The magnetic monopole problem of hot big bang model

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Abstract: Topological defects such as magnetic monopoles that may have formed during cosmological phase transitions in the early universe. The Kibble mechanisms govern the formation of this defects and their abundance is influenced by the correlation length and the temperature-dependent Higgs mass. The Magnetic monopoles are particularly interested but their excessive production poses the monopole problem which is challenge to the standard model. Many solutions including symmetry breaking patterns and thermal pair production have been proposed mitigated to the monopole problem. Detecting monopoles would offer valuable insights into high-energy physics and the early universe. Detection of (even) a single monopole could revolutionize our current knowledge of the universe.

Key words: Monopole, Cosmology, Problems of Hot Big Bang, GUT, Topological Defect

1. Introduction

The hot big bang theory is one of the acceptable explanations of the origin and evolution of our universe. It is started from the Planck epoch, at that time our universe was very hot and the density was very high. At that time, there was only a single force present from which all fundamental forces emerged and the space-time was as singularity. As the hot big bang happened, the universe was expanded and reached to the grand unification epoch[1]. During that epoch the separation of the strong force from weak forces occurred and in there happened the birth of monopole. This epoch was followed by the electroweak unification epoch, where the weak and electromagnetic forces were separated from each other, after that the formation of particles occurred called quark epoch which are the building blocks of ordinary matter. After the enough cooling of universe from the quark's epoch, the hadron formation started normally called hadron epoch. After that the lepton epoch, the formation of protons, neutrons, and atomic nuclei while electrons, positrons, and neutrinos played critical roles in the evolution of matter. The photon epoch is the last epoch, where the total decoupling of matter and radiation happened. From hot Big Bang until around 380,000 years after big bang, when the universe had cooled enough, the neutral atoms formed, allowing photons to travel freely and giving rise to the cosmic microwave background radiation[2]. The hot big bang theory, which is successful in many aspects, is facing number of problems. Some of the problems are the horizon problem, flatness problem, and the monopole problem.

1.2 Magnetic monopole problem.

The monopole problem poses a potential challenge to the hot big bang model. The problem is that the early universe might had sufficient magnetic monopoles-theoretical particles- with only one magnetic pole. These single pole magnets would have been created in significant quantities during the grand unification phase of the universe, these monopoles are envisioned to form during a phase transition, analogous to the way bubbles form during a boiling process. However, the observations we have not detected yet, number of these magnetic monopoles produced in the universe, due to absence of monopole the hot big bang model is facing the magnetic monopole problem. It is an open question that why these particles are so rare and not be detected[3]. This discrepancy between theory and observation poses the monopole problem. There are many approaches to solve the monopole problem but one of them is the Kibble mechanism, to solve the monopole problem involves the formation of topological defects known as cosmic strings. During the phase transition in early universe, when the breaking symmetry occur of the fundamental forces.

The expansion of cosmic strings stretches and thin significantly their energy density was reducing[4]. In the same way and their number density remains unchanged and the energy density of cosmic strings become negligible as compared to the energy density of magnetic monopoles after inflation was end. These phenomena resist the overabundance of magnetic monopoles explaining and that is why they have not been observed in the current universe. The problem of overabundance of the magnetic monopole shows the persistence and maintaining consistency with our observations of the universe today[5]. To look for the magnetic monopoles many scientists works and using different methods to reach the magnetic monopole.

2. Solution to monopole:

2.1. Classical Electromagnetic interpretation:

According to classical electromagnetism, the monopoles are not independent entities. According to Maxwell's equations.

$$\nabla \times \mathrm{E} = \frac{\partial \mathrm{B}}{\partial \mathrm{t}} \qquad (1)$$

$$\nabla . \mathrm{E} = 0 \qquad (2)$$

$$\nabla \times \mathrm{B} = -\frac{\partial \mathrm{B}}{\partial \mathrm{t}} \qquad (3)$$

$$\nabla . \mathrm{B} = 0 \qquad (4)$$

In equation (4) "the divergence of magnetic field is zero" also means that the magnetic flux through a closed surface is always zero. Which means that there is no monopole or magnetic charge and also describes the behavior of magnetic fields of magnetic dipoles and their interaction with electric and magnetic charges and fields.

2.2. GUT Interpretation of monopole:

The interpretation of magnetic monopoles in classical electromagnetism shows a clash with the monopole view of grand unified theories (GUTs) and quantum field theory. The occurrence of magnetic monopoles in universe is one of the main predictions of grand unified theory[6].

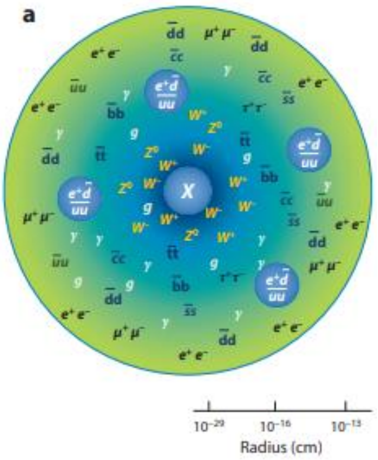


Figure a: This is the structure of GUT monopole, having different regions. The virtual X boson at the center ($r \sim 10^{-29}$cm), near to this the electroweak unification region present virtual $w^{\pm}$ and Z^0 boson at the region ($r \sim 10^{-16}$cm). After that followed by virtual photons, gluons, a fermion–antifermion condensate at the region($r \sim 10^{-13}$cm). For the Radius > 10^{-15}m having the field of point magnetic charge. $B = {}^{g}/_{r^2}$.[7]

The classical electromagnetic theory, despite being failed in predicting and providing details for magnetic monopoles, provides a good background for understanding the behavior of magnetic fields and their sources.

3. Monopole in Cosmology:

The Kibble mechanisms is one of the main concepts in cosmology which explain the formation of topological defects in phase transitions of early universe[8]. The Kibble mechanisms focus on variations in vacuum configurations of different phase transitions as the universe gradually cooled and undergone spontaneous symmetry-breaking phenomena. The correlation length of developing fields during the phase transitions become limited by the horizon of the particle. While these defects are non-perturbative in nature, and their formation during cosmological phase transitions is deemed inevitable[9]. The particle horizon refers to maximum possible distance traveled by a massless particle may have after the Big Bang is given by:

$$d_H = R(t) \int_0^t \frac{d(t')}{R(t')} \qquad (5)$$

$$\text{If } \; R > t^n \; (n > 1),\; d_H = \left(\frac{t}{1-n}\right)$$

The maximum possible distance over which the Higgs field can show correlationξ is called as correlation length. Correlation length is dependent on certain properties and temperature of each phase transition.

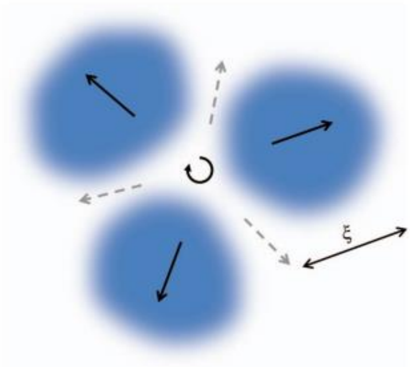


Figure 1: Take example of two dimensional in kibble mechanism, the randomness in Higgs field is longer thanξ. The Higgs field interpolates continuously between any pair of two domains but meet to three domains, it is unable to destroy in middle. When the topological defect formed, similarly in three dimensions, monopoles can be formed where four domains meet.[10]

The relation between correlation length and the temperature-dependent Higgs mass,$\xi \sim M_H^{-1}(T)$, is such that $M_H^{-1}(T)$ controls the properties of the correlation length of a phase transition[11]. The horizon distance is finite and Higgs field don't have correlation over horizon distance scale(d_H) during the phase transition (at $t = t_c$, T =Tc), (5). Let's consider monopoles as topological defect, the vacuum expectation value (VEV) of the Higgs field is (Φ^a) is supposed to be directed randomly over the range higher than the horizon distance (d_H). This indicates that in different volume the universe, the direction of (Φ^a) will experience variations, on the scale of the Hubble horizon. The change of (Φ^a) direction about a central point is known as a hedgehog configuration

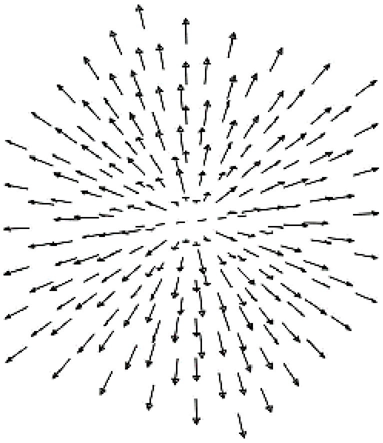


Figure 2: The hedgehog configuration, vector of Higgs field point away from origin in all direction and this configuration does not turned continuously into the uniform vacuum state.[10]

Therefore, it is acceptable to expect the monopole configuration probability as one, which arise from a Higgs (VEV) with uncorrelated directions above the Hubble horizon (H^{-1}). As a result, about one monopole (or antimonopole) is considered to be formed per horizon volume should be in order of $\mathrm{n_M \sim d_H^{-3} \sim \frac{T_c^6}{m_{pi}^3}}$. The density of entropy at critical temperature T_c is order of $T_c^3 \sim s$. So the ratio of monopole-to-entropy ($\frac{n_M}{s}$) can also be shown as $\left(\frac{T_c}{m_{pi}}\right)^3$, which shows that the monopoles number relative to the entropy density is found by the cube of the ratio of the critical temperature to the monopole mass ($\frac{T_c}{m_{pi}}$).

Supposing that the formation of monopoles and the ratio of monopoles to entropy don't change with time and the number of monopoles present in the universe could be determined by it. But this produce a major problem as the relic monopole abundance for canonical values of Grand Unified Theory (GUT) symmetry breaking, such as$T_c \sim 10^{14}$ GeV and $m_M \sim 10^{16}\,GeV$,$\frac{n_M}{s} \sim 10^{-13}$ represents a current monopole mass density of about $10^{11}\,\rho c$ times that of critical density. The given value is creating problems and hints about some major cosmological problems in cosmological phase transition. The phase transition cause rise in entropy which dilutes the initial monopole abundance, exponentially reducing the value of $\frac{n_M}{s}$.

4. Monopole at birth:

To calculate the number of magnetic monopoles formed during Kibble mechanism in phase transition, consider the correlation length. During phase transition, the correlation length is inversely proportional to the age of the Universe[12],

$$t \cong t_c \text{ , where } t_c \cong 0.3 g_*^{-1/2}\, m_{pi}/T_c^3$$

For an SU(5) grand unified theory (GUT), we have $T_c \sim 10^{14}$Gev temperature at the phase transition, $m_M \sim 10^{16}$Gev mass of the monopole and $t_c \sim 10^{14}$sec the age of the Universe at T_c, so taking the Correlation length = t_c^{-1}.

$$\frac{n_M}{s} \cong 10^2 \left(\frac{T_c}{m_{pi}}\right)^3 \cong 10^{-13} \qquad (6)$$

Let's consider the Preskill's conditions in order to approximate monopole abundance relative to the entropy density ($^{n_M}/_{s}$). According to him, the $^{n_M}/_{s} > 10^{-10}$, the annihilations of monopole-antimonopole significantly decrease the initial monopole abundance. So, in Kibble mechanism, the initial monopole abundance is less than10^{-10} , so we can ignore annihilations of monopole-antimonopole pair. Since T≅ T_c, we can estimate $^{n_M}/_{s}$ by considering adiabatic expansion. As entropy is conserved in adiabatic expansion. So;

$$\frac{n_M}{s} \approx n_M\left(\frac{T_c}{s(T_c)}\right), \qquad (7)$$

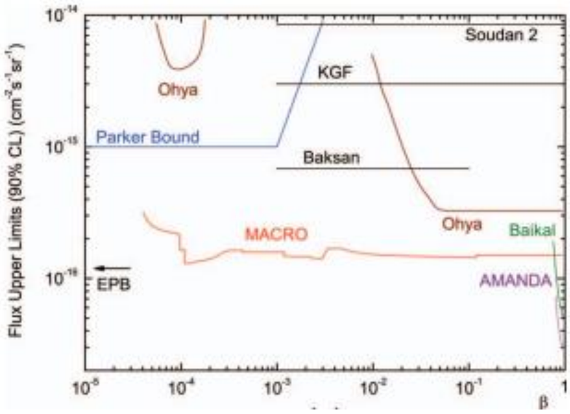
Where $n_M\,(T_c)$ is the number of monopole during phase transition and entropy density is represented by s(T_c). The specific value of n_M (T_c) arising from the Kibble mechanism. If there are more information or if we assume the value of n_M (T_c), then the estimated $\mathrm{n_M}/\mathrm{s}$ ratio can be calculated.

5. Flux of magnetic monopole:

The combination of a flux that is easy to detect and an unacceptably large mass density (unless T_c <<10^{14} GeV) results in a cosmic catastrophe known as the "Monopole Problem." The age of the Universe restricts $\Omega_0 h^2$ to be less than 1, which poses a significant challenge when attempting to reconcile the simplest Grand Unified Theories with standard cosmology[13]. This problem arises due to the excessive production of magnetic monopoles during the phase transition associated with GUTs, where T_c is the critical temperature. If T_c is not much less than 10^{14} GeV, the predicted abundance of magnetic monopoles would far exceed the limits set by the Universe's age, leading to a profound inconsistency between theory and observation driving ongoing research to find a compelling solution. The limit on Ω_M based on the age of the Universe implies T_c < 10^{11} GeV.

$$< F_M > \cong 10^{-3}\left(\frac{T_C}{10^{14}GeV}\right)^3\left(\frac{V_M}{10^{-3}c}\right) cm^{-2} sr^{-1} sec^{-1}$$
(8)

$$\Omega_M h^2 \cong 10^{11}\left(\frac{T_C}{10^{14}GeV}\right)^3\left(\frac{m_M}{10^{16}GeV}\right).$$
(9)



Graph 1: Flux of grand unified monopole which is less than 1.[10]

Additionally, if the forces were not unified, such as if the gauge group is G= SU(3) × SU(2) × U(1), or if the grand unification symmetry doesn't happen in the early Universe, there would have been no monopole. It is important to find the solution of the monopole problem, which provides us the valuable insight in physics at very high energy physics and the moments occurring in earliest Universe which offering a "window" to energy approximately equal to $10^{14}\,GeV$ and times approximately equal to 10^{-34} seconds[14].

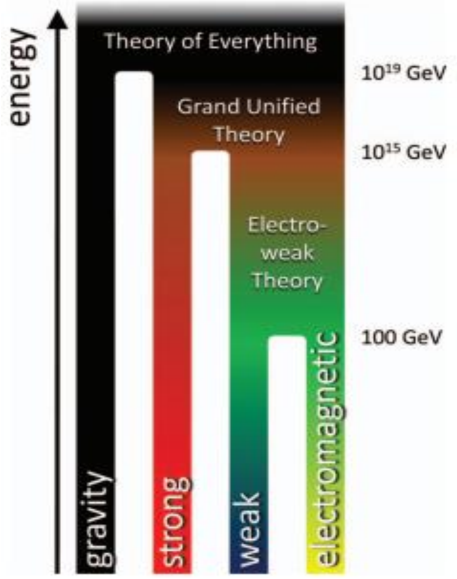


Figure 3: The Grand Unified Theory (GUT) at the scale of 15 GeV where the electroweak force unified to strong force. Below the GUT at the 100 GeV is the unification of electromagnetic force and weak force and above to GUT at 19 GeV combine all the fundamental forces called Theory of everything, which are may be a superstring or the M-theory. [10]

The deficiency of monopoles arise due to a specific symmetry breaking pattern: SU(5) → SU(3) × SU(2) × U(1) → SU(3) × U(1). The particular symmetry breaking phenomenon happens through a number of transitional phases in early universe. The first one occurs at $T_c \approx 10^{14}\,GeV$, the second phase transition happens at T_1, and the last one happen at T_2 . The important is the spontaneous breaking of U(1) gauge symmetry in intermediate epoch ($T_1 > T > T_2$) . The U(1) gauge symmetry corresponds to electromagnetism . This symmetry breaking brings the Universe to a superconducting phase in which magnetic flux becomes confined to flux tubes[15]. Due to this, magnetic monopoles and antimonopoles annihilate within these flux tubes, formed during the GUT transition. The remaining magnetic monopole abundance is about one per horizon volume at the end of superconducting phase. This indicate that approximately one magnetic monopole may exist in the whole observable Universe. A possible answer to the problem suggests that during the superconducting phase, the significant decrease in monopole abundance is due to the confinement of magnetic flux within flux tubes. Langacker and Pi made precise calculations for the abundance monopoles.

$$\frac{n_M}{s} \cong 10^2 \left(\frac{T_2}{m_{pi}}\right)^3 \cong 10^{-46} \left(\frac{T_2}{10^3 GeV}\right)^3$$

(10)

If the Kibble mechanism is not the efficient phenomenon for monopole production, the only possible production source is the formation of monopole-antimonopole pair due to the collisions of high energy particles. This happens due to the collision of highly energetic particles and antiparticles, causing a monopole - antimonopole to form. But the monopoles production through pair production is small because before the spontaneous symmetry breaking, monopole configurations do not exist in the theory. In SSB the temperature of phase transition is represented by T_c, and the monopoles mass is given by $m_M \cong \frac{M}{\alpha} \cong T_c$, where M represents the scale of SSB and is the scale factor.

The formation of magnetic monopoles through thermal pair production results in the relic monopole abundance. To calculate the abundance of monopole due to pair production, we need the detailed dynamics and energy scales involved in particle collisions. However, the important thing is that if the efficient formation of monopoles through the Kibble mechanism cannot happen, the relic monopole production due to the Kibble mechanism would be greater compared to the monopole production thermal pair production process.

$$\frac{n_M}{s} \cong 10^2 \left(\frac{m_M}{T_{max}}\right)^3 e^{-\left(2m_M/T_{max}\right)} \quad (11)$$

$T_{\max}$ represents the highest temperature after spontaneous symmetry breaking. The ratio $\left(\frac{m_M}{T_{\max}}\right)$ is expected to be at least 100, which means that the magnetic monopole mass (m_M) to the inverse of the highest temperature ($T_{\max}$) would be in order of $\Omega_M \le 10^{-40}$ (9). Due to this, a relic monopole density (F_M) on the order of $10^{-32} \times cm^{-2} \times sr^{-2} \times \sec^{-1}$ (8), which represents a rare number of monopoles. However, the ratio $\left(\frac{m_M}{T_{\max}}\right)$ exponentially effect the number of monopoles formed due to thermal production.

6. Conclusion:

After the happening of hot big bang and occurring of some transitional phases. Grand unifications one of the phases during which the monopoles were produced. Monopoles problem was raised during the Kibble mechanism of cosmological phase transitions. The topological defects are an important concept and aspect of particle physics and cosmology. After the symmetry-breaking processes when the universe cooled enough, various vacuum configuration regions unlinked due to the finite speed of light and thus monopole formation halted. These theoretical entities have single magnetic charge and have potential challenge for our understanding of subatomic interactions of standard model of particle physics. However, the theoretical interpretation, existence and abundance of monopole are beyond the cosmological constraints, thus creates a huge challenge to unite theoretical knowledge with observations.

Some possible solutions have been put forwarded to the monopole problem like intermediate superconducting phase during which the initial monopole was significantly reduced. The monopoles observation and detection would lead to valuable information of the very first time of the universe and various phenomenon during that time. Despite being very important in theoretical physics, it is very challenging to detect monopoles due to its rarity in universe. The search for detecting a super heavy monopole remains a very hot topic for researchers because its discovery can advance our knowledge of the early universe and fundamental physics.

The magnetic monopoles formed due the Kibble mechanism during the phase transitions in early universe are considered as mysterious cosmological entities with vast implication for particle physics and cosmology. The detection and observation of monopoles is very challenging and provide the potential to unlock new areas in understanding the early universe and the fundamental nature of the cosmos.

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