Modeling and Optimization of Bus Bunching in Rapid Transit Route using Petri Nets and Max_Plus Algebra

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Abstract - In developing cities, Bus Rapid Transit (BRT) has rapidly become an attractive alternative in the urban transport system owing to its cost effectiveness, environmentally friendly, flexible implementation, and traffic congestion reduction and control. For these reasons, many public transport providers, such as Trans-Peshawar in Pakistan operate BRT routes with high frequency to ensure high urban mobility in Peshawar city. One of the serious problems pertaining to the high-frequency BRT routes called Bus Bunching, which arises in a situation when two or more buses of the same route arrive at the same station at the same time at a bus stop. In the proposed study, a Petri Net based modeling of BRT Peshawar corridor has been carried out for optimal future stations arrival timing with the help of Max-Plus Algebra and optimal number of buses on each route using Integrated Net Analyzer (INA) and Linear, Integer, Non-linear and Global Optimization (LINGO) software. This technique, Petri nets and Max_Plus Algebra are combined as a solution to the proposed situation of BRT from Bus bunching and to improve the service quality.

Keywords: Bus Bunching, Petri Nets, Max-Plus Algebra, Bus Rapid Transit (BRT), LINGO, INA

I. INTRODUCTION

Nowadays numerous cities are facing the problem of constantly growing traffic density and congestion, which causes delays and disturbs transportation services. In addition, the growing population implying the number of private vehicles and bus/wagon public transport service are now causing many roads to be too narrow to accommodate those vehicles and increases unnecessary travel time. To overcome this problem the city residents must be motivated to use public transportation instead of private modes of transportation. Many cities are unable or have no conditions to build public rail transportation, the only option remaining is Bus Rapid Transient (BRT) which has many advantages of substantial passengers' capacity, high quality transient service, environmental friendliness, reduce queue delays and can bring traffic congestion reduction, which common transportation system cannot do [1]. Because of this, many public transportation companies, like in Pakistan Trans-Peshawar, have BRT routes that run often so that people can get around Peshawar quickly. With high-frequency BRT routes comes a fundamental problem called "bus bunching", which occurs when two or more buses on the same route arrive at a bus stop at the same time. When a bus arrives late from the scheduled time and the previous bus has departed from the same stop on scheduled time then there will be more passengers waiting at the stop, so the boarding and disembarking of passengers will take more time than the scheduled dwell times. The increase in the dwell times is a cascade of delay up and down the BRT route.

Bus bunching is one of the serious problems in BRT and it has not only negative effects on passengers but also on public transport services providers. It leads to longer waiting time for passengers who wait on the bus stops and longer travel time for passengers who are inside the bus due to longer boarding/disembarking and dwell times on the stops. When the waiting times of passengers increases, this instigates additional serviceable cost for transient provider to restore reliable service by providing more vehicles and drivers.

Transportation networks can be put in the circle of Discrete Event Dynamic System (DEDS) due to its behavior, characterized by synchronization, concurrency, parallelism, and conflicts. For the purpose of solving and analyzing these complex systems, many methods and techniques have been developed. On one hand, Petri Net (PN) has been a proven tool because of its conceptual simplicity, intuitive graphical representation, and a powerful modeling formalism for various kinds of DEDS. A Petri net consists of a set of places, transitions, and arcs. Places are represented as circles, transitions as rectangles, and arcs as directed lines that connect places to transitions and transitions to places. The tokens are represented by small black dots that are placed inside the places. A Petri net models the system behavior as a state transition diagram. At any given time, the system is in a particular state, which is represented by the distribution of tokens among the places. The transitions represent the actions that can be taken by the system, and they can be enabled or disabled depending on the state of the system. On the other hand, Max-plus algebra is one of the most determined mathematical formalisms for analyzing certain type DEDS. It is a mathematical framework that operates on real numbers, where the usual addition and multiplication operations are replaced by the maximum and addition operations, respectively. In other words, given two real numbers "a" and "b", the max-plus sum (denoted by $\oplus$) and max-plus product (denoted by $\otimes$) are defined as follows:

$$a \oplus b = \max(a, b) \ldots\ldots\ldots\ldots (i)$$

$$a \otimes b = a + b \ldots\ldots\ldots\ldots (ii)$$

Max-plus algebra is useful in a wide range of applications, including optimization, control theory, scheduling, and telecommunications. It has some interesting properties, such as being commutative, associative, and distributive over the max operation. There is a little literature available where these tools are combined to solve the bunching phenomenon of a BRT network.

II. LITERATURE REVIEW

For consumers, fast and dependable transportation service is necessary; additionally, travel time consistently ranks first in transportation services survey along with consumers waiting time [2][3][4]. In this manner, both travel and waiting times are imperative factors from the service provider perspective. Boyle and Hollander study found that reduced travel and waiting times attract and maintain passengers [5]. Besides, passengers also give preference to the fast transit service [6].

There are many factors which influence the travel time of BRT network, this is the time taken by bus from departure to arrival stations. Passenger's activities like boarding and disembarking, time of the day and unpredictable circumstances due to accident and force majeure can impact the travel time [7][8]. Transit services provider endeavor to control some variables to optimize travel time. Dwell time is also an imperative donor to travel time, defined as the amount of time spent at bus stops to serve passengers. Barr et al focused-on transit times and conclude that between 10-30% of dwell time anywhere contributes to travel time [9]. Knowing the factors underlying dwell time can help transit services provider to minimize dwells and raise up transit operations [7].

While there has been extensive research on dwell and travel times, bus bunching and its operational impacts have been relatively overlooked. This phenomenon was come in consideration of Newell & Potts studies in 1964 by proving the instability of a bus route which causes bus bunching [10]. Bunching happens when planned headways between buses are hindered, resulting in wasted capacity for service operators as well as longer wait times for passengers due to overcrowded lead buses.

[11][12][13] generated theoretical holding techniques using mathematical approaches to eliminate bus bunching. [14] Investigated some possibilities to reduce bus bunching with the help of petri nets and max-plus algebra. Petri nets was invented in 1962 by Dr. Carl Adam Petri in his doctoral thesis titled as "Kommunikation mit automaten", later it became widely modeling graphical tool used for describing distributed system [15]. Dynamical behavior of systems can be modeled by petri nets, which combines a well-defined mathematical theory with a graphical representation. The mathematical theory of petri nets permits exact modeling and analysis of the system behavior, whereas the graphical representation of petri nets empowers visualization of the change of states of the modeled system. The enormous success of petri net can be attributed to this combination. Petri nets have been used to model several kinds of DEDS like manufacturing shops [16], control systems and command [17], computer networks [18], communication system [19], real time computing systems and workflow [20]. The wide range of applications is followed by a wide range of different facets which have been considered in petri nets research.

In the beginning of 1960s, An English professor described and analyzed industrial processes with the help of Max plus algebra [21]. Cohen et al [22] studied linear theory of DEDS using max plus algebra and then transformed a class of deterministic DEDS to a linear system.

Many past researchers suggested different possible techniques to eliminate bus bunching which are presented in the literature. But in this study modeling bus bunching addressed in a unique way, which are future stopping times prediction for all stations of the route and the optimal number of buses on each route based on the real data of Peshawar BRT, which, to the best of our knowledge, is the first effort in this domain.

III. METHODOLOGY

To model bus bunching in BRT Peshawar, a detailed methodology is proposed here. This methodology is composed of some major steps which are given below:

A. Build petri net model

The graphical petri net model or timed event graph is being developed with the help of gathered data, the model means transforming a public transportation into a logical topology. To simulate the service of a public transit route using petri nets the platforms at stations are typically named and represented by transitions. The firing of transition represents bus leaving from the station, tokens at a place represents buses and the moment of token from place to place represents bus traveling or dwelling at a station, and the token holding time at place is the bus travel time.

B. Future stations times using max-plus algebra

Timed Petri nets or Time event graphs can be examined using max-plus algebra to produce the firing times of transitions using recurrence equation. A comprehensive study of these equations can be found in [23] .

A given timed event graph having maximum initial marking with respect to all places, the transitions size $|Q|\times|Q|$ matrices A_0, A_1 .....A_M are built as follows. Regarding transitions q_j and qi, the matrix $[A_m]_{ij}$ is assumed to be the maximum holding times for places between transitions q_j and q_i with *m* initially. That is, if there are N places in timed event graph p_1, p_1, ………p_n , every place with m token initially and holding times between transitions q_j and q_i then;

$$[A_m]_{ij} = \begin{cases} \max\{t_1, t_2 \dots \dots t_n\} & if\ M_0(p_{qjqi}) = m \\ \in & otherwise \end{cases} \quad \dots (iii)$$

To generate a Matrix

$$A_0^* = \sum_{i=0}^{|Q|-1} A_0^i \qquad \dots\dots\dots\dots\dots \qquad (iv)$$

Define new state vector,

$$\tilde{x}(k) = \begin{pmatrix} x^T(k) \\ x^T(k-1) \\ \vdots \\ x^T(k-M+1) \end{pmatrix} \qquad \dots\dots\dots. \qquad (v)$$

And

The matrix $(|Q| \times M) \times (|Q| \times M)$;

$$\tilde{A} = \begin{pmatrix} A_0^* \otimes A_1 & A_0^* \otimes A_2 & \cdots & A_0^* \otimes A_{M-1} & A_0^* \otimes A_M \\ E & \varepsilon & \vdots & \varepsilon & \varepsilon \\ \varepsilon & E & \ddots & \varepsilon & \varepsilon \\ \vdots & \vdots & & \vdots & \vdots \\ \varepsilon & \varepsilon & \cdots & E & \varepsilon \end{pmatrix} \ldots (vi)$$

With this, the standard autonomous equation, we can now anticipate the behavior of the timed event graph;

$$\tilde{x}(k) = \tilde{A} \otimes \tilde{x}(k-1) \qquad \ldots \ldots \qquad (vii)$$

C. Analysis through Integrated Net Analyzer (INA), Linear Programing (MS Excel) & LINGO for Optimized number of buses on each route

After constructing the PN model, the next step is to find all possible basic circuits, which are called invariants. For small systems, the invariants can be found manually by selecting circuits from the Petri net diagram, but for complex systems this process is exceedingly difficult to do manually. For this purpose, the INA program is used to find invariants from the Petri net diagram. A notepad file is created as an input file for the INA program. The known invariants are then imported to Excel spreadsheet which is then programmed in such a way that it can automatically determine BRT performance parameters. For the linear programming lingo software will be used; the input data to the lingo program will be the constraints from the excel sheet and to get the desired parameters based on those constraints the lingo will give us the optimal values.

IV. RESULTS AND DISCUSSION

For the model construction, the data of BRT Peshawar were obtained from the *Transpeshawar* office. The gathered data primarily consists of travel time for each route, total number of buses assigned to each route, total number of stations and stops, proposed travel speed in km/h and length of each route in Km. The average travel time for each route of BRT was obtained from the calculations of five surveys. As the return trip of each route is identical so we have selected one way trip from *Chamkani* to *Karkhano* for modeling and analysis. The Petri net model of BRT Peshawar is shown in the following Figure 1.

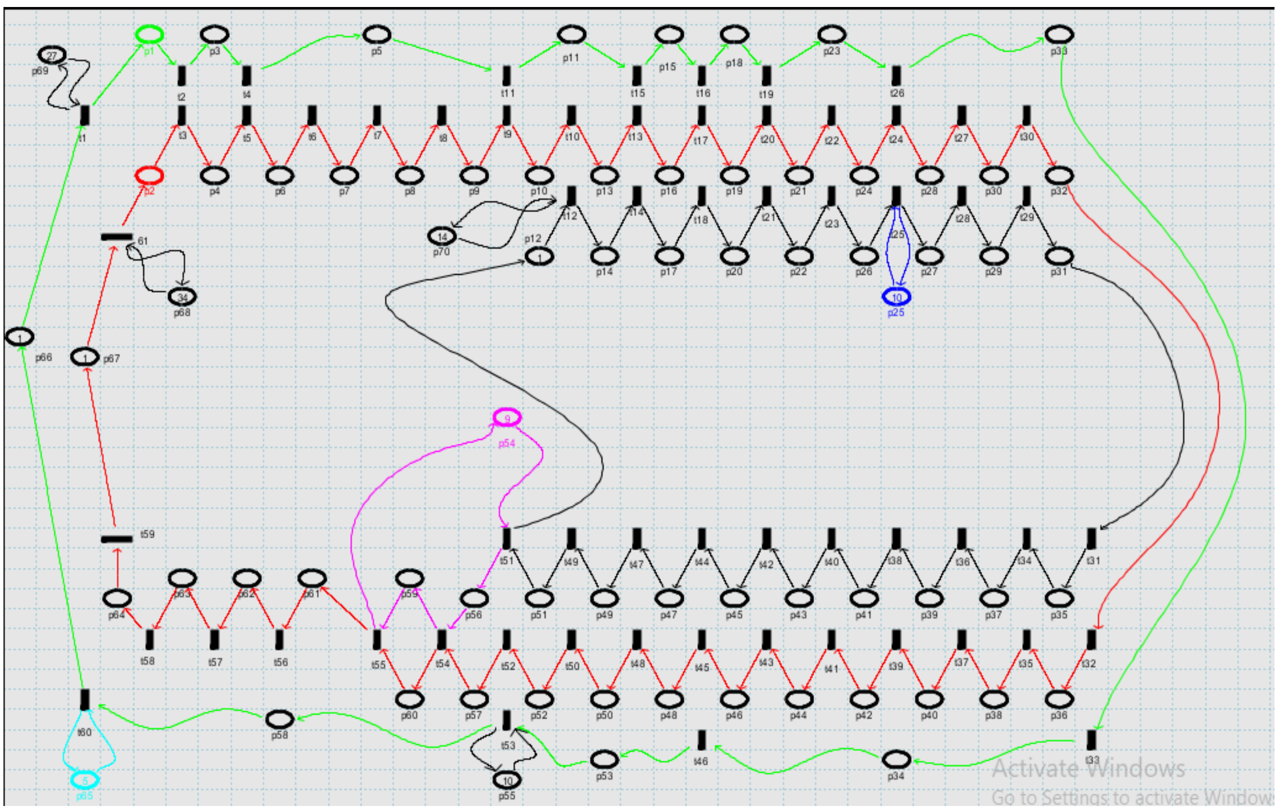


Fig.1 Petri Net graphical model of BRT Peshawar, Pakistan

Future stations' times for each route have been derived from Max-plus algebra, the gathered data used to build the timed petri net and then converted into Max-plus algebra. For the concreate analysis of the transportation network using max-plus algebra the hand calculations are cumbersome even for small problems having a little complexity, fact that fully motivates this work in using a computer aided software to calculate parameters. A software tool of this kind has been designed and implemented as an instrument under the name of the Max-plus algebra Toolbox for MATLAB. This was done to take advantage of the knowledge and experience we gained through using MATLAB's capabilities within the context of discrete event systems.

The method describes in section *C* have been implemented in Max-Plus algebra tool for MATLAB M. file editor. The MATLAB editor contains the system matrices and initial conditions for a BRT Peshawar.

The optimized timing for each route consists travel time and dwell time, for example for stopping route SR-02 the BRT Bus must reach from Chamkani to sardar Garhi (bus stations) in 3 minutes and 41 seconds, and from the optimal timing tables it can be seen that the service route SR-02, SR-08 and express route ER-01 should take 52:25, 27:52 and 41:54 minutes respectively to complete one trip.

Table 1: Stopping Route SR-02 optimal station arrival timings

S.No	Departure Stop	Stopping Route- SR-02 Arrival Stop (station)	Optimized Time (mm:ss)
1.	Chamkani	Sardar Garhi	**03:41**
2.	Sardar garhi	Chughal Pura	**05:49**
3.	Chughal Pura	Faisal Colony	**07:28**
4.	Faisal Colony	Old Haji camp	**09:34**
5.	Old Haji camp	Lahore adda	**10:47**
6.	Lahore adda	Gulbahar Chawak	**12:02**
7.	Gulbahar Chawak	Hashtnagri	**13:17**
8.	Hashtnagri	Malik saad Shaheed	**14:54**
9.	Malik saad Shaheed	Khyber Bazaar	**16:09**
10.	Khyber Bazaar	Shoba Bazar	**17:56**
11.	Shoba Bazar	Dabgari Gardens	**19.43**
12.	Dabgari Gardens	Railway Station	**20:48**
13.	Railway Station	FC Chowak	**22:32**
14.	FC Chowak	Saddar Bazar	**23:42**
15.	Saddar Bazar	Mall Road	**25:05**
16.	Mall Road	Tehkal Payyan	**26:13**
17.	Tehkal Payyan	Tehkal Bala	**30:19**
18.	Tehkal Bala	Abdara Road	**32:01**
19.	Abdara Road	University Town	**33:30**
20.	University Town	University of Peshawar	**34:30**
21.	University of Peshawar	Islamia College	**35:52**
22.	Islamia College	Board Bazar	**37:18**
23.	Board Bazar	Mall of Hayatabad	**38:31**
24.	Mall of Hayatabad	Bab-e-Peshawar	**40:44**
25.	Bab-e-Peshawar	Hayatabad Phase 3	**42:22**
26.	Hayatabad Phase 3	Tatara Park	**43:45**
27.	Tatara Park	PDA Hayatabad	**46:09**
28.	PDA Hayatabad	Hospital Chowk	**47:02**
29.	Hospital Chowk	Karkhano	**51:35**
30.	Karkhano	Deport	**52:25**

Table 2: Stopping Route SR-08 optimal station arrival timings

S.No	Departure Stop	Stopping Route- SR-08 Arrival Stop	Optimized Time (mm:ss)
1.	Gulbahar Chawak	Hashtnagri	**02:05**
2.	Hashtnagri	Malik saad Shaheed	**03:53**
3.	Malik saad Shaheed	Khyber Bazaar	**04:55**
4.	Khyber Bazaar	Shoba Bazar	**06:01**
5.	Shoba Bazar	Dabgari Gardens	**07:07**
6.	Dabgari Gardens	Railway Station	**08:50**
7.	Railway Station	FC Chowak	**10:00**
8.	FC Chowak	Saddar Bazar	**11:19**

9.	Saddar Bazar	Mall Road	**12:27**
10.	Mall Road	Tehkal Payyan	**16:29**
11.	Tehkal Payyan	Tehkal Bala	**18:10**
12.	Tehkal Bala	Abdara Road	**19:37**
13.	Abdara Road	University Town	**20:55**
14.	University Town	University of Peshawar	**22:16**
15.	University of Peshawar	Islamia College	**23:38**
16.	Islamia College	Board Bazar	**24:49**
17.	Board Bazar	Mall of Hayatabad	**27:02**
18.	Mall of Hayatabad	Deport	**27:52**

Table 3: Express Route ER-01 optimal station arrival timings

S.No	**Departure Stop**	**Express Route- ER-01 Arrival Stop**	**Optimized Time (mm:ss)**
1.	Chamkani	Sardar Garhi	**02:39**
2.	Sardar garhi	Lahore adda	**05:05**
3.	Lahore adda	Hashtnagri	**10:19**
4.	Hashtnagri	Malik saad Shaheed	**13:17**
5.	Malik saad Shaheed	Khyber Bazaar	**14:32**
6.	Khyber Bazaar	Dabgari Gardens	**16:22**
7.	Dabgari Gardens	Saddar Bazar	**18:14**
8.	Saddar Bazar	University of Peshawar	**31:55**
9.	University of Peshawar	Mall of Hayatabad	**36.00**
10.	Mall of Hayatabad	Karkhano	**41:04**
11.	Karkhano	Deport	**41:54**

In a Petri net model, an elementary circuit is a directed path that begins at one node, place, or transition and ends back at the same node, place, or transition without repeating any other nodes; and always moving in the direction of the arcs. In the case of small systems, it is simple to locate the invariants manually by selecting the possible circuits from the petri net diagram. On the other hand, finding the invariants manually for complex or large systems that contain many variables is an extremely challenging task. Integrated Net analyzer (INA) is one of the freely alphanumeric petri net executers that can resolve such problems. From the petri net diagram, a notepad file is being prepared to feed into the INA software. This will allow the software to determine the total number of invariants.

The output file that is being produced by the INA provides information regarding the total number of invariants as well as all the places that are contained within each invariant. The input PN model to INA has 12 elementary circuits in this case can be counted from the first column. The places of the elementary circuits are given by the matrix spanned by the 12 rows and 70 columns containing only 0's and 1's. Each row of the matrix corresponds to elementary circuits. A "1" in the matrix indicates that the corresponding place is the included while "0" denotes that the corresponding place is not the part of elementary circuit.

To optimize the number of buses on each route of the BRT corridor, linear programming techniques were used. To achieve the said goal the output results from the INA were imported to MS Excel spreadsheet. The Excel spreadsheet is then programmed in such a way that it can automatically determine the parameters for optimization software called Linear, Integer, Non-linear and Global Optimization (LINGO). These parameters were used as input conditions to LINGO which is shown in Figure 2. These parameters include objective function, constraints, and initial conditions.

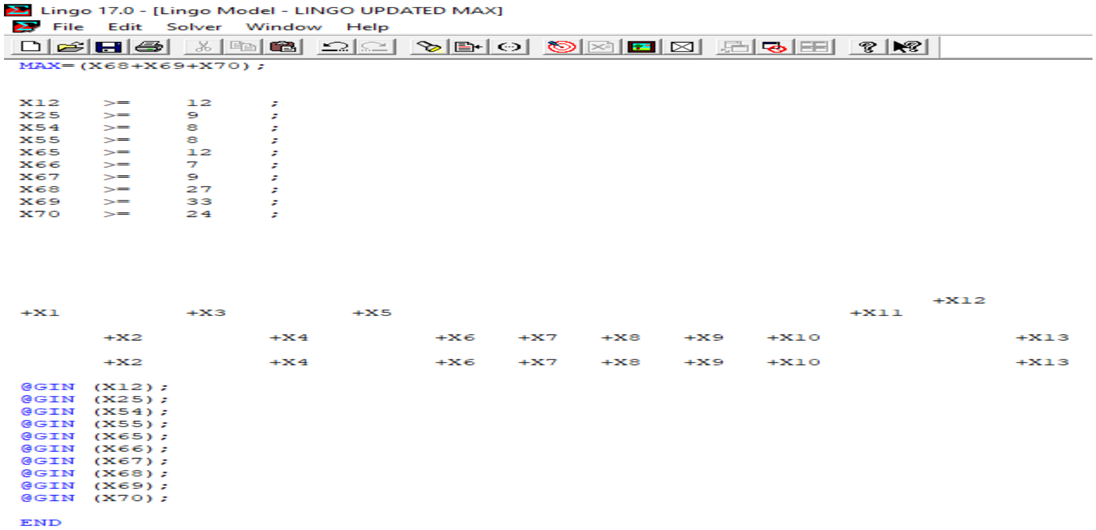
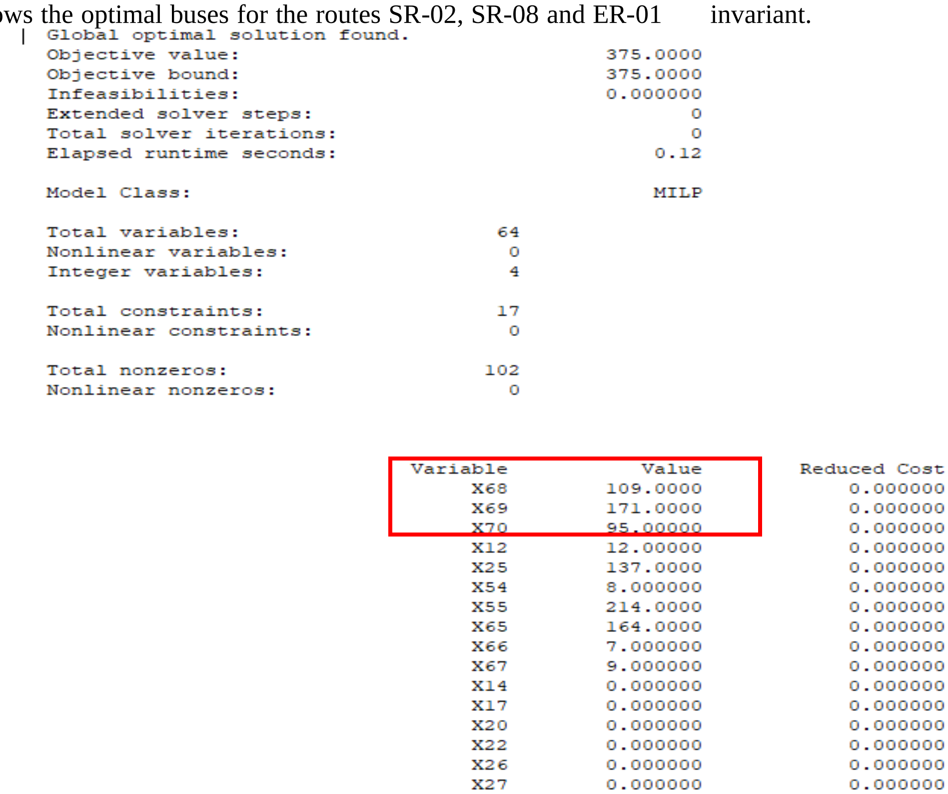


Fig.2 Input data to LINGO

The input data has been executed for optimal result and the output results from the LINGO can be seen in Figure 3, which shows the optimal buses for the routes SR-02, SR-08 and ER-01



on BRT corridor. After determining the optimal quantity of tokens to use at Places through LINGO, these results are entered into the LP row of the Microsoft excel file. The cycle timings are then recalculated using the optimal quantity of tokens for each invariant.

Fig.3 Optimized results from LINGO

This study emphasized bus bunching as clearly related to stop arrival within the calculated optimal time to destination stop and optimal number of buses on each route. One major benefit of a BRT system is that it operates on a dedicated route, separate from regular traffic. This means that travel times can be accurately predicted and shared with passengers, which builds trust in the system and attracts more customers to use the service. Hence, by taking into account variables such as the average speed based on the optimal calculated time between consecutive stops and the optimal quantity of buses on the route, this system can be effectively utilized to establish a mass transportation solution, leading to improved mobility conditions in city by reducing the number of private vehicles on the road and enabling passengers to reliably estimate their travel times.

V. CONCLUSION

In this study we presented Petri net and Max_Plus based modeling for bus bunching in BRT Peshawar. Acquisition of actual data, prediction of future stop timing, and optimization procedures were all addressed, which, to the best of our knowledge, is the first effort in this domain. By this we abstracted the BRT Peshawar as a graphical PN model and then translated to mathematical modeling with the help of Max_Plus Algebra and found that the Stopping Route (SR)-02, Stopping Route (SR)-08 and Express Route (ER)-01 must complete one trip in 52:25, 27:52 and 41:54 minutes, respectively. Furthermore, the analysis was done using INA and LINGO and found that the current BRT corridor routes are operating below than the minimum required Buses for each route and suggested that BRT Peshawar needs the maximum 171, 109 and 95 Buses for SR-02, SR-08 and ER-01 route on BRT corridor respectively for reduction of Bus Bunching.

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