Solution of multi-objective assignment problems in the distance-based method using improved Hungarian algorithm

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Abstract: This approach solves MOAP based on the IHA while using distance-based approaches. The weighted goal programming approach is developed, which is focused on decreasing the distances between ideal objectives and achievable objective space. It presents finest compromise approach for MOLPP. MOLPP is addressed by the suggested model by addressing a sequence of single goal sub-problems, where the objectives are turned into constraints. The provided compromise solution can be improved by setting priority in terms of weights. A criterion for determining the best compromise solution is also proposed. The proposed methodology will tackle the multi-objective assignment problems while using the improved Hungarian algorithm used for mono-objective assignment problems. The suggested technique will be useful in solving transportation and assignment problems with various and competing objectives.

Key Words: Multi-objective programming; Distance-based Method; Optimization,

INTRODUCTION

Optimization has been introducing and expanding all around at successive rates. New algorithmic and theoretical techniques have been introducing for the development of other disciplines rapidly especially in the field of machine learning, artificial intelligence, and quantum computing. One of the most notable trends in optimization is the emphasis on the interdisciplinary nature of the field. Optimization is using as a basic tool in areas in many areas of applied mathematics, economics, computer science, engineering, medicine, and other sciences. Moreover, we tackle multi-objective problems in which involve more than one objective that can be minimized or maximized simultaneously for an aggregate solution [22]. The uses of multi-objective optimization in many fields of science, engineering, economics, and logistics where optimal decisions are needed are remarkable. Here are the following examples of multi-objective optimization problems involving two and three objectives, respectively: Minimizing cost while maximizing comfort while buying a car. Maximizing performance while minimizing fuel consumption and emission of pollutants of a vehicle. For further understanding about multi-objective optimization, one can see the recent articles [1, 2, 3] and references therein.

We deal with numerous replacements specified by a certain number of linear constraints in a finite-dimensional space in a multi-objective optimization problem [4]. In a multi-objective optimization problem, we interact with multiple substitutes described by a specific number of linear constraints in a finite-dimensional space [4]. Overall, there is no ideal approach that maximizes all priorities simultaneously. The concept of an optimum solution gives birth to the concept of non-dominated solutions, in which no change in any objective function is feasible without compromising at least one of the other objective functions. This multi-objective linear programming is given in [4].

In multi-objective optimization, one of the known problems is TSP. Anyone who has conducted an organizational study has undoubtedly encountered the subject of travelling salesmen (also known as TSPs). The TSP's inevitability stems from both its ease of understanding and its significant links to fundamental and complex concerns. TSP has remained a source of consternation for many mathematicians for many years. Mathematicians have yet to find a suitable solution to the TSP. TSP math was created in the 1800s by Irish and British mathematicians, Sir William Rowan Hamilton and Thomas Pennington Kirkham, respectively. It is the highest TSP ever agreed to. TSP is classified into two types: symmetric TSP and asymmetric TSP. For further detail see [5, 6]. For the solution of TSPs, mathematicians introduced many algorithms like, branch and bound algorithm, nearest neighbor approach, greedy approach etc., one can see [6, 14-18] and references therein.

Another problem is the assignment problem, which is like the TSP but in addition, it is restricted such that the salesman starts from his city, visit each city once, and returns to the initial point (home city) so that the total distance (cost or time) is minimum.

The assignment issue is a type of LPP that includes determining the most effective allocation of People on plans [7]. It is utilized all throughout the world to tackle real-world problems [8]. The assignment problem is one of the most notable problems in mathematical programming since it involves assigning various roles (tasks or jobs) to an equal number of machines (people) based on their performance. To tackle assignment difficulties, there exist particular algorithms. The Hungarian solution is likely the most well-known [9].

Hungarian method

The Hungarian allocation approach allows us to identify the best answer without comparing each allocation alternative directly. It works on the basis of a matrix reduction theory. This indicates that the necessary values are removed and added to the matrix, reducing the issue to an opportunity cost matrix. In contrast to the greatest or cheapest allocation, opportunity costs represent the relative limits associated with the assignment of each individual in a project. We wish to make assignments in which the opportunity cost is negligible. One can see [9] for the procedure of Hungarian methodology and algorithms for Hungarian methodology see [10].

The management has many objectives for tasks allocation to workers. The multi-objective assignment model considers time, cost, safety, quality, etc. simultaneously one can see [11]. So, we give some detail of the multi-objective assignment problem.

Multi-objective assignment problem

Real-world issues involving multiple objectives pose practical difficulties, such as pricing, time, distance, and so on. Consolidating a set of priorities into a unified value function or overarching objective allows for optimization in specific scenarios. In certain instances, finding a viable method for consolidation may be challenging. Nevertheless, decision-makers (DM) often face a myriad of possibilities, and it falls upon institutional researchers to narrow down the options within the attainable set. This involves leveraging the decision-maker's knowledge of preferences to streamline the available choices.

Expressed in mathematical terms, the problem of multi-objective assignment can be stated as: [12]:

$$Minimize \sum_{i=1}^{n}\sum_{j=1}^{n} V_{ij}C_{ij}$$

$$Minimize \sum_{i=1}^{n}\sum_{j=1}^{n} V_{ij}T_{ij}$$

$$Minimize \sum_{i=1}^{n}\sum_{j=1}^{n} V_{ij}Q_{ij}$$

Where $X_{ij} = \begin{cases} 1 \; if \; the \; i^{th} \; worker \; is \; assigned \; to \; j^{th} \; task \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad otherwise \end{cases}$

$$\sum_{j=1}^{n} v_{ij} = 1 \; ; j = 0,1,2, \cdots, n \; (Only \; one \; person \; is \; assigned \; to \; j^{th} task)$$

$$\sum_{j=1}^{n} v_{ij} = 1 \; ; i = 0,1,2, \cdots, n \; (Only \; one \; task \; is \; done \; by \; i^{th} \; Person)$$

The inconsistency among objectives adds complexity to the resolution of multi-objective problems. Typically, decision-makers assign preference ratings to prioritize objectives, prioritizing, for example, consistency over time sensitivity. Mathematicians grapple with these challenges using various techniques, but encounter difficulties due to inherent flaws in their approaches. For further exploration, refer to [12, 19-23].

Method for compromise solution in multi-objective optimization problems based on distances

A distinctive strategy is proposed for the weighted approach to goal programming, emphasizing the reduction of the distance between ideal objectives and the feasible objective space. This approach leads to the optimal compromise solution for Multiple Objective Linear Programming Problems (MOLPP). The method involves addressing a set of single objective sub-problems, where objectives are converted into constraints. By incorporating weights to denote importance, the resulting compromise solution is enhanced. Additionally, a criterion for selecting the best compromise option is provided. The application of this technique is explored in scenarios involving transportation and assignment issues with diverse and conflicting objectives.

Introduction

The study of choices in MCDM is constrained by a set of constraints known as MOP programming. Goals are sometimes contradicting, making quick goal optimization challenging. MOP involves the exploration of a collection of efficient or Pareto optimal solutions. Consequently, decision-makers (DMs) strive for a compromise option that represents the best choice rather than pursuing the ideal one. Multi-objective decision-making has become a compelling field in recent times. The general formula for a multi-objective program is outlined in [13].

$$Minimize \; F = \{f_1(v), f_2(v), f_3(v), \cdots, f_M(v)$$

Subject to:

$$v \in S, x_{ij} \geq 0.$$

Here, S represents a non-empty and bounded region enclosed in $R^n$and $f_i, i = 1,2, \dots, M$ are functions with real values in R^n. Charnes and Cooper (1961) introduced an objective programming approach, providing Decision-makers (DMs) with a valuable method to simultaneously evaluate multiple objectives (goals) as a compromise, yielding a satisfactory solution. The advantage of the objective programming technique lies in its ability to precisely define the specified value for each criterion. The outcome of the OP strategy is dependent on the weighting technique of the various objectives. There are two common techniques to weighing objectives: setting the order of objectives and using weights on objectives to minimize the weighted total of objective deviations. Because of its variety in displaying and ease in concepts, the OP technique in MOP has acquired significance. It is true that different objectives in OP are of varying priority and urgency, and significant work is being done on this issue. For further research, see [13].

Solving Techniques for MOOP

Regarding MOOP, solution strategies can be categorized into two types: preference-based techniques and ideal procedures. Preference-based solutions prove beneficial when only the desired factors of the goals are known. Ideal techniques address the acquisition of a wide range of solutions as well as the selection of the appropriate knowledge base. [13] describes techniques for solving multi-objective problems. For the MOOP, the following concepts are necessary to understand:

Definition 2. For a MOOP (1), a solution v^* is known as a Pareto optimal solution if and only if there is no $x \in S$ such that $f_i(v) < f_i(v^*), for\ all\ i = 1, 2, \dots, n, for\ at\ least\ one\ i.$

Definition 3. The MOLP's compromise solution is a feasible solution that DM chooses over all other viable alternatives, considering all the parameters included in the multi-objective functions.

Proposed Criteria for Best Compromise Solution

Researchers have put forth methodologies and algorithms to identify optimal compromise alternatives. By employing the notion of the maximum compromise distance and selecting the solution with the minimum ideal compromise distance, one can determine the optimal compromise solution. While ideal objectives may be challenging to attain, they can be conceptualized as points in Euclidean space. The optimal objective is situated in the objective space, outside the feasible region, where conflicting objectives coexist. In the Euclidean space of n dimensions, when the set of ideal solutions is $\{f_1^{Ideal}, f_2^{Ideal}, f_3^{Ideal}, \dots, f_n^{Ideal}\}$, and the collection of desired values for the i^{th} compromise solution is $\{f_1^{*i}, f_2^{*i}, f_3^{*i}, \dots, f_n^{*i}\}$ within the feasible objective space, the compromise ideal distance D^{Ideal} and the optimal compromise ideal distance D^{Ideal} are defined as:

$$D_i^{Ideal} = \sqrt{\left|f_1^{Ideal} - f_1^{*i}\right|^2 + \left|f_2^{Ideal} - f_2^{*i}\right|^2 + \left|f_3^{Ideal} - f_3^{*i}\right|^2 + \dots + |f_n^{Ideal} - f_n^{*i}|^2}$$

$$= \sqrt{\left(f_1^{Ideal} - f_1^{*i}\right)^2 + \left(f_2^{Ideal} - f_2^{*i}\right)^2 + \left(f_3^{Ideal} - f_3^{*i}\right)^2 + \dots + (f_n^{Ideal} - f_n^{*i})^2}$$

$$D^{Ideal*} = Min\{D_i^{Ideal}, i = 1,2,3, \cdots, k$$

The Weighted Sum Method

The transformation of MOOP into SOOP involves assigning weighting coefficients to each criterion. This is achieved by minimizing the weighted sum of objectives, where the weights $w_i, i = 1,2, \dots, M$ correspond to the objective functions, thus satisfying the following conditions.

$$\sum_{i=1}^{N} w_i = 1, \ \ w_i \geq 0, \qquad i = 0,1,2,3, \cdots, N$$

Let

For the j^{th} objective, let $V_k^{(j)^*}$ represent the ideal solution.

Then solutions Z_j^* and the objective values are described as follows [1]:

$$\begin{matrix} Z_1^* & = & v_1^{(1)} & v_2^{(1)} & \cdots & v_K^{(1)} \\ Z_2^* & = & \vdots & \vdots & \vdots & \vdots \\ \vdots & = & \vdots & \vdots & \vdots & \vdots \\ Z_k^* & = & v_1^k & v_2^k & \cdots & v_K^{(K)} \end{matrix}$$

The normalized Single Objective Optimization Problem (SOOP) is obtained through the weighted sum method.,

$$Minimize\ F = \sum_{i=1}^{M} w_i f_i$$

Subject to:

$$v \in S, v_{ij} \geq 0$$

Weights are assigned to objective functions in accordance with the mentioned characteristics. Employing the aforementioned technique, a single solution point is derived for diverse weights that encapsulate the preferences of the decision-maker.

Additive Model

In real-life MOLP difficulties with constraints, certain objectives take precedence over others. According to the specifications, the DM will have a preference hierarchy. When no priorities are stated, the proposed framework delivers the best consensus option. In the proposed framework, decision-makers (DMs) may identify diverse priorities through the weighting assigned to different goals.

$$Minimize\ F = \{f_1(v), f_2(v), f_3(v), \cdots, f_M(v)$$

Subject to:

$$v \in S, v_{ij} \geq 0$$

Expanding the proposed model can be accomplished by initially determining the optimal value for each aim or goal within a specified set of constraints. In this framework, we streamlined the Multiple Objective Linear Programming Problem (MOLPP) into a new single-objective transport problem with the aim of minimizing it.

$$\sum_{i=1}^{M} \left|f_i - f_i^{ideal}\right|(1 - w_i)d$$

The aim is to minimize the weighted sum of deviations from the ideal objective values within the feasible objective space. In this context, 'd' signifies the general deviation variable for all objectives, while 'w_i' denotes the weight assigned to the i^{th} goal. Each objective is converted into a constraint with an upper limit of $f_i^{ideal} + d(1 - w_i)$.

The optimal goal, f_i^{Ideal}, is attained by solving the aforementioned linear transport problem separately for each objective, distinct from the other goals.

The Multiple Objective Problem (MOP) is constrained to the subsequent single-objective problem:

$$Minimize\ F = \sum_{i=1}^{M} \left|f_i - f_i^{ideal}\right|(1 - w_i)d$$

Subject to:

$$f_i \leq f_i^{Ideal} + d(1 - w_i), v \in S, v_{ij} \geq 0$$

Where $w_i \in W = \{w \in R^n | 0 \leq w \leq 1\}$, $f_i^{Ideal}$is the optimum value of i^{th} objective gained as a single objective problem or the ideal value of the i^{th} objective, and d is the general deviational variable. In this function, bigger w_k values will result in smaller $d(1 - w_i)$ values that are required by allocating greater weight, so that the objective value can get close to the ideal objective value by adding upper boundary constraints. Figure 3 signifies the flow chart of the suggested framework:

Step 1: Solve all functions of the M objective as SOLPP, ignoring any other objects related to the constraints.

Step 2: Quantify each M goal and obtain the desired objective value for the ideal solutions. Create the multi-objective optimization model as a single-objective optimization model using the given technique.

Step 3: The suggested algorithm is for finding the best answer.

Step 4: If the decision-maker is satisfied with the answer obtained, the process ends; otherwise, it continues.

Step 5: Request that the decision-maker classify the weights of each objective and resume from Step 3 to Step 5 until the process yields an optimal answer.

The Proposed model's use in multi-objective traveling salesman Problems

Because this approach is recommended for MOLPP, we indicate implementation of the aforesaid technique in MOLTP here. In real-world circumstances, decision-makers frequently encounter numerous and conflicting goals, and such transportation issues are referred to as MPs. In multi-objective transport issues, the item should be transported to n destination from m origin points. Transporting cost, a unit to endpoint j form, source i is represented by C_{ij} , this can be known as a time of delivery, rate of destruction, or delivery protection, and so on. A variable v_{ij} represents the unidentified capacity to be transported to endpoint j from source i. Let their capacities be $a_1, a_2, \dots, a_m$ and$b_1, b_2, \dots, b_n$, correspondingly. The objects are to minimize the time of delivery, rate of destruction, and/or total cost of transportation. Let $f_1, f_2, \dots, f_K$ be K goals to be minimized. The MOLTP may be formulated with these suppositions as below:

$$Minimize\ F_k = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} v_{ij},\ \ k = 1, 2, 3, \cdots, k$$

Subject to:

$$\sum_{i-1}^{n} v_{ij} = a_i,\ \ i = 1,2,3, \cdots, n$$

$$\sum_{j-1}^{m} v_{ij} = b_i,\ \ j = 1, 2, 3, \cdots, m$$

$$v_{ij} \geq 0, i = 1,2,3, \cdots, n\ and\ j = 1,2,3, \cdots, m$$

For each objectiveF_k, $k = 1,2, \dots, K$, solve the above-mentioned linear transport problem independently to formulate the above MOLTP referring to the suggested framework. For optimum solutions calculate the value of each objective function. Let $f_1^{Ideal}, f_2^{Ideal}, f_3^{Ideal}, \cdots f_k^{Ideal}$ be the acquired optimum values of the K objective functions.

For all objectives, let the general deviational variable be"d" and to the k^{th} objective let w_k be the weight assigned. Then the model is formulated as:

$$Minimize\ F = \sum_{j=1}^{M} \left| f_k - f_k^{Ideal} \right| (1 - w_i) d$$

Subject to:

$$f_k \leq f_k^{Ideal} + d(1 - w_k), k = 1,2,3, \cdots K$$

$$\sum_{i-1}^{n} v_{ij} = a_i,\ \ i = 1,2,3, \cdots, n$$

$$\sum_{j-1}^{m} v_{ij} = b_i,\ \ j = 1,2,3, \cdots, m$$

$$v_{ij} \geq 0, i = 1,2,3, \cdots, n\ and\ j = 1,2,3, \cdots, m$$

Numerical Illustration

We would solve various mathematical situations to identify the formulation and solution procedure of the proposed framework. For this, we devised a MOTP and a MOAP. With the aid of the provided method, the findings disclose the ideal compromise option with the perfect distance of minimal compromise.

Example 1:

The following matrix shows a salesman's travel distance to five different cities. Using the given methods, reduce the salesman's overall journey distance.

$$C^1 = \begin{pmatrix} 10 & 8 & 15 \\ 13 & 12 & 13 \\ 8 & 10 & 9 \end{pmatrix}$$

$$\begin{matrix} c^1 & c^2 & c^3 \end{matrix}$$

$$\begin{matrix} C^1 \\ C^2 \\ C^3 \end{matrix} \begin{pmatrix} 10 & 8 & 15 \\ 13 & 12 & 13 \\ 8 & 10 & 9 \end{pmatrix}$$

$$\begin{matrix} \text{Sums} & 31 & 30 & 37 \end{matrix}$$

$$\begin{matrix} K_j & 1.03 & 1 & 1.23 \end{matrix}$$

The net cost matrix become

$$\begin{pmatrix} 10 & 8 & 15 \\ 13 & 12 & 13 \\ 8 & 10 & 9 \end{pmatrix}$$

To find the minimum row entries

$$\begin{pmatrix} 10 & 8 & 15 \\ 13 & 12 & 13 \\ 8 & 10 & 9 \end{pmatrix}$$

As a result, we have,

$$\left(\begin{array}{ccc|c} 10 & 8 & 15 & 8 \\ 13 & 12 & 13 & 12 \\ 8 & 10 & 9 & 8 \end{array}\right)$$

To find Minimum column entries

$$\begin{pmatrix} 2 & 0 & 7 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

$$\begin{matrix} \text{Min} & 0 & 0 & 1 \end{matrix}$$

To subtract the minimum value from C^3 As a result, we have,

$$\begin{pmatrix} 2 & 0 & 6 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{pmatrix}$$

$$\text{Min} \quad 0 \quad 0 \quad 0$$

$$\begin{pmatrix} 2 & \textcircled{0} & 6 \\ 1 & \textcircled{0} & 0 \\ \textcircled{0} & 2 & 0 \end{pmatrix}$$

$$N = n$$

As result, we can state that

The salesman traveled from

$$C_1 \ to \ C_2, C_2 \ to \ C_3, C_3 \ to \ C_1$$

$$=C_1 \ C_2 \ C_3 \ C_1$$

$$= 8 + 13 + 8$$

$$= 29$$

Example 2:

A sales representative is tasked with delivering products to five distinct cities. The goal is to minimize the overall travel distance using the proposed methodology.

$$C^1 = \begin{pmatrix} 13 & 15 & 8 \\ 10 & 20 & 12 \\ 15 & 10 & 12 \end{pmatrix}$$

$$\begin{matrix} & c^1 & c^2 & c^3 \\ C^1 & 13 & 15 & 8 \\ C^2 & 10 & 20 & 12 \\ C^3 & 15 & 10 & 12 \end{matrix}$$

$$Sums \quad 38 \quad 45 \quad 32$$

$$K_j \ 1.18 \quad 1.4 \quad 1$$

The net cost matrix become

$$\begin{pmatrix} 13 & 15 & 8 \\ 10 & 20 & 12 \\ 15 & 10 & 12 \end{pmatrix}$$

To find the minimum row entries

$$\begin{pmatrix} 13 & 15 & 8 \\ 10 & 20 & 12 \\ 15 & 10 & 12 \end{pmatrix}$$

As a result, we have

$$\left(\begin{array}{ccc|c} 13 & 15 & 8 & 8 \\ 10 & 20 & 12 & 10 \\ 15 & 10 & 12 & 10 \end{array}\right)$$

To find Minimum column entries

$$\begin{pmatrix} 5 & 7 & 0 \\ 0 & 10 & 2 \\ 5 & 0 & 2 \end{pmatrix}$$

$$Min \quad 0 \quad 0 \quad 0$$

$$\begin{pmatrix} 5 & 7 & 0 \\ 0 & 10 & 2 \\ 5 & 0 & 2 \end{pmatrix}$$

$$N = n$$

$$\begin{pmatrix} 5 & 7 & \textcircled{0} \\ \textcircled{0} & 10 & 2 \\ 5 & \textcircled{0} & 2 \end{pmatrix}$$

$$C_1 \ to \ C_3 \quad C_3 \ to \ C_2 \quad C_2 \ to \ C_1$$

$$= C_1 \ C_2 \ C_3$$

$$= 8 + 10 + 10$$

$$= 28$$

Example 3:

Examine the three-objective assignment problem, characterized by three cost matrices.

$$C_1 = \begin{pmatrix} 2 & 5 & 4 & 7 \\ 3 & 3 & 5 & 7 \\ 3 & 8 & 4 & 2 \\ 6 & 5 & 2 & 5 \end{pmatrix}$$

$$C_2 = \begin{pmatrix} 3 & 3 & 6 & 2 \\ 5 & 3 & 7 & 2 \\ 5 & 2 & 7 & 4 \\ 4 & 6 & 3 & 5 \end{pmatrix}$$

$$C_3 = \begin{pmatrix} 4 & 2 & 5 & 3 \\ 5 & 3 & 4 & 3 \\ 4 & 3 & 5 & 2 \\ 6 & 4 & 7 & 3 \end{pmatrix}$$

Now we can find $C = C_1 + C_2 + C_3$

$$\begin{pmatrix} 9 & 10 & 15 & 12 \\ 13 & 9 & 16 & 13 \\ 12 & 13 & 16 & 8 \\ 16 & 18 & 14 & 10 \end{pmatrix}$$

Applying the assignment method proposed earlier to the single-objective assignment problem with the cost matrix C, the problem is solved, and the optimized point is obtained.

$$v = \begin{cases} v_{11} = v_{22} = v_{34} = v_{43} = 0 \\ v_{ij}, \ \ is\, nonzero \ \ otherwise \end{cases}$$

The optimal points for the given objective function are Z(v*) = (9, 13, 16). Based on the findings and demonstrations, v* is identified as the super-efficient point, and Z(v*) stands as the best non-dominated solution for the tri-objective assignment problem, as straightforwardly indicated by reference [25].

The first objective is minimized as

$$C_1 = \begin{pmatrix} 2 & 5 & 4 & 7 \\ 3 & 3 & 5 & 7 \\ 3 & 8 & 4 & 2 \\ 6 & 5 & 2 & 5 \end{pmatrix}$$

$$Sum\text{:} \ \ 14 \ 21 \ 15 \ 21$$

$$k_j \quad 1 \ \ 1.5 \ 1.07 \ 1.5$$

From step 3 we have, i.e., the minimum row elements are given by;

Min

$$\left(\begin{array}{cccc|c} 2 & 5 & 4 & 7 & 2 \\ 3 & 3 & 5 & 7 & 3 \\ 3 & 8 & 4 & 2 & 2 \\ 6 & 5 & 2 & 5 & 2 \end{array}\right)$$

$$\begin{pmatrix} 0 & 3 & 2 & 5 \\ 0 & 0 & 2 & 5 \\ 1 & 6 & 2 & 0 \\ 2 & 3 & 0 & 3 \end{pmatrix}$$

Following step 4, i.e., the minimum column elements are provided by:

$$\begin{pmatrix} 0 & 3 & 2 & 5 \\ 0 & 0 & 2 & 5 \\ 1 & 6 & 2 & 0 \\ 2 & 3 & 0 & 3 \end{pmatrix}$$

Min: 0 0 0 0

$$\begin{pmatrix} 0 & 3 & 2 & 5 \\ 0 & 0 & 2 & 5 \\ 1 & 6 & 2 & 0 \\ 2 & 3 & 0 & 3 \end{pmatrix}$$

$$\boldsymbol{N = n}$$

In this context, N signifies the total count of lines that incorporate at least one zero either in a row or a column, whereas n denotes the order of the distance matrix.

Now we obtain our goal.

$$\begin{pmatrix} 0 & 3 & 2 & 5 \\ 0 & 0 & 2 & 5 \\ 1 & 6 & 2 & 0 \\ 2 & 3 & 0 & 3 \end{pmatrix}$$

The second objective is minimized as

$$C_2 = \begin{pmatrix} 3 & 3 & 6 & 2 \\ 5 & 3 & 7 & 2 \\ 5 & 2 & 7 & 4 \\ 4 & 6 & 3 & 5 \end{pmatrix}$$

$$Sum: \; 17 \quad 14 \quad 23 \quad 13$$

$$k_j \qquad 1.3 \;\; 1.07 \;\; 1.7 \quad 1$$

From step 3 we have, i.e. the minimum row elements are given by:

$$\begin{pmatrix} 3 & 3 & 6 & 2 & & 2 \\ 5 & 3 & 7 & 2 & & 2 \\ 5 & 2 & 7 & 4 & & 2 \\ 4 & 6 & 3 & 5 & & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 4 & 0 \\ 2 & 1 & 5 & 0 \\ 3 & 0 & 5 & 2 \\ 1 & 3 & 0 & 2 \end{pmatrix}$$

From the completion of step 4, the minimum column elements are derived as follows:

$$\begin{pmatrix} 1 & 1 & 4 & 0 \\ 2 & 1 & 5 & 0 \\ 3 & 0 & 5 & 2 \\ 1 & 3 & 0 & 2 \end{pmatrix}$$

Min: 1 0 0 0

$$\begin{pmatrix} 1 & 1 & 4 & 0 \\ 2 & 1 & 5 & 0 \\ 3 & 0 & 5 & 2 \\ 1 & 3 & 0 & 2 \end{pmatrix}$$

$$\boldsymbol{N = n}$$

In this context, N denotes the number of lines containing at least one zero in either a row or a column, and n represents the order of the distance matrix. Having established this, we have successfully reached our goal.

$$\begin{pmatrix} 1 & 1 & 4 & 0 \\ 2 & 1 & 5 & 0 \\ 3 & 0 & 5 & 2 \\ 1 & 3 & 0 & 2 \end{pmatrix}$$

The third objective is minimized as

$$\begin{pmatrix} 4 & 2 & 5 & 3 \\ 5 & 3 & 4 & 3 \\ 4 & 3 & 5 & 2 \\ 6 & 4 & 7 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 2 & 5 & 3 \\ 5 & 3 & 4 & 3 \\ 4 & 3 & 5 & 2 \\ 6 & 4 & 7 & 3 \end{pmatrix}$$

$$Sum: \quad 19 \quad 10 \quad 21 \quad 11$$

$$k_j \quad 1.9 \quad 1 \quad 2.1 \quad 1.1$$

Subtracting 1 from each element in column 3 is performed to reduce each sum smaller than 2. The resulting new distance matrix is as follows:

$$\begin{pmatrix} 4 & 2 & 4 & 3 \\ 5 & 3 & 3 & 3 \\ 4 & 3 & 4 & 2 \\ 6 & 4 & 6 & 3 \end{pmatrix}$$

$$Sum: 19 \quad 10 \quad 17 \quad 11$$

$$k_j \quad 1.9 \quad 1 \quad 1.7 \quad 1.1$$

And the net cost matrix is as follows:

$$\begin{pmatrix} 4 & 2 & 4 & 3 \\ 5 & 3 & 3 & 3 \\ 4 & 3 & 4 & 2 \\ 6 & 4 & 6 & 3 \end{pmatrix}$$

From step 3 we have, i.e. the minimum row elements are given by:

Min

$$\left(\begin{array}{cccc|c} 4 & 2 & 4 & 3 & 2 \\ 5 & 3 & 3 & 3 & 3 \\ 4 & 3 & 4 & 2 & 2 \\ 6 & 4 & 6 & 3 & 3 \end{array}\right)$$

$$\begin{pmatrix} 2 & 0 & 2 & 1 \\ 2 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 \\ 3 & 1 & 3 & 0 \end{pmatrix}$$

From step 4 we have, i.e. the minimum column elements are given by:

$$\begin{pmatrix} 2 & 0 & 2 & 1 \\ 2 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 \\ 3 & 1 & 3 & 0 \end{pmatrix}$$

Min: 2 0 0 0

$$\begin{pmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 1 & 3 & 0 \end{pmatrix}$$

$$\boldsymbol{N = n}$$

Here, N denotes the count of lines containing at least one zero either in a row or a column, and n represents the order of the distance matrix.

Now we obtain our goal.

$$\begin{pmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 1 & 3 & 0 \end{pmatrix}$$

$$f_1 = 2v_{11} + 5v_{12} + 4v_{13} + 7v_{14} + 3v_{21} + 3v_{22} + 5v_{23} + 7v_{24} + 3v_{31} + 8v_{32} + 4v_{33} + 2v_{34} + 6v_{41} + 5v_{42} + 2v_{43} + 5v_{44}$$

$$f_2 = 3v_{11} + 3v_{12} + 6v_{13} + 2v_{14} + 5v_{21} + 3v_{22} + 7v_{23} + 2v_{24} + 5v_{31} + 2v_{32} + 7v_{33} + 4v_{34} + 4v_{41} + 6v_{42} + 3v_{43} + 5v_{44}$$

$$f_3 = 4v_{11} + 2v_{12} + 5v_{13} + 3v_{14} + 5v_{21} + 3v_{22} + 4v_{23} + 3v_{24} + 4v_{31} + 3v_{32} + 5v_{33} + 2v_{34} + 6v_{41} + 4v_{42} + 7v_{43} + 3v_{44}$$

Subject to:

$$\sum_{i=1}^{4} v_{ij} = 1, \;\; j = 1,2,3,4$$

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$$\sum_{i=1}^{4} v_{ij} = 1, \;\; j = 1,2,3,4$$

$$v_{ij} = 0 \; or \; 1, \quad i = 1,2,3,4 \;\; and \; j = 1,2,3,4$$

Determine the ideal objective value by solving each objective individually, resulting in the following:

$$v_{12} = 1, \quad v_{23} = 1, \quad v_{13} = 1, \quad v_{21} = 1, \quad v_{32} = 1,$$

$$f_1^{Ideal} = 9, \quad f_2^{Ideal} = 13, \quad f_3^{Ideal} = 16$$

Formulating this can be achieved as a single-objective optimization problem utilizing the proposed algorithm, as outlined below:

Minimize:

$$F = (f_1 - 9)(1 - w_1)d + (f_2 - 13)(1 - w_2)d + (f_3 - 16)(1 - w_3)d$$

Subject to:

$$2v_{11} + 5v_{12} + 4v_{13} + 7v_{14} + 3v_{21} + 3v_{22} + 5v_{23} + 7v_{24} + 3v_{31} + 8v_{32} + 4v_{33} + 2v_{34} + 6v_{41} + 5v_{42} + 2v_{43} + 5v_{44} \leq 9 + (1 - w_1)d$$

$$3v_{11} + 3v_{12} + 6v_{13} + 2v_{14} + 5v_{21} + 3v_{22} + 7v_{23} + 2v_{24} + 5v_{31} + 2v_{32} + 7v_{33} + 4v_{34} + 4v_{41} + 6v_{42} + 3v_{43} + 5v_{44} \leq 13 + (1 - w_2)d$$

$$4v_{11} + 2v_{12} + 5v_{13} + 3v_{14} + 5v_{21} + 3v_{22} + 4v_{23} + 3v_{24} + 4v_{31} + 3v_{32} + 5v_{33} + 2v_{34} + 6v_{41} + 4v_{42} + 7v_{43} + 3v_{44} \leq 16 + (1 - w_3)d$$

$$\sum_{i=1}^{4} v_{ij} = 1, \;\; j = 1,2,3,4$$

$$\sum_{i=1}^{4} v_{ij} = 1, \;\; j = 1,2,3,4$$

$$\sum_{i=1}^{4} v_{ij} = 1, \;\; j = 1,2,3,4$$

$$v_{ij} = 0 \; or \; 1, \quad i = 1,2,3,4 \;\; and \; j = 1,2,3,4$$

Table 1. Compromise objective values corresponding to priorities

	Weights assigned	Z_1, Z_2, Z_3	D^{ideal}	X_{ij}
1	$w_1 = 0.0, w_2 = 0.0, w_3 = 1.0$	9, 13, 16	0	$v_{11} = v_{22} = v_{34} = v_{43} = 1$
2	$w_1 = 0.0, w_2 = 1.0, w_3 = 0.0$	18, 20, 13	11.789	$v_{12} = v_{23} = v_{31} = v_{44} = 1$
3	$w_1 = 1.0, w_2 = 0.0, w_3 = 0.0$	15, 17, 16	7.211	$v_{34} = v_{13} = v_{22} = v_{41} = 1$
4	$w_1 = 0.3, w_2 = 0.3, w_3 = 0.3$	11, 14, 19	3.741	$v_{43} = v_{33} = v_{22} = v_{11} = 1$
5	$w_1 = 0.1, w_2 = 0.3, w_3 = 0.6$	18, 20, 13	11.789	$v_{12} = v_{23} = v_{31} = v_{44} = 1$
6	**Without preference**	11,14,19	3.741	$v_{43} = v_{33} = v_{22} = v_{11} = 1$

Conclusion

In this study, a new approach to the weighted sum method is outlined, providing an optimal aggregate solution while the DM lacks information about the relevance of associated objectives. The methodology involves handling the ideal objective through an enhanced Hungarian algorithm, while the compromised solution is computed using the weighted sum technique. Explicitly specifying the relevance of each objective through assigned weights allows the proposed approach to generate compromise solutions in the absence of predefined priorities for each aim. Moreover, when the priority of each aim is unspecified, the compromise distance approach is employed to derive the optimal aggregate solution. This methodology offers an advantage over previous methods by delivering the optimal compromise option even when individual aims are considered unimportant. Importantly, it doesn't rely on simulation tools like LINGO. The proposed approach is versatile enough to handle large-scale transportation and assignment challenges.

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