Application of goal programming for optimal routing investment: A case study in Pakistan Railways

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Abstract- In this study, we propose a weighted integer goal-programming model for investment prioritization of railway projects. The model's objective is to maximize objectives while staying within the budgetary constraints for capital investments while prioritizing the mentioned schemes for investment. The case study also estimates how more passengers would utilize rail if ticket costs were reduced. The proposed model reduces the objectives' target deviations. The model's objectives cover both quantitative and qualitative traits. Even if the objective is to maximize all traits, the investment choice is based on available funds. When making a decision relies on economic advantages, revenue, or qualitative target scores, the research suggests investment alternatives at various capital investment levels. In order to increase the outcome of the Pakistani railway department while staying within the allotted budget, the proposed model presents a case study to upgrade the current railway tracks and, in addition, to create a new proposed railway track network. The proposed approach is specifically used to priorities the upgrading of the railway track between Peshawar and Karachi for capital expenditure, which was recognized and studied by Pakistan's department of transport. Furthermore, we have compared the results of the existence railways track and the proposed optimal railway tracks in Pakistan for the case study by using classical optimization techniques. The obtained results for the case study indicate the effectiveness of the proposed railway model for the user benefits as compared to the existing railway model.

Keywords: mathematical modeling; weighted integer goal-programming; optimal railway model; profit maximization; cost minimization

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Introduction

Linear Programming

Optimization methods are used for decision making (DM), mainly in the selection of an optional subset of projects selected for investment [1]. The degree of objective and dependence on data, as well as the structure of their results which differs widely across these methods. Linear programming (LP) is a popular optimization approach. One of the most significant limitations of LP (linear programming) is that it can only optimize a single objective and cannot attach soft constraints. In order to enhance resource efficiency, linear programming was created during World War II. New war-related initiatives were needed attention and spread resources [2]. The word "Programming" was a military word and it was related to operations such as effectively organizing schedules or optimally deploying soldiers. In 2002, George Dantzig, a United States Air Force member, invented the Simplex technique of optimization to provide an effective solution for programming problem's solution with linear structures. Since, then specialists from different fields particularly Economics and Mathematics, had created and investigated the idea of "linear programming" and its applications [3]. Leonid Kantorovich was a Soviet Economist and Mathematician, who in 1960 created a formulation for a problem that was comparable to the general linear programming problem, also with a technique for solving it [4]. He worked on this method, during World War-II to plan expenses and refunds in order to minimize Army budget while increasing losses put on the enemy. F. L. Hitchcock defined public transport issues as linear programmes in 1941 and provided a solution that was very close to the later simplex technique [5]. George B. Dantzig worked alone from 1946 to 1947 created a generic linear programming approach for usage in the United States Air Force planning issues [6]. Dantzig also created the simplex technique in 1997, which efficiently handled the linear programming issue in most situations for the first time [7]. On January 5, 1948, Dantzig approved an unpublished paper titled "A Theorem on Linear Inequalities." Dantzig's work was originally published his work in 1951 [8]. Many companies used it in their routine planning in the postwar years. Leonid Khachiyan demonstrated in 1979 that the linear programming problem is solvable in Mathematical terms [9], however, Narendra Karmarkar proposed a new interior-point approach for identifying linear-programming problems in 1984, which was a greater theoretical and practical development in the area [10].

Goal programming

The fundamental psychological and economic challenge of fulfilling boundless needs is one of man's core problem. This has always been the most challenging human issue. Man can choose between two options for bridging the gap between needs and availability. The first strategy which a person might choose is to struggle for more resources this strategy demands laborious tasks to get more money so that he can expand his resources by means of which can satisfy the majority of his desires. The second strategy a person might choose is to impede his needs to a certain limit so that to make the available sources sufficient for him. The aforementioned two strategies are completely contradictory; however, both have an ample range of applications concurrently. The problem about making decisions is this. Any problem that calls for a wise decision might be categorized as an operations research topic. [11]. It was not until World War-II. Operations Research is a term that was developed. George B. Dantzig (1963) suggested a method for addressing these decision-making problems during World War II. [12]. The creation of approaches for handling logistical challenges in military planning was a major focus of Dantzig's work. [13].

His investigation was backed by other scholars working on a related subject, according to L. Hurwicz (1961) [14]. The term "linear programming" was first used to refer to the approach, which was originally known as "the programming of interdependent activities on a linear structure."

After World War II, a number of researchers joined Dantzig in refining the method and exploring the potential uses of linear programming. However, the 1968 introduction and use of the technique to a business problem was of interest to the W. W. Cooper and A. Chornes team [15]. The fact that linear programming only allows for one goal is a significant criticism of the method. The public sector and economy are both dynamic in today's society, the dilemma of various conflicting objectives is significant as a result, In the real world, a decision maker is someone who makes an effort to accomplish a variety of goals to the maximum extent possible in the face of competing interests, a lack of information, and limited resources.

Goal programming was originally used in 1955 by Ferguson, Cooper, and Charnes [16]. One of the main important contributions that sparked interest in the application of goal programming was the organizational designs and industrial applications of goal programming created by W. W. Cooper (1968) in connection to intractable linear programming issues [17]. Additionally, they discussed the difficulty in achieving goals and the value of goal programming in enabling goals to be adaptable and included in model development.

Another significant contribution to the development of goal programming models and their use (1965) was made by Y. Ijiri throughout the 1960s [18]. goals for control accounting and management Y. Ijiri, R. K. Jaedicke, and Chames and Cooper (1966) laid the foundation for goal programming analyses' later growth [19]. According to their importance in the objective function Y, he described how to employ preemptive priority factors to solve a variety of conflicting objectives. Goal programming was established as a distinctive mathematical programming methodology by Ijiri, who also offered the extended inverse approach as a way of solution [20].

Goal programming did not become a widely used strategy for making decisions until the late 1960s and early 1970s [21]. The area of application saw the most advancements in the 1970s. Sang M. Lee created goal programming for decision analysis in a paper that was published in 1972 [22]. It made a significant contribution to the usage of goal programming's ongoing expansion. In his paper, Lee [23] explains way to resolve goal programming problems by utilizing a modified simplex method and gives computer software he developed. This computer software's accessibility made it possible to apply goal programming to complicated problems that were previously stressful and time-consuming [24]. Goal programming has been a managerial tool in decision science since Lee's essay established the foundation for it.

Multi-Criteria Analysis

In the literature, Multi-criteria analysis (MCA) is also known as Multiple decision-making (MDM), Multiple-criteria decision analysis (MCDA), Multi-objective decision analysis (MODA), Multi-dimensional decision-making (MDDM) or Multiple-attribute decision-making (MADM), includes a variety of strategies, procedures, and tools (of varying complexity) that specifically take into account a variety of goals and standards (or characteristics) when making decisions. MCA techniques have attracted the interest of scholars as well as practitioners that operate in a variety of disciplines, including transportation planning and policy, since the late twentieth century.

The structure and techniques of multi-criteria analysis (MCA) are provided to measure the here, the potential costs of interest, that is the values assigned to specified in terms of foregone investment, qualitative or political objectives [25]. The multi-criteria field provides various ways for inducing preferences of a decision maker between projects depending on the mutual significance of the several criteria. The methods' principal purpose is to handle the challenges that human decision-makers has be demonstrated to do in dealing with huge volumes of complicated information in a consistent direction.

The GAM (goal achieving matrix) is now widely accepted as an effective approach for weighing the advantages and costs of various strategies. The GAM method is a helpful technique for considering situations whose benefits and costs cannot be measured in cash terms and can't be included in standard benefit-cost analyses. The GAM approach was utilized by Pakistan's Department of Transport (DOT) to assess the qualitative objectives of nominated railway investment projects.

A weighted integer goal-programming (WIGP) paradigm is proposed in this paper for prioritizing chosen projects for investment while optimising the objectives and staying within the capital investment budget limit. The objective constraints in this study are balanced ENT Euclidean normalizing techniques (ENT) in order a biased conclusion in the solution scenario of the proposed model owing to variations in quantity units and measure of degree among the characteristics. This study prioritizes railway projects for investment and examines project combinations for investment. The study also suggests options for investing at various capital investment levels if the selection is created based on economic advantages, income or qualitative goal scores.

Goal Programming Mathematical Model

The standard linear goal programming methodology with weights is as follows.

$$\text{min/max } Z = \sum_{l=1}^{n} (P_i W_{l,i}^{+} d_l^{+} + P_j W_{l,j}^{-} d_l^{-})$$

Subject to:

$$\sum_{m=1}^{n} M_{l,m} X_j + d_l^{-} - d_l^{+} = g_l, \qquad \mathrm{l} = 1, 2, 3\ \ldots\ \mathrm{n}$$

$$\sum_{l=p+1}^{p+n} a_m X_l \leq b_l, \qquad m = 1, 2, 3, \ldots n$$

$$X_m, d_l^{-}, d_l^{+} \geq 0 \qquad l = 1, 2, \ldots p \text{ and } m = 1, 2, \ldots n$$

Where d_l^{-}, d_l^{+} are deviation from the goal and P_i, P_j = Priority factors

$W_{l,i}^{+}$ = Relative weight of d_l^{+} in the K^{th} ranking

$W_{l,j}^{-}$ = Relative weight of d_l^{-} in the K^{th} ranking

There are additionally "m" non-goal limitations, "n" objectives, and "l" decision variables.

Goal programming has the ability to solve a problem with multi goals of a system of complex objectives rather than a single objective. In other words, goal programming is a method that can't handle decisions involving a single goal, a few sub-goals, or a goal with many sub-goals. Additionally, the objective function of a goal programming model can be made up of non-homogeneous evaluation values like pounds and dollars rather than just one kind of unit. In the optimization of a single of functions we optimize the objective function directly for instance in linear programming but on the other hand in goal programming variances between goals and what will be achieved within the provided set of constraints are minimized.

The values of the decision variables are determined by the objective function, same like as in the solution of linear programming approaches. However, unlike the linear programming objective function, the values of the slack variables do not usually include decision variables. Additionally, it consists primarily of deviational variables, which represent each type of objective as a sub-goal. The deviational variable in the objective function can be stated in two dimensions as a deviation from each sub-goal both positively and negatively, as well as for restriction. Based on the relative importance or priority of these variations, the target function is to decrease them.

In a relatively straightforward goal programming problem with a single goal, the model is not materially different from a linear programming model. When there are several goals, there is a major difference since the system may encounter competing and conflicting goals.

Formulation of Goal Programming

Goal programming problems are formulated in a manner that is strikingly similar to that of linear programming challenges. The primary differences include a clear evaluation of the numerous goals and the likelihood that they will be accomplished, as well as the varied priorities associated with the various goals.

1. Use a single-goal model
2. A model with several objectives
3. Multi-Objective Conflict Model

Properties of Goal Programming

1. The common linear programming model only allows for a single goal, which is expressed by an objective function in the model. Goal programming has the ability to solve issues with several conflicting goals.

2. The highest priority goal must be accomplished first, according to the goal programming solution method. The next highest ranked goal is then sought to be obtained, but it cannot be done at the expense of the higher rated goal. In order to achieve the best level of satisfaction for all goals, the problem-solving process is effective. As a result, an acceptable solution is obtained rather than a perfect one.

3. Typically, deviational variables rather than choice variables are used to specify each sort of target or sub-goal in the goal programming objective function. Most of the time, the objective function includes a deviational variable that combines overachieving d^+ and underachieving d^- in relation to the current goal. This is shown as $d^- + d^+$, meaning that $d_k^- + d_k^+$ calculates the discrepancy between the K^{th} aim's actual and expected success. Goal underachievement and goal overachievement cannot coexist, hence one or both of these variables must be equal to zero in the end result.

Goal programming will produce outcomes for the choice variables in the objection inside the scope of a minimization problem if it is desired to use them for any purpose.

4. In order to assess each objective's relative worth or usability, goal programming does not require an accurate measurement of its quality. For the addition of goals, the statement of the preemptive in an ordinal order, the priority of the desired accomplishment of each goal, and a rating scale would suffice as the model's limitations and objective function.

5. The essential factor of it being a goal programming feature, as explained above, provides a solution for problems involving several and conflicting purposes without necessitating an exact quantification of the utility of each goal. The outcome of a goal programming model reveals how well the goals were achieved with the given inputs and under the specified constraints. Goal programming can be used for sensitivity analysis because of this feature.

Statement of the Problem: A case study of Pakistan Railway

Passenger transit is the main focus of Pakistan's railway system. Because the train system is currently inefficient, improvements must be made to increase its dependability, security, and dependability in order to draw more passengers. People may prefer to travel by train than over long distances in a car or a coach since rail is more responsive to service changes than buses. A state's investment in rail expansion and upkeep is primarily made to enhance passenger transit. The Department of Transportation (DOT) investigated and evaluated the benefits and drawbacks of numerous railway routes/projects that might be worthwhile investments in Pakistan, dividing them into two categories: upgrading existing train routes and investing in existing train routes to improve train services. In this study, just the upgrading is covered. Table 1 shows the newly recognized railway projects, with values being yearly averages. The upgrade plan has identified 5 projects for investment, with a total capital investment cost of around Rs. 90 billion. Both quantitative and qualitative benefits are estimated. Five qualitative and five quantitative attributes are taken into account in the prioritizing method. Benefits to users, safety, and accidents advantages, advantages for the environment, vehicle cost of resources reductions, and projected yearly income generated by the projects are among the quantitative characteristics. When assessing the projects' capital investment cost, the DOT introduced a lot of capital cost categories such as building costs, equipment costs, land costs, decommissioning costs, rolling stock expenses, dislocation costs, and so on. Clearing, acquisition, site preparation, and demolition all included in the land expenses. The figures were computed using opportunity costs or market pricing. Labor expenses, maintenance expenses, energy/fuel use prices, and utility service expenses are all taken into consideration when calculating operational costs. A variety of user expenses were evaluated, including generic Penalty and cost values (such as having to wait for the finalities like time, journey, junction, egress, access charges as well as fares), penalty for disaster cost as well as modal shift. The DOT analyzed the qualitative characteristics using MCA methods. The GAM method was the name given to the MCA methodology that was used. Projects are rated according to the MCA analysis on a scale of 0 (no impact), 1 (remote achievement), 2 (temporary achievement), 3 (major achievement), and 4 (better/fully achievement). The findings of the MCA investigation are shown in Table 1. Using the net present value (NPV) criteria, the DOT prioritized the selected projects for funding. The projected values for the projects (Billion Rupees).

Table 1

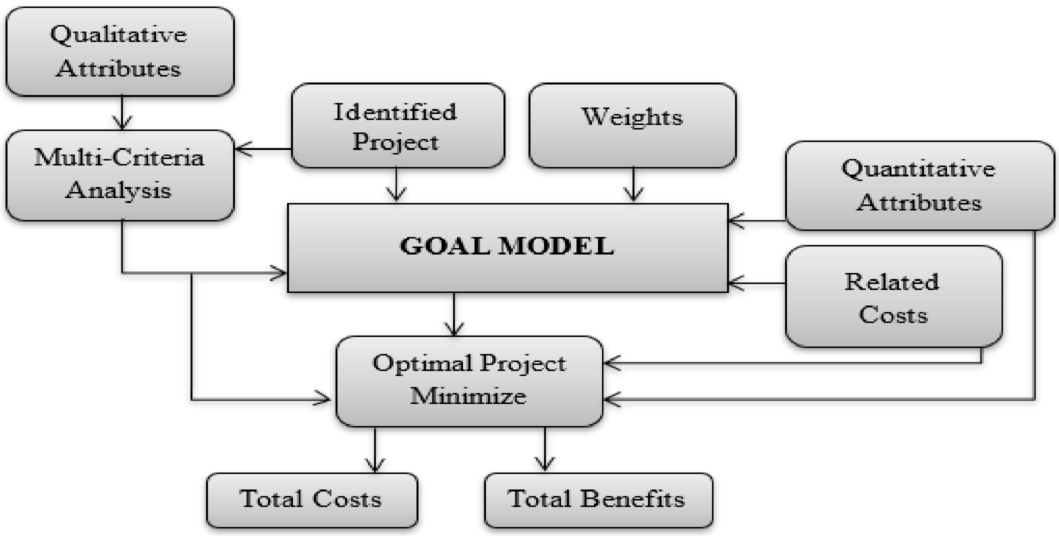
Projects	Revenue	User benefits	Qualitative goal 1	Qualitative goal 2	Qualitative goal 3	Investment cost
P1	5.131	10.395	2	3	2	39.111
P2	4.665	9.994	3	4	3	29.867
P3	2.897	4.191	4	3	3	14.533
P4	2.113	3.195	2	2	3	13.593
P5	1.998	2.995	3	3	2	10.281

P1 in the aforementioned table stands for the Lalamusa to Chaklala Railway Station, P2 for the Golra Sharif to Peshawar Cantt Railway Station, P3 for the Khanewal to Raiwind Railway Station, P4 for the Shahdara Bagh to Lalamusa Railway Station, and P5 for the Shahdara Bagh to Faisalabad Railway Station.

Development of the Model

For the purpose of prioritizing identified railroads projects for investments in Pakistan, a weighted integer goal programming (WIGP) model with just one or zero variables is being created. It's possible that the goal of the model is to lower the target deviation variables. According to Eq., only (-) negative deviations are taken into account for the objective function (1). Goal programming was developed with the aid of the LINGO software (GP). The flowchart below illustrates the process.

Flowchart or the methodology



Identification of the Project

The DOT (Department of Transport) identified and assessed the costs and advantages of various viable railway projects for investment throughout Pakistan, classifying them into two categories: improvements to existing rail lines for improved train services and creation of new projects. When calculating the capital expenditure of its projects, the DOT added a number of capital investment components, such as the cost of land, infrastructure costs, building expenses, moving stock expenditures, decommissioning expenditures, dislocation expenditures, and so forth. The costs associated with land acquisition, demolition, clearing, and site preparation are all included. The market price or the opportunity cost was used to determine the values. When calculating running expenses, labor costs are taken into account along with fuel/energy supply costs and utility service fees. Generalized cost estimations and charges, modelling shifting fees, tragedy fees, exchange fees, travel duration fees, accessibility and egress fees, and waiting time fees were all assessed.

Both quantitative and qualitative benefits are estimated. The DOT employed the multi-criteria analysis methodology to examine the qualities. The type of MCA methodology employed was the goal achievement matrix (GAM) approach. The GAM technique placed more emphasis on specific socioeconomic goals than it did on the repercussions for community groupings. Projects are rated according to the MCA analysis on a scale of 0 (no impact), 1 (remote achievement), 2 (temporary achievement), 3 (major achievement), and 4 (better/fully achievement). Using the net present value (NPV) criteria, the DOT prioritized the selected projects for funding.

Proposed Model

The main goal of the model is to minimize weighted negative goal deviations (-). The simultaneous minimizing of all objectives is referred to as "deviations are minimized."

Objective: $$\text{Min} \sum_{i=1}^{m} W_i d_i^-, \quad (1)$$

Where W_i is actually the attribute weight supplied by the user to take the importance of the characteristics into consideration for project planning?

Hard budget constraint: The total capital investment for the listed projects is below the available investment. The GP (goal programming) approach is used for capital expenditures at different spending plan levels, starting at 90 billion rupees and going up to 100 billion rupees. The spending plan was increased to 90 billion rupees.

$$\sum_{i=1}^{n} a_i X_{i,j} \leq A. \quad (2)$$

Soft constraints for different goals: The five (5) qualitative and five (5) quantitative parameters are taken into account by this soft constraint.

$$\sum_{i=1}^{n} b_{ij} X_i + d_j^- - d_j^+ = B_{j,} \quad \forall j \quad (3)$$

The following are the reasons why Eq. (3) has to be improved: The target constraints are normalized using Euclidean normalizing methods to correct for the units and magnitude mismatches. The model solution is skewed due to the variations in measuring units and magnitude scales between the characteristics.

$$\begin{bmatrix} b_{11} & b_{12....} & b_{ij....} & b_{1n} \\ b_{21} & b_{22....} & b_{2j....} & b_{2n} \\ . & . & . & . \\ . & . & . & . \\ b_{i1} & b_{i2....} & b_{ij....} & b_{in} \\ . & . & . & . \\ . & . & . & . \\ b_{m1} & b_{m2....} & b_{mj....} & b_{mn} \end{bmatrix} \tag{4}$$

One of the components of the ENDM d_{ij} can be calculated using Eq. (5) and the Euclidean normalization approach (Niemeier et al., 1995; Hwang and Yoon, 1981) [26, 27].

$$d_{ij} = {b_{ij}}\Big/{\sqrt{\sum_{i=1}^{m} b_{ij}^2}} \tag{5}$$

The Euclidean Normalized Goal (ENG) attributes' matrix form is displayed in Eq (6).

$$\begin{bmatrix} d_{11} & d_{12....} & d_{1j....} & d_{1n} \\ d_{21} & d_{22....} & d_{2j....} & d_{2n} \\ . & . & . & . \\ . & . & . & . \\ d_{i1} & d_{i2....} & d_{ij....} & d_{in} \\ . & . & . & . \\ . & . & . & . \\ d_{m1} & d_{m2....} & d_{mj....} & d_{mn} \end{bmatrix} \tag{6}$$

The Euclidean normalized version of the soft constraint is presented in Eq. (7) and can be obtained by using Equations (3) and (5).

$$\sum_{i=1}^{m} d_{ij} X_i + d_j^- - d_j^+ = D_{j,} \quad \forall j \tag{7}$$

The objective level D_j for every attribute is specified as in Eq (8). You want to have maximal values, to put it another way.

$$D_j = \sum_{i=1}^{m} d_{ij} \qquad \forall\, j \tag{8}$$

A constraint that is positive. The goal deviation variables have to have positive values.

$$d_j^-, d_j^+ \geq 0 \qquad \forall\, j \tag{9}$$

According to the physical interpretation of X_i, a decision variable with an integer outcome of "0" to "1" indicates whether or not a project is credited (referred to as a binary variable).

$$0 \leq X_i \leq 1; \qquad \forall\, i \tag{10}$$

Model Variables:

X_i Decision variable

d_{ij}^+ Goal j is Positive deviation variable

d_{ij}^- Goal j is negative deviation variable

Parameters

b_{ij} The numerical outcome of the i^{th} project in respect to the j^{th} attribute (indicating the value of the project i's characteristics j)

B_j The j^{th} characteristic is calculated while making a decision.

A Total amount of capital financing available

a_i Capital costs i's of the project

d_{ij} The project's i^{th} Euclidean normalized numerical output in relation to the j^{th} attribute

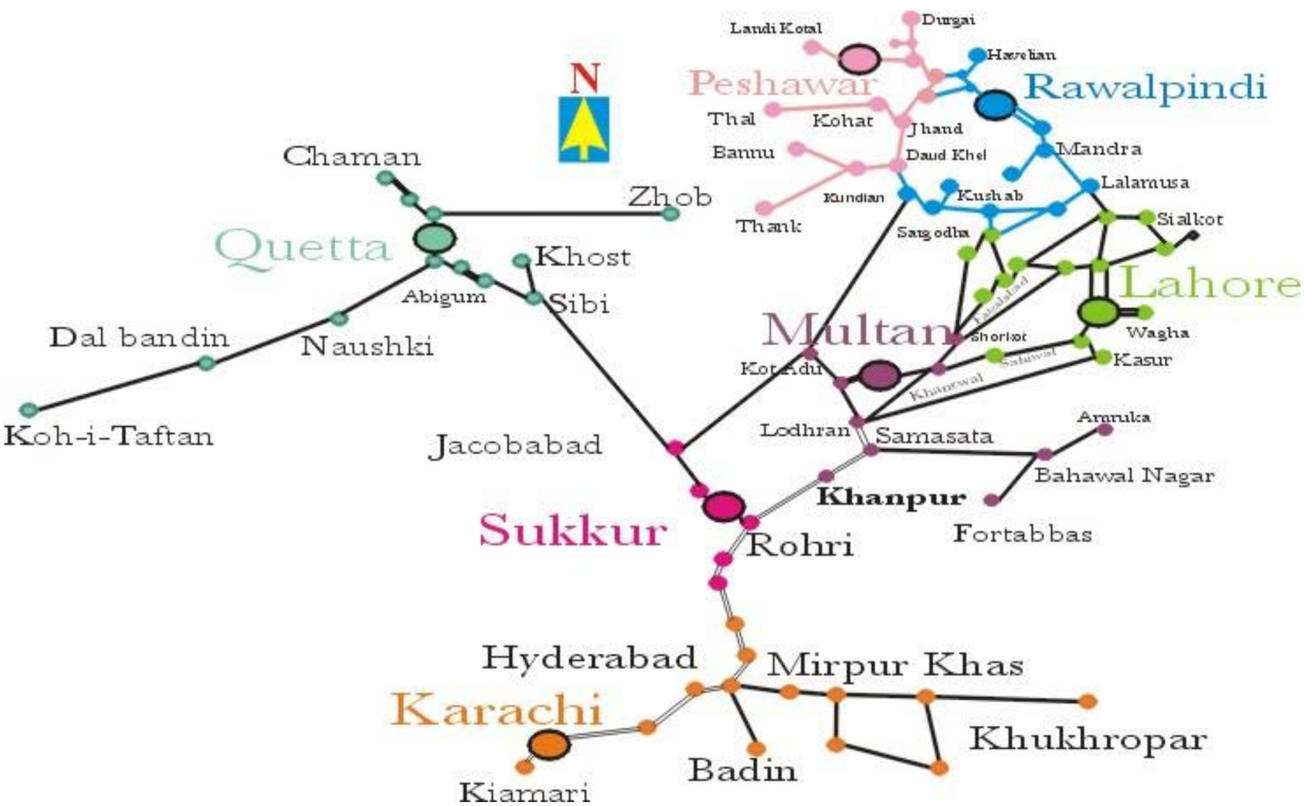
D_j The j^{th} attribute taken into account in the decision has a target

m Amount of objective characteristics considered

n The quantity of projects taken into consideration

W_{Bj} The attribute j has a user-assigned weight

Figure 1: Railway Tracks in Pakistan



Problem estimation

Table 2 provides values for the projects that have been estimated (in billions of rupees).

Table 2

Projects	Revenue	User benefits	Qualitative goal 1	Qualitative goal 2	Qualitative goal 3	investment cost
P1	5.131	10.395	2	3	2	39.111
P2	4.665	9.994	3	4	3	29.867
P3	2.897	4.191	4	3	3	14.533
P4	2.113	3.195	2	2	3	13.593
P5	1.998	2.995	3	3	2	10.281

In table 2, P1 stands for the railway station connecting Lalamusa to Chaklala, P2 for the station connecting Golra Sharif to Peshawar Cantt, P3 for the station connecting Khanewal to Raiwind, P4 for the station connecting Shahdara Bagh to Lalamusa, and P5 for the station connecting Shahdara Bagh to Faisalabad.

Consider the following example to explain the GP approach phases, which involves five (5) projects and five (5) attributes (objectives) as stated in Table. The objective function of the problem is depicted in Eq. (11) using Eq (1).

Objective $$Min = 0.240d_1^- + 0.251d_2^- + 0.174d_3^- + 0.173d_4^- + 0.170d_5^- \quad (11)$$

The hard capital cost constraint, Eq. (2), is visible in Eq (12). The following capital investment funds are readily available and are believed to be worth between 90 billion and 100 billion rupees.

$$39.111x_1 + 29.867x_2 + 14.533x_3 + 13.593x_4 + 10.281x_5 \leq 90 \quad (12)$$

The five (5) attributes described by Eq. (13) are subject to soft constraints presented by Eq (3).

Revenue

$$5.131x_1 + 4.665x_2 + 2.897x_3 + 2.113x_4 + 1.998x_5 + d_1^- - d_1^+ = 35.085$$

User benefit

$$10.395x_1 + 9.994x_2 + 4.191x_3 + 3.195x_4 + 2.995x_5 + d_2^- - d_2^+ = 30.967$$

Goal: 1

$$2x_1 + 3x_2 + 4x_3 + 2x_4 + 3x_5 + d_3^- - d_3^+ = 19.160$$

Goal: 2

$$3x_1 + 4x_2 + 3x_3 + 2x_4 + 3x_5 + d_3^- - d_3^+ = 17.260$$

Goal: 3

Where the above are qualitative objectives or goals

$$2x_1 + 3x_2 + 3x_3 + 3x_4 + 2x_5 + d_3^- - d_3^+ = 18.197 \quad (13)$$

Numbers are used to describe qualitative goals, but User Benefits and Revenue are quantified in billions of rupees. The data are normalized using the Euclidean normalizing process to address the scale and unit concerns. Following is the first step in normalizing, which is described in Eq. (5).

Revenue

$$\frac{5.131}{\sqrt{64.939}}x_1 + \frac{4.665}{\sqrt{64.939}}x_2 + \frac{2.897}{\sqrt{64.939}}x_3 + \frac{2.113}{\sqrt{64.939}}x_4 + \frac{1.998}{\sqrt{64.939}}x_5 + d_1^- - d_1^+ = 35.085$$

User benefit

$$\frac{10.395}{\sqrt{244.68}}x_1 + \frac{9.994}{\sqrt{244.68}}x_2 + \frac{4.191}{\sqrt{244.68}}x_3 + \frac{3.195}{\sqrt{244.68}}x_4 + \frac{2.995}{\sqrt{244.68}}x_5 + d_2^- - d_2^+ = 30.967$$

Qualitative goal 1

$$\frac{2}{\sqrt{42}}x_1 + \frac{3}{\sqrt{42}}x_2 + \frac{4}{\sqrt{42}}x_3 + \frac{2}{\sqrt{42}}x_4 + \frac{3}{\sqrt{42}}x_5 + d_2^- - d_2^+ = 19.160$$

Qualitative goal 2

$$\frac{3}{\sqrt{47}}x_1 + \frac{4}{\sqrt{47}}x_2 + \frac{3}{\sqrt{47}}x_3 + \frac{2}{\sqrt{47}}x_4 + \frac{3}{\sqrt{47}}x_5 + d_2^- - d_2^+ = 17.260$$

Qualitative goal 3

$$\frac{2}{\sqrt{35}}x_1 + \frac{3}{\sqrt{35}}x_2 + \frac{3}{\sqrt{35}}x_3 + \frac{3}{35}x_4 + \frac{2}{\sqrt{35}}x_5 + d_2^- - d_2^+ = 18.197 \quad (14)$$

Simplify Eq. (14) we have

Revenue

$$0.637x_1 + 0.567x_2 + 0.360x_3 + 0.262x_4 + 0.248x_5 + d_1^- - d_1^+ = 35.085$$

User benefit

$$0.665x_1 + 0.639x_2 + 0.268x_3 + 0.204x_4 + 0.191x_5 + d_2^- - d_2^+ = 30.967$$

Qualitative goal: 1

$$0.309x_1 + 0.463x_2 + 0.617x_3 + 0.309x_4 + 0.363x_5 + d_3^- - d_3^+ = 19.160$$

Qualitative goal: 2

$$0.438x_1 + 0.583x_2 + 0.438x_3 + 0.292x_4 + 0.438x_5 + d_3^- - d_3^+ = 17.260$$

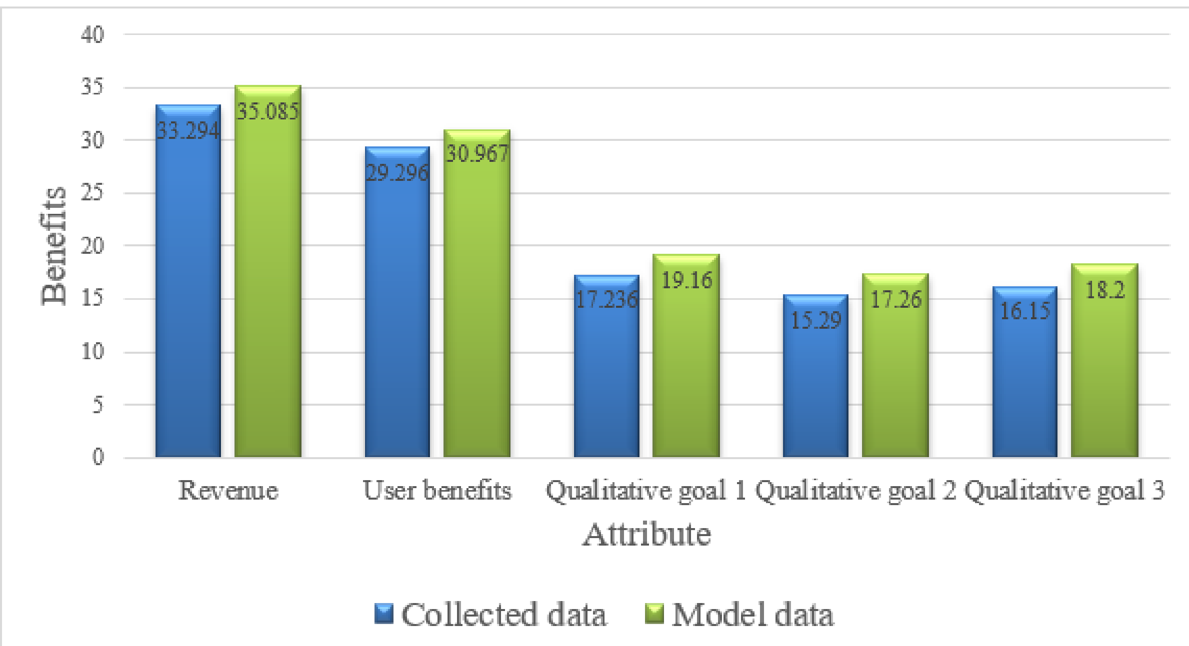
Qualitative goal: 3

$$0.338x_1 + 0.507x_2 + 0.507x_3 + 0.507x_4 + 0.338x_5 + d_3^- - d_3^+ = 18.197 \quad (15)$$

In Eq. (15) $x_1, x_2, x_3,\ x_4\ and\ x_5$ are integer variables

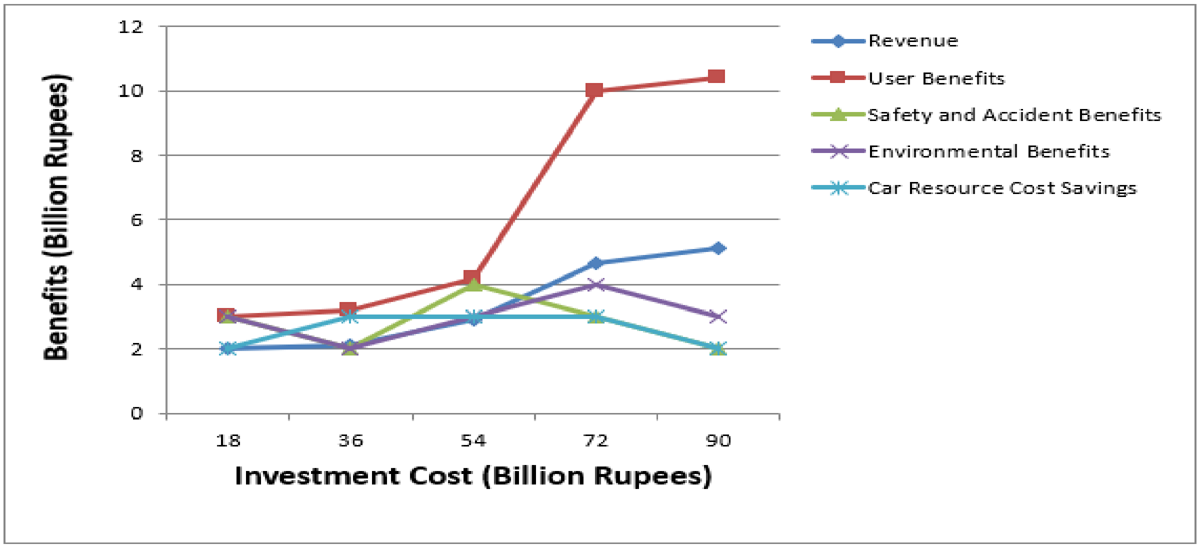
The projects P1, P2, P3, P4, and P5 are those that are advised for investment while solving equations (10), (11), (12), and (15) with the help of LINGO software. To complete the projects that have been specified, 85.04 billion Rupees in capital expenditures are required. The following graph shows the overall advantages of the chosen project as well as the objective values.

Fig:1 Benefits brought forth by the investments chosen for the projects



By optimizing the objective qualities, the WIGP (weighted integer goal programming) technique calculates the ideal number of projects to invest in at various investment rates (chosen project at various budget levels). The benefits of quantitative features are illustrated in Figure 1 and include user benefits, security and accident benefits, environmental benefits, lower vehicle resource costs, and anticipated annual income at various capital investment levels. In the figures with the data label, the project implementation expense cost is shown.

Fig: 2 Benefits at various investment levels (Billion Rupees)



Data collection for the case study

The proposed model is created on the novel railway projects recognized by the Department of Transportation (DOT), data on capital expenditures, user advantages, safety advantages, environmental advantages, yearly revenue, and quality advantages approach were gathered from the DOT (Department of Transportation) publication. Anandarajah provided the amount of significance based on weighted attributes of goals in prioritization of railway projects [28, 29]. The questionnaire was distributed to the DOT (Department of Transportation) professionals, academics, and transportation researchers. The questionnaire's respondents included transportation professionals, such as those from Pakistan Rail, as well as academics and researchers in the field. The questionnaire was completed by 150 participants. A questionnaire survey was used to evaluate the amount of importance of the qualities in the analysis. In the questionnaire, the importance was graded on a scale of 1–4, with 1 being trivial and 4 being critical.

Outcomes and discussion

The proposed WIGP model calculates the best project mix for different budget levels maximizing the objective characteristics (chosen projects at various budget levels as shown in table 1). Figure 2 illustrates the benefits of quantitative qualities, such as user benefits, safety and accident benefits, environmental benefits, resource cost savings for vehicles, and predicted annual revenue at various capital expenditure ratios. The numbers with the data label display the investment cost for the entire project. Because the model maximizes the objective characteristics, the advantages are expected to rise as the investment amount increases. However, although the advantages, according to the conclusions of the WIGP model, economic growth is rising on average advantages (user advantages, accident and safety advantages, environmental advantages, and car resource reduced costs) and one investment level, both economic advantages and revenue are reduced; as the capital investment cost is increased, both economic advantages and revenue drop. It is not to say that the advantages are not being maximized. The advantages are optimized since the model's objective is to maximize the advantages, which include both qualitative and quantitative advantages. The difference between the weighted sum of the goal targets (D_j) and the objective value (weighted goal deviations) at each investment level is the total benefit at each investment level. The variance in qualitative goal scoring of the ideal project mix at various investment levels, as opposed to quantitative features, economic advantages, and income, qualitative goals reduce as capital expenditure cost increases at different expenditure levels. Unless the model indicates the best a mix of projects with varying degrees of investment while maximizing objective characteristics, making a selection is difficult since qualitative attributes produce quantitative features offer relatively low ratings at various investment levels, whereas qualitative attributes give comparatively high scores. The estimated economic benefit expenditure relation and revenue expenditure relation for various levels of investment, the economic situation varies depending on the level of investment benefit expenditure relation is calculated by dividing the overall economic advantages (sum of advantages of safety and accident prevention, user advantages, vehicle resource reduced expenditures, and environmental benefits) in terms of overall capital investment expenditure. On the same lines, the revenue ratio of expenditure is evaluated by dividing revenue by capital investment expenditure.

Conclusion

To assist investors in selecting railroad projects with the aim of maximizing benefits, including qualitative features, a WIGP model is being developed. The ideal group of projects has been identified, taking into account different budgetary restrictions for capital spending. The investment decision in this method is dependent on the decision maker's thoughts and whether they are largely focused on economic benefits, income, or objective target values, despite the fact that the goal is to optimize these aspects. Additionally, the decision is based on the capital expenditure budget's accessibility. The proposed findings show that the weighted integer goal-programming model's sensitivity to user-assigned weights is influenced by the number of attributes. A weighted integer goal-programming model is employed in the research to show, however, that the normalizing strategy has no impact on how sensitive the user-specified weights are. The target deviations of the model's principal flaw are devoid of cost components. The only benefits are optimized as a consequence. In addition, it is not possible to do sensitivity analyses to the uncertainties in the estimated cost values of the projects mentioned. The model's derived benefits, such as user benefits, revenue, benefits related to user benefits, benefits related to safety and accident prevention, and benefits related to the environment, are illustrated visually to help find the best option.

Author's Contributions

All authors participated equally in the preparation of this manuscript. All authors verified the final version of this manuscript. All authors contributed equally to this work. All authors read and approved the final manuscript.

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