T.

If $Locdim_{FNT}(T) \leq n$, P is an FNP in T and a fuzzy neutrosophic open set U such that $P \subseteq U$, then there exists a fuzzy neutrosophic set V such that $P \subseteq V \subseteq U$ and $dim_{FNT}(\overline{V}) \leq n$. For there exists an fuzzy neutrosophic open set W such that $P \subseteq W$ and $dim_{FNT}(\overline{W}) \leq n$.

Hence $Locdim_{FNT}(T) \leq n$, if and only if for every finite fuzzy neutrosophic open cover of $1_T^{\sim}$ has an fuzzy neutrosophic open refinement $\mathbb{U} = \{U_\lambda : \lambda \in \Lambda\}$, say, such that $dim_{FNT}(\overline{U}) \leq n$, for each

Clearly $Locdim_{FNT}(T) \leq dim_{FNT}(T)$.

According to the above definition we prove the following open subset theorem for fuzzy neutro-sophic regular space.

**Theorem 5.2** If $(Y, \psi_Y)$ is a fuzzy neutrosophic open subspace of fuzzy neutrosophic regular space $(T, \psi)$, then $Locdim_{FNT}(Y) \leq Locdim_{FNT}(T)$.

**Proof** Suppose that $Locdim_{FNT}(T) = n$. To prove that $Locdim_{FNT}(Y) \leq n$. Let P be a fuzzyneutrosophic point in Y, then there exists a fuzzy neutrosophic open set U of P in X such that $\dim_{FNT}(\overline{U}) \leq n$

Let $T_Y$ be a fuzzy neutrosophic set in Y, then $T = U \cap T_Y$, and since $(T, \psi)$ be a fuzzy neutrosophic regular space there exists an fuzzy neutrosophic open set V in T such that $P \subseteq V \subseteq \overline{V} \subseteq T$ then V is a FN-nbd of P in $\psi_Y$ and $\overline{V}$ is the closure of V in Y, and since $\overline{V} \subseteq \overline{T} \subseteq U \cap T_Y$ this implies that $\overline{V} \subseteq \overline{U}$, this show that $\overline{V}$ is a fuzzy neutrosophic closed in $\overline{U}$ it follows that $\dim_{FNT}(\overline{V}) \leq n$ then $Locdim_{FNT}(Y) \leq$ n and hence $Locdim_{FNT}(Y) \leq Locdim_{FNT}(T)$.■

**Definition 5.3** Let $(T, \psi)$ and $(Y, \delta)$ be two FNRTSs. A continuous function $f : (T, \psi) \to (Y, \delta)$ is said to be fuzzy neutrosophic local homeomorphism denoted (FN-Lochome.), if for each fuzzy neutrosophic point P in T has an open FN-nbd U such that $f(U)$ is fuzzy neutrosophic open in Yand the function $f|U : U \to f(U)$ is an FN- home.

**Theorem 5.4** Let $(T, \psi)$ and $(Y, \delta)$ be two FNRTSS, and $f : (T, \psi) \to (Y, \delta)$ is a surjective FN-Lochome. Then $Locdim_{FNT}(T) = Locdim_{FNT}(Y)$.

**Proof** Clearly if $Locdim_{FNT}(T)) = -1$ and **Y** is homeomorphic to **T**, then $Y = 0_Y^{\sim}$ and $Locdim_{FNT}(Y) = -1$, also $Locdim_{FNT}(T) = \infty$ follows directly.

Suppose that $Locdim_{FNT}(T) = n$, and let P be a fuzzy neutrosophic point in T, there exists anopen FN-nbd U of P such that $f(U)$ is an fuzzy neutrosophic open set in Y and $f$ maps U home- omorphically onto $f(U)$, since Y is FNRS. there exists a fuzzy neutrosophic open set H in Y such that $P \subseteq H \subseteq \overline{H} \subseteq f(U)$, and $\dim_{FNT}(\overline{H}) \leq n$. Since H is a fuzzy neutrosophic open set Y then $f^{-1}(H)$ is an fuzzy neutrosophic open set in T. If $V = f^{-1}(H) \cap U$ then V is a fuzzy neutrosophic open FN-nbd of P and $\overline{V}$ is homeomorphic with $\overline{H}$ and since T is an FNRTS,

We have $P \subseteq V \subseteq \overline{V} \subseteq U$ so that $\dim_{FNT}(\overline{V}) \leq n$ then by definition we have $Locdim_{FNT}(T) \leq n$. Henc $Locdim_{FNT}(T)) \leq Locdim_{FNT}(Y)$....(1)

Now, suppose that $Locdim_{FNT}(T)\ ) \leq n$, and let q be fuzzy neutrosophic point in Y and $f : (T, \psi) \to (Y, \delta)$ surjective there exists a fuzzy neutrosophic point P in X such that

$f(P) = q$, and there exist, a fuzzy neutrosophic open FN-nbd U of P such that $f$ maps U homeomorphicallyonto $f(U)$, which is an open FN-nbd of q, and since $(T, \psi)$ is FNRs., there exist a neutrosophic open set **V** in **X** such that $p \subseteq V \subseteq \overline{V} \subseteq U$ and by definition $\dim_{FNT}(\overline{V}) \leq n$ .

Since $V = f^{-1}(H) \cap U$ then $f(V) = f(f^{-1}(H) \cap U) = H \cap f(U) = H$ ,, (since $f(H) = U$ )

therefore $f$(V ) = H, then H is an open FN-nbd of q and $\overline{H}$ is homeomorphic with $\overline{V}$ ,so that

$\dim_{FNT}(\overline{H}) \leq n$, ThusLocdimFNT (Y ) ≤ n. Then LocdimFNT (Y )≤ LocdimFNT (T). … (2)

From(1) and (2), we get $Locdim_{FNT}(T)) = Locdim_{FNT}(Y)$.■

# 6.Conclusions.

In the present research, we introduced the concept of covering dimension and zero dimensionality of fuzzy neutrosophic space, We studied some of their properties and theorems. Also, we intro- duced the concepts of locally covering dimension ,we extended and discussed some theorem to fuzzy neutrosophic space. As a more generalization, in our next work, we will use fuzzy neutrosophic space to define the fuzzy neutrosophic local small inductive dimension and fuzzy neutrosophic local large inductive dimension with study of relationship between the concepts of dimension and local dimension for covering and inductive dimensions, in particular, we will study dimensionality and locally dimensionality for fuzzy neutrosophic normal and regular spaces.

## Conflict of interest.

No potential conflict of interest relevant to this article was reported.

.

## REFERENCES

[1] L. A. Zadeh.. Fuzzy sets, Information and control 8(1965) 338-353.
[2] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems., 20 (1986), 87-96.
[3] F. Smarandache, A unifying field in Logics: Neutrosophic Logic. Multiple-Valued Logic 2002; 8: 385-438.
[4] F.Smarandache, An introduction to Neutrosophy, neutrosophic Logic, neutrosophic set, and neutrosophic prob- ability and Statistics. In Proc.First Intern. Conf. Neutrosophy, neutrosophic Logic, neutrosophic set, and neu- trosophic probability and Statistics. Phoenix, AZ: Hexis; 2002, 5-21.
[5] A.A. Salama and S. AL-Blowi, Neutrosophic Set and Neutrosophic Topological Spaces, IOSR Journal of Math- ematics, 3(4) (2012), 31-35.
[6] Y.Veereswari,An Introduction To Fuzzy Neutrosophic Topological Spaces, IJMA,Vol.8(3), (2017), 144-149.
[7] AbdulGawad Al-Qubati, On Intuitionistic Fuzzy $\beta$ Generalized $\alpha$ Normal Spaces, International Journal of Math- ematics and Applicatins, V (20),no 4 (2022),1-11.
[8] B. Amudhambigai, N. Krithika and R. Dhavaseelan, Covering Dimension of Fuzzy Normal Spaces, The Journal of Fuzzy Mathematics 25(2)(2017)575-584.
[9] N. Ajmal and J. K. Kohil, Overlapping families and covering dimension in fuzzy topological spaces, Fuzzy Sets and Systems, 64 (1994), 257-263.
[10] N. Ajmal and J. K. Kohil, Zero dimensional and strongly zero dimensional topological spaces, Fuzzy Sets and Systems, 61 (1994), 231-237.
[11] T. Baiju and Sunil Jacob John, Covering dimension and normality in L topological spaces, Springer Mathemat- ical Sciences, (2012), 6:35 doi:10.1186.6251. 7456-635
[12] AbdulGawad, A. Al-Qubati, Covering dimension of intuitionistic fuzzy topological spaces, Annals of Fuzzy Mathematics and Informatics.vol 7,no.3, pp. 485-493, 2014.http://www.afmi.or.kr/

[13] C.L.Chang, Fuzzy Topological Spaces, J. Math. .Anal. Appl.24(1968),182-190.
[14] G.C.Ray, and S Dey. Neutrosophic point and its neighbourhood structure. Neutrosophic Sets and Systems, 43(2021), 156-168.
[15] G.C.Ray, and S Dey. Relation of Quasi-coincidence for Neutrosophic Sets. Neutrosophic Sets and Systems, 46(2021), 402-415.

## AUTHOR

ABDULGAWAD. A. Q. AL-QUBATI.

**Correspondence Author** ABDULGAWAD. A. Q. AL-QUBATI.