THE USE OF LAGRANGIAN INTERPOLATION POLYNOMIAL TO PREDICT THE SAFE DISTANCES FOR A BOREHOLE LOCATION IN THE CEMETERY VICINITY

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Abstract

Leachate formation and migration from cemeteries is a major threat to underground water bodies due to toxic chemicals released during decomposition. This study uses Lagrangian Interpolation Polynomial (LIP) to predict safe distances for locating hand-dug wells in the vicinity of a cemetery in Ede (Latitude 7.7349°N and Longitude 4.4439°E). Eleven (11) soil samples were collected using hand augers at 20.0 m intervals from the center of the cemetery to the outside of the sloppy side of it at a maximum depth of 2.0 m; they were analyzed in the laboratory for the presence of leachate ionic concentrations (LIC) (nitrate, sulphate, and carbonate). The analysis results were substituted into LIP to obtain a Model Equation (ME) for the predicted concentration (PC) of each ion. The graphs of LIC and PC against distance were plotted for each ionic component to compare and test for accuracy. The ME of PC for different ionic components was combined to obtain a General Model Equation (GME) for PC. The results of the correlation coefficient (CC) of LIC and PC for each model revealed that there is a strong correlation between them for nitrate (R_{RN} = 0.9278, R_{PN} = 0.9273), sulphate (R_{RS} = 0.9394 and R_{PS} = 0.9384), and carbonates (R_{RC} = 0.9706 and R_{PC} = 0.9701). The CC of GME of LIC and PC indicated a close correlation. Hence, LIP can be used to calculate safe distances from cemeteries for hand-dug wells and boreholes.

Keywords: Leachate, Cemeteries, Model Equation (ME), Lagrange Interpolation Polynomial (LIP), and General Model Equation (GME)

INTRODUCTION

The sixth sustainable development goal is to ensure clean water and sanitation for all. Just as we breathe air, water is another vital natural resource that must be protected. Domestic use, agricultural purpose, industrial use, and commercial purpose are some of the aspect of life that relies on water for their sustenance. Pollution of underground water as a result of human activities is a major threat to the availability of clean potable water. Such activities may include but are not limited to waste from industries, location of the dumpsite, contamination by leaking pipes and tanks, and leachate generation from cemeteries (Oliveira *et al, 2013*). Leachates are referred to fluids released from decomposed substances; excessive production of such fluids from areas with accumulated buried organisms can percolate the soil and move to the groundwater over time and possibly contaminate its chemical composition. The rate of leachates seepage is determined by the direction of flow, the topography of the graveyard, the number of buried bodies, the depth of the buried bodies, the size of the cemetery, soil type, porosity, and permeability of the soil, closeness of buried cadavers to the water table, and how often coffin is used (Engelbrecht 1998, EPA 2015).

According to Oyelami *et al* (2021), Silva and Filho (2011), and Fiedler et al. (2012), different sources of contaminants from body decomposition include; chemical substances used in chemotherapy and embalming processes (e.g., arsenic, formaldehyde, and methanol), makeup (e.g., cosmetics, pigments, and chemical compounds), and various additional items such as fillings, cardiac pacemakers, paints, varnishes, metal hardware elements, iron nails are all sources of contaminants from body decomposition. The contamination processes are caused by the sequential processes of a corpse's decomposition stages, such as autolysis, putrefaction, and decay (Fiedler et al. (2012)). CO, CO_2, H_2S, CH_4, Mg^{2+}, NO_3^-, SO_4^{2-}, PO_4^{3-}, CO_3^{2-} and NH_4^+, and so on (Fakunle *et al* (2021), Oyelami *et al* 2021) are some of the gaseous and chemical constituents of the leachates released from decomposing bodies; such chemicals may affect the chemistry of water and have adverse effects on health after consumption.

When the soil characteristics at burial sites are unsuitable for leachate retention and filtration, the liquid migrates through the unsaturated zone and reaches the underlying aquifer. The unsaturated zone conditions govern attenuation processes and the eventual elimination of chemicals of necro-leachate, as well as the contamination of surface and groundwater bodies (Oliveira *et al*., 2013).

To prevent leachate contamination of nearby water sources, in developed countries cemeteries typically have setback distances, which are regulations or by-laws that specify how far cemeteries must be located from water sources, drainage, and ditches must be implemented, however, this is not the practice in the developing countries. Mathematical equations can be used to model safe distance for citing hand-dug wells and borehole's cemetery vicinity.

Therefore this study is designed to use Lagrangian Interpolation Polynomial (LIP) to predict the safe distance for locating hand-dug wells or boreholes in the vicinity of the cemetery, this is achieved by the collection of soil samples at different intervals within and outside the cemetery to determine leachate ionic components (LIC) (NO_3^{2-}, SO_4^{2-} and CO_3^{2-}) in the soil samples, LIP was used to develop a model for the predicted concentration (PC) of NO_3^{2-}, SO_4^{2-} and CO_3^{2-} generated and general model equations (GME) for migration of leachate were estimated.

Methodology

Description of the study site

The study area is a cemetery located at Ede South Local Government and accessed by a tarred road, the area is dominated by Yoruba-speaking indigenes. The cemetery is surrounded by a residential area and the area experience rapid development as a result of urbanization, the community has a population size of 159,866 according to the 2006 Population census. As a result of the river flowing nearby the study area, the main activities of the locals are farming and fishing on the remaining pieces of land around the cemetery as well as the agribusiness. The area is characterized by two major seasons; the rainy season for about a period of eight months (March to October) and the dry season lasting for three months (November to February). The study area lies within latitude 07° 36′N and 07° 46′N of the equator and on longitude 04° 22′E and 04°34′E of the Greenwich meridian respectively in the Guinea Savannah zone of Nigeria.

The selection of the burial site was based on the fact that in the Islamic religion, embalmment is not allowed in any form; it is usually customary to wash the dead body and wrap it up with a cloth in a ritual manner before it is buried, which should be accomplished within 24 hours of death (Gatrad A. R. 2020). The burial practice was adopted because the embalmment procedures slow down the process of decomposition of a cadaver and may influence the activities of microorganisms.

Geological Description of the Study Site

Geologically, the rock in the study area is underlain by the crystalline Basement complex of southwestern Nigeria and it is of two main rock types which include pegmatite and schist (Audu *et al*., 2015, Salufu *et al*. 2019). The Geotechnical characterization of lateritic soils in the study area by Oyelami and VanRooy (2018) shows that the pegmatites, which occur as near vertical dykes and intrude into the older lithology of banded gneiss, and strike primarily in the NNE-SSW direction. Banded gneiss is a massive rock composed of alternating bands of felsic minerals, particularly plagioclase feldspars and quartz, and dark bands of biotite and hornblende (Oyelami *et al* 2021).

Soil Samples Collection

The study location was surveyed using GIS to determine elevation, topography, coordinates, and flow patterns. Eleven (11) soil samples were further collected at a distance of 20.0m intervals to cover a distance of 200.0m away from the center of the cemetery, from the topsoil at the depths of 2.0m using soil auger and labeled A_1 – A_{11} along the direction of hydrostatic flow which normally takes the path of the surface topography. A control sample was also collected at distance above 240m at the same depths. The leaves, roots, and stones are manually removed and the samples were finally transferred into polyethylene bags and transported to the laboratory for analysis.

Physicochemical Analysis of Soil Sample

At the laboratory, the soils sample were dried in a 40 °C oven for 24 hours, the samples were crushed with a mortar and rubber pestle and passed through a 2.0mm sieve to eliminate stones and other materials extraneous to the soil. 2.5 g of the sample was weighed using an analytical balance; the weighted sample was then placed in a 125-mL Erlenmeyer conical flask. The samples were then placed in a digestion tube and chemical fume hood and 25 mL of concentrated H_2SO_4, HNO_3-$H2O_2$, aqua regia (HNO3-HCl 1:3v/v) were added for extraction of the high concentrations of sulphate, nitrate, and carbonate respectively. The solutions were placed in a shaker for 15 minutes and filtered, and were then analyzed by the flame atomic absorption unit of the Absorption Atomic Spectrophotometer (AAS).

Lagrange Interpolation Equation (LIP)

Interpolation is a mathematical function that estimates the values at locations where no measured values are available (Baillargeon S. 2005). The Lagrange interpolation polynomial (LIP) is a mathematical equation (ME) developed to find a polynomial that takes on certain values at arbitrary points. Senol C. (2018) applied LIP to determine the crop and milk production in agriculture, the result shows the reliability of LIP to predict values of physical quantities in the absence of measured data.

The Lagrange Interpolation Polynomial (LIP) is defined as:

$$P(x) = y_1 + \frac{(x-x_1)(x-x_3).....(x-x_n)}{(x_2-x_1)(x_2-x_3).....(x_2-x_n)} y_2 + ... + \frac{(x-x_1)(x-x_3).....(x-x_{n-1})}{(x_n-x_1)(x_n-x_2).....(x_n-x_{n-1})} y_n \tag{1}$$

$P(x)$ represents the predicted concentration (PC) of ionic components, y_1,y_2 and y_n are the LIC, x_1, x_2, x_2 and x_n are the measured distance from the centre of the cemetery and x denotes the unknown distance.

In this study, Lagrange Interpolation Polynomial (LIP) was used to predict the concentration of leachate migrating from the cemetery at a different distance to determine the safe distance for siting of hand-dug wells and boreholes around a cemetery. The result of the LIC obtained from the laboratory analysis and the measured distance from the centre of the cemetery are the variables substituted into equation (1) to generate a polynomial equation that can be used for the prediction of the ionic concentrations in a situation where laboratory-analyzed concentrations are not available.

Result and Discussion

The result of the LIC with the measured distance is shown in table 1.0. It can be deduced in table 1.0 that the concentration of the analyzed nitrate is higher than that of sulphate and carbonate. The carbonate concentration is extremely low compared to the sulphate and nitrate. The concentration variation is in the decreasing order $nitrate > sulphate > carbonate$. The analysis also gives a gradual attenuation of the ionic concentrations with an increase in distance from the centre of the cemetery. The summation of all ionic components gives the total LIC present in the soil sample as presented in table 1.0. Moreover, it was observed that the LIC of the decomposed bodies tend to zero at the control point (i.e. at 240.0 m) and this implies that hand-dug wells and boreholes cited in the region are safe from cemetery contamination.

Table 1.0: The result of the analyses of the various ionic components and the measured distance

Distance (m)	Lab. analysis of NO_3^{2-} concentration (mg/L)	Lab. analysis of SO_4^{2-} concentration (mg/L)	Lab. analysis of CO_3^{2-} concentration (mg/L)	Total concentration (mg/L)
0.00	49.60	36.40	2.22	88.22
20.00	45.46	32.67	2.13	80.26
40.00	40.15	30.15	2.10	72.40
60.00	36.28	28.36	1.96	66.60
80.00	24.80	27.58	1.93	54.31
100.00	19.89	18.87	1.92	40.68
120.00	18.29	12.52	1.74	32.55
140.00	18.15	12.40	1.58	32.13
160.00	18.05	12.10	1.55	31.70
180.00	10.02	9.04	1.53	20.59
200.00	10.00	9.00	1.34	20.34
240.00	0.05	0.00	0.00	0.05

Predicted Models and Analyses

Equations (2), (3), and (4) are the developed models for the prediction of the concentration of nitrate, sulphate and carbonate respectively using Maple 18 software.

$$NO_3^{2-} = 7.21\times10^{-8}x^4 - 1.72\times10^{-4}x^3 + 6.23\times10^{-4}x^2 - 2.13x + 49.6 \quad (2)$$

$$SO_4^{2-} = 2.48\times10^{-6}x^4 - 9.27\times10^{-4}x^3 + 0.10x^2 - 3.54x + 36.40 \quad (3)$$

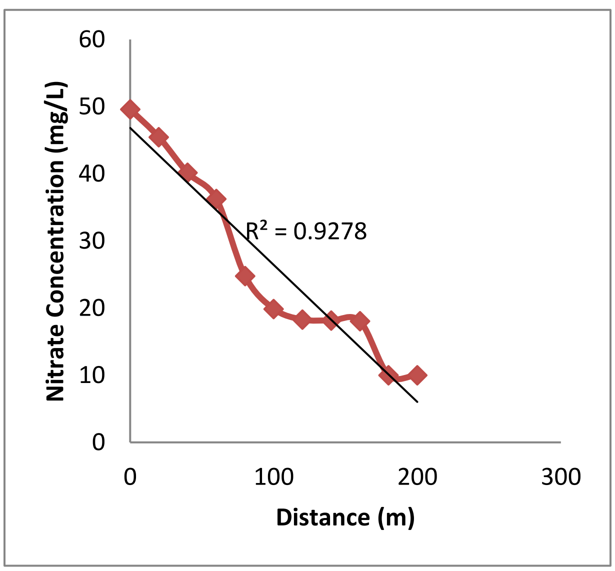
$$CO_3^{2-} = -4.96\times10^{-10}x^4 + 1.13\times10^{-7}x^3 - 2.30\times10^{-6}x^2 - 4.50\times10^{-3}x + 2.22 \quad (4)$$

The result and discussion of the LIC of Nitrate obtained from LIP

Table 2.0: LIC and the PC of the nitrates

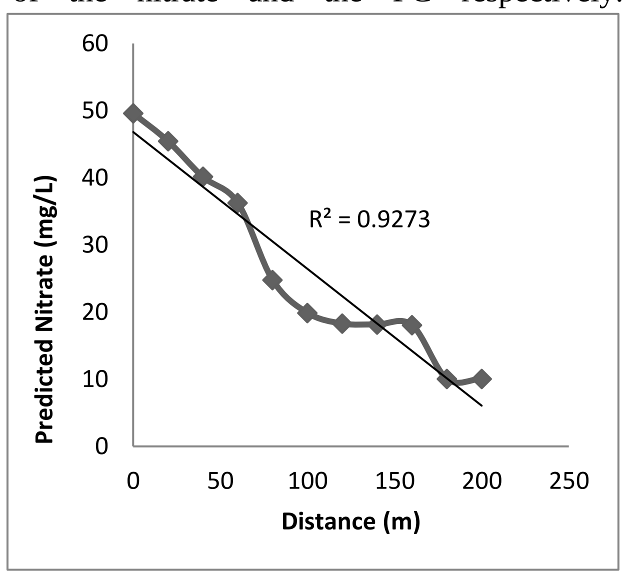
Distance (m)	LIC of NO_3^{2-} (mg/L)	PC of NO_3^{2-} concentration (mg/L)
0.00	49.600	49.600
20.00	45.460	45.460
40.00	40.150	40.149
60.00	36.280	36.279
80.00	24.800	24.799
100.00	19.890	19.890
120.00	18.290	18.291
140.00	18.150	18.155
160.00	18.050	18.064
180.00	10.020	10.053
200.00	10.000	10.070

The LIC and the PC of the nitrates concentration (from equation 2) as obtained from Maple 18 are presented in Table 2.0. The table reveals a close relationship between the LIC of the nitrate concentrations and the PC. Figure 2.0 represent the plot of the LIC of the nitrate concentrations and the PC against the distance away from the centre of the cemetery. It was observed that the LIC of the nitrate and the PC values decreased with an increase in the distance away from the centre of the cemetery (as in Figures 2(a) and 2(b)). A strong correlation of $R^2 = 0.9278$ and $R^2 = 0.9273$ was obtained for the LIC of the nitrate and the PC respectively.



2(a)

Fig 2 (a): The plot of the analyzed nitrate concentration with distance



2(b)

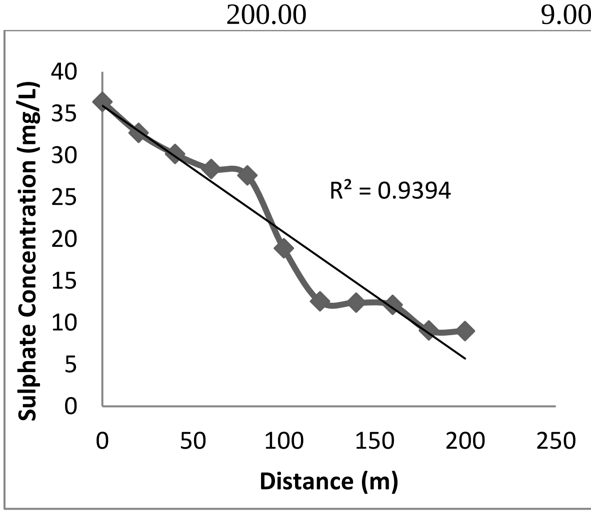
Fig 2 (b): The plot of the predicted concentration of nitrate obtained from LIP with distance

The result and discussion of the concentration of Sulphate obtained from LIP

Table 3.0 shows the result of LIC of the sulphate and the PC as obtained from equation (3). Figure 3.0 presents the plot of LIC of the sulphate and the PC against distance, it was noted that there is a strong correlation of $R^2 = 0.9394$ and $R^2 = 0.9384$ for the PC and the LIC [as observed in Figures 3(b) and 3(a)], it was also observed that the LIC of the sulphate decreased as the distance increased from the centre of the cemetery.

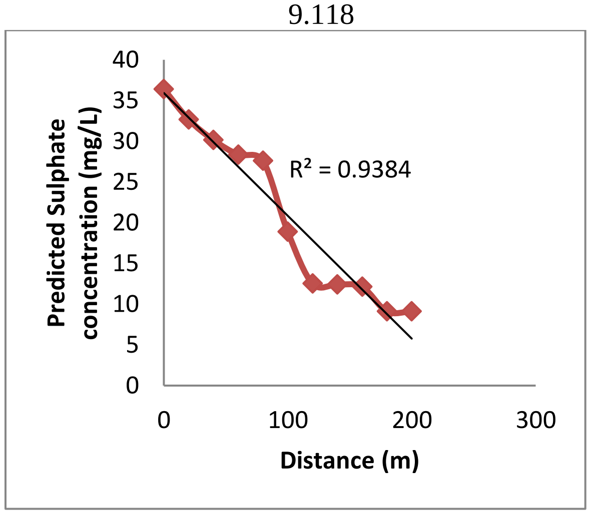
Table 3.0: LIC and the PC values of the sulphate

Distance (m)	LIC of SO_4^{2-} (mg/L)	PC of SO_4^{2-}concentration (mg/L)
0.00	36.400	36.400
20.00	32.670	32.670
40.00	30.150	30.149
60.00	28.360	28.359
80.00	27.580	27.580
100.00	18.870	18.871
120.00	12.520	12.523
140.00	12.400	12.409
160.00	12.100	12.124
180.00	9.040	9.098
200.00	9.000	9.118



3(a)

Fig. 3 (a): The plot of the analyzed sulphate concentration with distance



3(b)

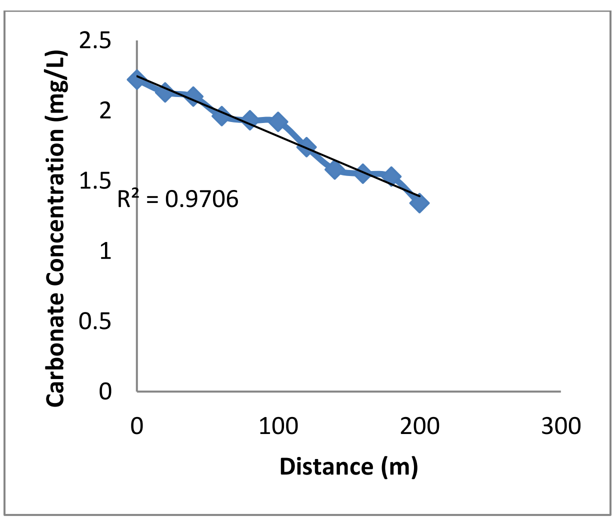
Fig. 3 (b): The plot of the predicted concentration of sulphate obtained from LIP with distance

The result and discussion of the concentration of carbonate obtained from LIP

Equation (4) was analyzed using Maple 18 and the results of the PC of the carbonate and the LIC of the carbonate are presented in Table 4.0. The plot of the comparison of the result of the LIC of the carbonate concentration and it PC is shown in Figure 4.0. The plot shows the gradual reduction in the carbonate concentration with an increase in the distance away from the centre of the cemetery. In figure 4(a) and 4(b), it was noted that there is strong correlation of $R^2 = 0.9706$ and $R^2 = 0.9701$ between the carbonate LIC result and the PC of the carbonate.

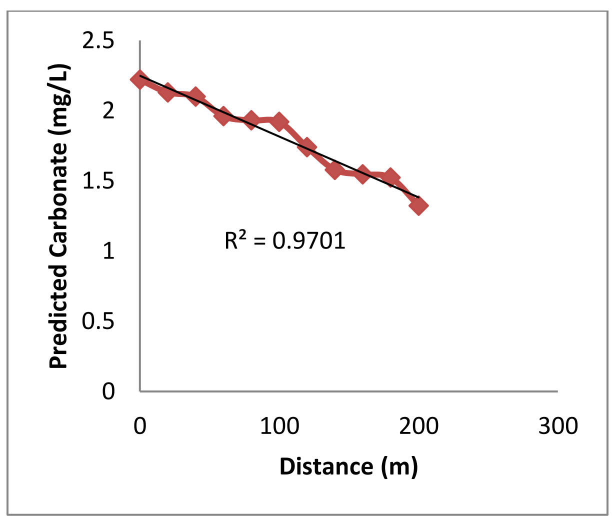
Table 4.0: LIC and the PC of the carbonate

Distance (m)	LIC of CO_3^{2-} (mg/L)	PC of CO_3^{2-} (mg/L)
0.00	2.220	2.220
20.00	2.130	2.130
40.00	2.100	2.100
60.00	1.960	1.959
80.00	1.930	1.929
100.00	1.920	1.919
120.00	1.740	1.739
140.00	1.580	1.579
160.00	1.550	1.548
180.00	1.530	1.527
200.00	1.340	1.345



4(a)

Fig. 4 (a): The plot of the analyzed carbonate concentration with distance



4(b)

Fig. 4 (b): The plot of the predicted concentration of carbonate obtained from LIP with distance

Result and Discussion of the combined concentration of the leachate

The results of the LIC of nitrate, sulphate, and carbonate were added to give the resultant concentration, this is because the combine leachate ionic concentration (CLIC) make-up the leachate content of the soil. The CLIC was substituted into LIP to obtain the general mathematical equation (GME) for the prediction of the leachate content in the soil, the result is as shown in equation (5).

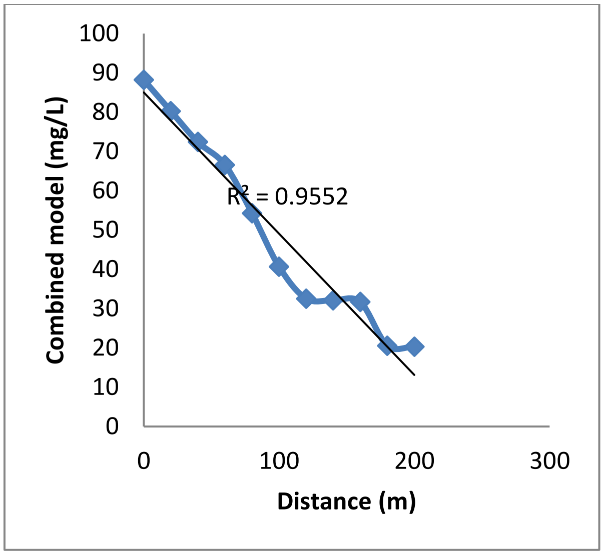
$$C_M = 1.54\times10^{-6}x^4 - 5.38\times10^{-4}x^3 + 5.22\times10^{-2}x^2 - 1.247x + 88.22 \quad (5)$$

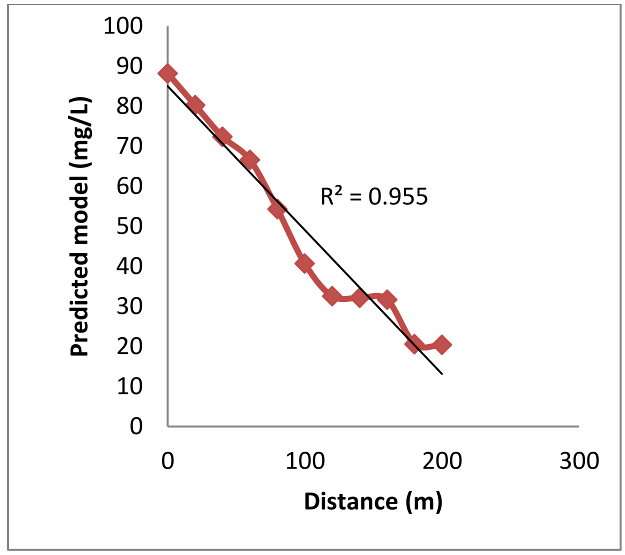
The result of the CLIC and the PC of the leachates are presented in Table 5.0, it was observed that the concentrations decrease with an increase in the distance from the centre of the cemetery.

Table 5.0: CLIC and the PC values of the carbonate

Distance (m)	CLIC (mg/L)	PC of Leachates (mg/L)
0.00	88.220	88.220
20.00	80.260	80.259
40.00	72.400	72.399
60.00	66.600	66.599
80.00	54.310	54.309
100.00	40.680	40.678
120.00	32.550	32.548
140.00	32.130	32.129
160.00	31.700	31.704
180.00	20.590	20.609
200.00	20.340	20.400

The plot of the CLIC and the PC of the leachates with distance are presented in Figures 5(a) and 5(a), it was deduced that is an insignificant difference in concentration. A strong correlation of the $R^2 = 0.9552$ and $R^2 = 0.9550$ was obtained for CLIC and the PC obtained from LIP. It was also noted that the CLIC and that of the PC fade away with an increase in the distance from the center of the cemetery. The outcome of the result of the PC model presented in equation (5), and the LIC shown in Table 5.0 and Figure (5) confirmed that LIP can be used to predict the leachate content migrating from the centre of the cemetery to its vicinity.





5(a) 5(b)

Fig. 5 (a): The plot of the combined leachate concentration with distance

Fig. 5 (b): The plot of the predicted leachate concentration obtained from LIP with distance

CONCLUSION

This paper presents the reliability of mathematical equations (ME) in the prediction of contaminants migration from a contaminated area. Lagrangian Interpolation Polynomials (LIP) was used to predict safe distance for citing hand-dug wells and boreholes in the cemetery vicinity. The models show a gradual reduction in the concentration level of the contaminants with an increase in the distance away from the centre of the cemetery. The coefficient correlation of the predicted models and the actual measurement shows a close relationship. It was deduced that groundwater from hand-dug wells and boreholes situated at distance of 240.0m is safe for drinking. The model can therefore be used to determine the concentration of leachate migrating from the cemetery vicinity and in extension, it can be applied to predict the leachate quantity migrating from a decompose site. However, it is recommended that more data from other locations be used to revalidate the performance of predicted models and other soil parameters such as porosity and permeability can be integrated into the LIP models to account for the leachate quantity from a selected soil type.

Declarations

''All authors have read, understood, and have complied as applicable with the statement on "Ethical responsibilities of Authors" as found in the Instructions for Authors and are aware that with minor exceptions, no changes can be made to authorship once the paper is submitted.''

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Availability of data and materials

All data generated or analysed during this study are included in this published article.

Ethics approval, guidelines and consent to participate

Not Applicable

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