Application of q-Rung Orthopair Fuzzy Entropy Measures to Multicriteria Decision Making and Medical Diagnosis

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Abstract

The notion of generalized q-rung Orthopair fuzzy sets (q-ROFSs) was introduced by Yager in 2016, which gives a new way to model uncertainty and vagueness with more precision and correctness than intuitionistic fuzzy sets (IFSs) and Pythagorean fuzzy sets (PFSs) respectively. The idea was concretely designed to characterize uncertainty and vagueness in mathematical approach and to demonstrate a formalized way for modeling uncertainty to real world problems. In this manuscript, we propose novel entropy measures for q-ROFSs and then establish an axiomatic definition for suggested entropy measures for q-ROFSs. This gives the decision makers more confident in conveying their belief about membership grade thus q-ROFSs increases the space of acceptance in the uncertain environment. Several examples are discussed to exhibit suitability and reliability of suggested methods particularly for selecting the superlative one/ones in structured linguistic variables. Moreover, Orthopairian Fuzzy Technique for ordering preference by similarity to ideal solution (OF-TOPSIS) based on the suggested entropy measures is developed. Finally, we utilized our suggested OF-TOPSIS technique to calculate the criteria weights and obtain a raking of alternatives to treat with the problems relating complex multi criteria decision making processes amicably. We also developed an application of proposed entropy measure related to Covid-19. Numerical examples illustrate the practicality, validity and applicability of the suggested methods.

Key Words: Fuzzy sets, Intuitionistic Fuzzy Sets, Pythagorean Fuzzy Sets, q-Rung Orthopair Fuzzy Sets, Entropy Measures, TOPSIS, Multicriteria Decision Making.

1. Introduction

Fuzzy set and its generalizations are being used in almost every area of science and technology including mathematics, economics, business management, computer science, artificial intelligence, pattern recognition and robotics etc. The term fuzzy sets (FSs) were first coined by the famous mathematician Zadeh [1]. The fuzzy sets play a vital role to compute uncertainties in more influential and reasonable way than classical crisps sets. Fuzzy sets give us an important way to express imprecise concepts in natural language. The majority of the systems are based on classical set theory but they are somehow inconvenient or inadequate in dealing with imprecise and vague information amicably. For instance, in set theory an element can either belong to a set or not. Whereas, fuzzy sets model vague, imprecise and uncertain information are associated to real life with high accuracy and precision. The theory of fuzzy logic gives us a mathematical strength to compute the uncertainties linked with human cognitive processes, such as thoughts and reasoning. Fuzzy sets, their extension and generalization have various applications in almost every field connected to daily life situations such as pattern recognition, image processing, decision making, water quality, mathematical programming, clustering and so on. The classification of fuzzy set was based on membership grades in the unit interval $[0,1]$ and the non-membership grade degree is obtained by one minus membership grade.

Traditionally, entropy was used to measure the disordering of a system. In this modern era, fuzzy entropy is being used to measure the fuzziness of a FS. De Luca and Termini [2], pioneer of the axiomatic structure for entropy of FSs with suggestion to Shannon's probability entropy. Fuzziness measures of FSs in terms of lack of distinction between the fuzzy set and its compliment based on l_p norm is given by Yager [3]. IVFSs based on normal forms are suggested by Turksen and Burhan [4]. A measure of fuzziness between FSs by means of a ratio of distance between the FS and its nearest set to the distance between the FSs and its farthest set are provided Kosko [5]. Some axiomatic definitions of entropy and σ-entropy are given by Liu [6]. The idea of exponential entropies is given by Pal and Pal [7]. Fuzzy sets are also useful to construct distance or dissimilarity between two objects. Distances between fuzzy sets are given by Rosenfel [8] and fuzzy Hamming distance: a new dissimilarity measure is proposed by Bookstein et al. [9].

Since the idea of FSs was based on the degree of membership and the degree of non-membership grade are obtained by one minus membership grade. On the other hand, in real, it may not for all time be correct that the degree of non-membership of an element in a fuzzy set is 1 minus the membership degree because there might be some uncertainty degree. Therefore, Atanassov [10] generalized the idea of FS theory and introduced the notion of intuitionistic fuzzy sets. The characterization of IFSs Atanassov [11] is based on the degree of membership denoted by μ and the degree of non-membership represented by ν such that $\mu + \nu \leq 1$ lies in $[0,1]$. The degree of indeterminacy is represented by π and calculated as $\pi = 1 - (\mu + \nu)$. Hence, it is obvious that $\mu + \nu + \pi = 1$. Previous studies reveal that the IFSs model uncertainty better than the FSs but this new development didn't reduce the importance of FSs. With the passage of time many extensions and generalizations of fuzzy sets have been made by researchers, like Belief and Plausibility Measures on IFSs with construction of Belief-Plausibility TOPSIS is proposed by Yang and Hussain [12], IVFSs Atanassov and Gargov [13], Entropy measures for IFSs are suggested by Szmidt and Kacprzyk [14], Entropy for IFSs and IVFSs suggested by Burillo and Bustince [15], Entropy on IFSs is put forwarded by Hung and Yang [16]. Another candid approach of fuzzy sets called hesitant fuzzy sets (HFSs) initially developed Torra and Narukawa [17] and Torra [18] which allow a possible set of values for each x in universal set X. Distance and similarity measures of HFSs suggested by Yang and Hussain [19]. Entropy for hesitant fuzzy sets with construction of HF-TOPSIS is suggested by Hussain and Yang [20]. Another amazing generalization of FSs, the Pythagorean fuzzy sets (PFSs), was proposed by Yager [21]. Yager & Abbasov [22] states that the sum of squares of membership grade and non-membership grades lies in [0, 1] called Pythagorean fuzzy sets. In PFSs, the membership values are ordered pairs (μ, ν) such that $\mu^2 + \nu^2 \leq 1$. The freedom of all Pythagorean membership values (PMVs) contains intuitionistic membership values (IMVs). For illustration, the condition with the numbers $\mu = 0.86603$ and $\nu = 0.5$ in such situations IFSs cannot be used but PFSs can handle this situation amicably. This is because $\mu + \nu = 1.36603 > 1$. On the other hand, we can use PFSs as $\mu^2 = 0.75$ and $\nu^2 = 0.25$ because the condition $\mu^2 + \nu^2 = 1$ is fulfilled. Thus, we may intuitionally say that PFSs are much wider than IFSs in solving daily life problems. A lot of researchers are actively busy in the progress of this new generalization of fuzzy set theory. Yager [23] gave Pythagorean membership grades in MCDM. Distance and similarity measures of PFSs are suggested by Hussain and Yang [23]. Pythagorean TODIM method for MCDM was given by Ren et al. [24]. Extension of TOPSIS to MCDM with PFSs is given by Zhang and Xu [25]. Pythagorean LINMAP technique supported on entropy for the railway project investment decision making was suggested by Xue et al. [26]. Fuzzy entropy for PFSs with application to MCDM is given by Yang and Hussain [27].

Most recently, a new and stunning generalization of FSs, the q-rung Orthopair fuzzy sets (q-ROFSs) was proposed by Yager [28] is more flexible and comparatively covered much larger space than IFSs and PFSs respectively. Hence, q-ROFS is the generalization of both IFSs and PFSs respectively. The q-ROFSs are illustrated by membership degree μ and non-membership degree ν and degree of hesitancy π such that $\mu^q+\nu^q+\pi^q=1$. In this sense, the generalized q-ROFSs are extra capable than IFSs and PFSs respectively in handling vague and uncertain information related to daily life settings. For example, if $\mu=0.9$ and $\nu=0.5$ then such problem can neither be described by IFSs $0.9+0.5>1$ nor by PFSs $(0.9)^2+(0.5)^2>1$. These types of problems can easily be tackled by q-ROFSs because $(0.9)^3+(0.5)^3<1$. It demonstrate that q-ROFS are extra flexible than IFS and PFS respectively. We can change the value of the parameter q to settle on the information expression range and hence q-ROFSs are extra adjustable and high desirable for the uncertain environment. With this new generalization, many researchers are engaged in the development of q-ROFSs and its applications such as Minkowski-type distance measures for generalized orthopair fuzzy sets are given by Du [29]. Some q-ROF Bonferroni mean operators by application to MAGDM is forwarded by Liu and Liu [30]. Some q-ROF Heronian mean operators in MADM introduced by Wei et al. [31]. Similarity measures of q-ROFSs on cosine function and its applications are given by Ping et al. [32]. Information measures for q-ROFSs are suggested by Peng and Liu [33]. Some new partitioned Bonferroni means operators under q-ROF environment presented by Wei and Pang [34]. Some Dombi aggregations of q-ROFNs and their applications in MADM are presented by Jana et al. [35]. Some q-ROF point weighted aggregation operators for MADM by Xing et al. [36]. A q-ROFMCGDM technique for supplier selection based on a new distance measure is proposed by Adem et al. [37].

The rest of this manuscript is set as follow. In section 2, we discuss various well-known fundamental ideas about q-ROFSs. In section 3, we proposed some novel entropy measures for q-ROFSs and introduce an axiomatic definition of the notion of entropy of q-ROFSs. We establish some numerical examples to illustrate the validity of our suggested entropy measures. We also expound how the uncertainty within q-ROFSs can be measured using our suggested entropy measures. In section 4, we developed a novel q-rung Orthopairian TOPSIS based on the suggested entropy measures. We then apply our suggested q-rung Orthopairian

TOPSIS to daily life problems with Covid-19 involving complex multicriteria decision making processes to rank the alternatives in preferred order. We wind up our discussion in section 5.

2. Preliminaries

In this section, we discuss fundamental definitions and operations of q-ROFSs.

Definition 1. A fuzzy set $\tilde{F}$ in the fixed set X is defined by Zadeh [1] as:

$$\tilde{F} = \left\{ \left(x, \mu_{\tilde{F}}\left(x \right) \right);\ x \in X \right\}$$

Where $\mu_{\tilde{F}}(x) : X \rightarrow \left[0,1\right]$ is called membership function and $0 \leq \mu_{\tilde{F}}\left(x\right) \leq 1.$

Definition 2. An intuitionistic fuzzy set (IFS) $\tilde{S}$ in the fixed set X is proposed by Atanassov [10] as:

$$\tilde{S} = \left\{ \left(x, \mu_{\tilde{S}}\left(x \right), \nu_{\tilde{S}}\left(x \right) \right); x \in X \right\}$$

Such that $\mu_{\tilde{S}}\left(x\right) + \upsilon_{\tilde{S}}\left(x\right) \leq 1,$ where $\mu_{\tilde{S}}(x) : X \rightarrow \left[0,1\right]$ and $\upsilon_{\tilde{S}}(x) : X \rightarrow \left[0,1\right]$ called membership grades and non-membership grades of x to $\tilde{S}$ respectively. The degree of indeterminacy or uncertainty of x to $\tilde{S}$ is denoted by $\pi_{\tilde{S}}\left(x\right) = 1 - \mu_{\tilde{S}}\left(x\right) - \upsilon_{\tilde{S}}\left(x\right).$

Definition 3. A Pythagorean fuzzy set (PFS) $\tilde{G}$ in the fixed set X proposed by Yager [21] with the following mathematical construction:

$$\tilde{G} = \left\{ \left(x, \mu_{\tilde{G}}(x), \upsilon_{\tilde{G}}(x) \right);\ x \in X \right\}$$

Such that $\mu_{\tilde{G}}^2(x) + \upsilon_{\tilde{G}}^2(x) \leq 1,$ where $\mu_{\tilde{G}}(x) : X \rightarrow \left[0,1\right]$ and $\upsilon_{\tilde{G}}(x) : X \rightarrow \left[0,1\right]$ called membership grades and non-membership grades of x to $\tilde{Y}$ respectively. The degree of indeterminacy or uncertainty of x to $\tilde{G}$ is obtained by $\pi_{\tilde{G}}(x) = \sqrt{1 - \mu_{\tilde{G}}^2(x) - \upsilon_{\tilde{G}}^2(x)}$. Obviously, $0 \leq \pi_{\tilde{G}}\left(\mathrm{x}\right) \leq 1.$

Definition 4. A q-rung orthopair fuzzy set (q-ROFS) $\tilde{Q}$ in X is defined by Yager [28] with the below mathematical formation:

$$\tilde{Q} = \left\{ \left(x, \mu_{\tilde{Q}}(x), \upsilon_{\tilde{Q}}(x) \right) \right\}$$

Such that $\mu_{\tilde{Q}}^{q}(x)+\upsilon_{\tilde{Q}}^{q}(x)\leq 1$, $q>2$ where $\mu_{\tilde{Q}}(x):X\rightarrow[0,1]$ and $\upsilon_{\tilde{Q}}(x):X\rightarrow[0,1]$ called membership grades and non-membership grades of $x\in X$ to $\tilde{Q}$ respectively. The degree of indeterminacy or uncertainty is calculated as $\pi_{\tilde{Q}}(x)=\sqrt[q]{1-\mu_{\tilde{Q}}^{q}(x)-\upsilon_{\tilde{Q}}^{q}(x)}$. The q-ROFSs are the generalization of both IFSs and PFSs respectively.

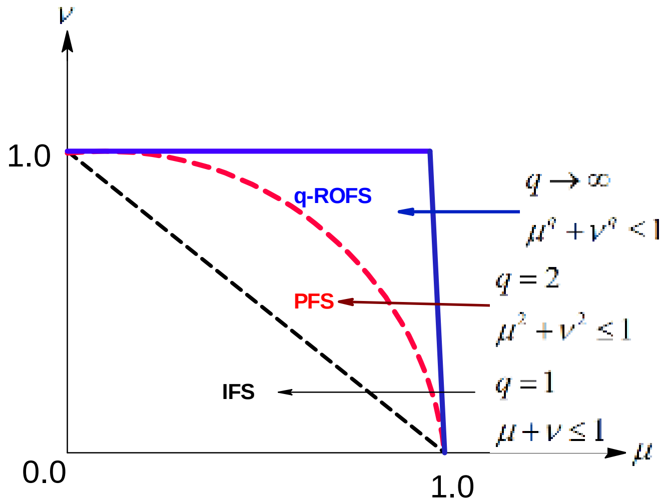

Fig 1. Space of acceptance of generalization of fuzzy sets.

Remark 1. If $q=1$ in q-ROFS $\tilde{Q}$ in Definition 4 reduces to an IFS.

Remark 2. If $q=2$ in q-ROFS $\tilde{Q}$ in Definition 4 reduces to a PFS.

Hence every IFS and PFS grades are also q-ROFS but converse is not necessary true.

Definition 5 [32]. Consider two q-ROFSs $\tilde{A}$ and $\tilde{B}$ on the finite discrete universe of discourse $X=\{x_1,x_2,...,x_n\}$ then

1) $\tilde{A}U\tilde{B}=\left\{\left(x,Max\left(\mu_{\tilde{A}}^{q}(x),\mu_{\tilde{B}}^{q}(x)\right),Min\left(\upsilon_{\tilde{A}}^{q}(x),\upsilon_{\tilde{B}}^{q}(x)\right)\right)\right\}$;

2) $\tilde{A}\cap\tilde{B}=\left\{\left(x,Min\left(\mu_{\tilde{A}}^{q}(x),\mu_{\tilde{B}}^{q}(x)\right),Max\left(\upsilon_{\tilde{A}}^{q}(x),\upsilon_{\tilde{B}}^{q}(x)\right)\right)\right\}$;

3) $\tilde{A}^c=\left\{\left(x,\upsilon_{\tilde{A}}^{q}(x),\mu_{\tilde{A}}^{q}(x)\right):x\in X\right\}$;

4) $\tilde{A}\leq\tilde{B}$ *if* and *if* $\forall\ x\in X$, $\mu_{\tilde{A}}^{q}(x)\leq\mu_{\tilde{B}}^{q}(x)$ and $\upsilon_{\tilde{A}}^{q}(x)\geq\upsilon_{\tilde{B}}^{q}(x)$;

5) $\tilde{A}=\tilde{B}$ *if* and *if* $\forall\ x\in X$, $\mu_{\tilde{A}}^{q}(x)=\mu_{\tilde{B}}^{q}(x)$ and $\upsilon_{\tilde{A}}^{q}(x)=\upsilon_{\tilde{B}}^{q}(x)$.

3. New Fuzzy Entropy Measures for q-Rung Orthopair Fuzzy Sets

In this section, first we give the axiomatic definition of entropy for q-ROFSs. Since the q-ROFSs introduced by Yager [28] which is the generalization of both IFSs and PFSs respectively, therefore, we may use similar notion to define the entropy of q-ROFSs. Assume that the q-ROFS(X) denotes the set of all q-ROFSs in X.

Definition 6. A function $E: q-ROFS\left(X\right)\to\left[0,1\right]$ is known as entropy on X if E fulfills the following axioms;

$\left(E_1\right): 0\le E\left(\tilde{A}\right)\le 1;$

$\left(E_2\right): E\left(\tilde{A}\right)=0, \mathit{iff}\ \tilde{A}$ is crisp set;

$\left(E_3\right): E\left(\tilde{A}\right)=1, \mathit{iff}\ \mu_{\tilde{A}}^{q}(x_i)=\upsilon_{\tilde{A}}^{q}(x_i), \forall\ x_i\in X;$

$\left(E_4\right): E\left(\tilde{A}\right)\le E\left(\tilde{B}\right)\ \mathit{iff}\ \tilde{A}$ is less fuzzy than $\tilde{B}$, i.e.,

$\forall\ x\in X,\ \mu_{\tilde{A}}^{q}(x_i)\le\mu_{\tilde{B}}^{q}(x_i)$ and $\upsilon_{\tilde{A}}^{q}(x_i)\ge\upsilon_{\tilde{B}}^{q}(x_i)\ \mathit{for}\ \mu_{\tilde{B}}^{q}(x_i)\le\upsilon_{\tilde{B}}^{q}(x_i)$

or $\mu_{\tilde{A}}^{q}(x_i)\ge\mu_{\tilde{B}}^{q}(x_i)$ and $\upsilon_{\tilde{A}}^{q}(x_i)\le\upsilon_{\tilde{B}}^{q}(x_i)$ for $\mu_{\tilde{B}}^{q}(x_i)\ge\upsilon_{\tilde{B}}^{q}(x_i);$

$\left(E_5\right): E\left(\tilde{A}\right)=E\left(\tilde{A}^c\right)$, where $\tilde{A}^c=\left\{\left(x,\upsilon_{\tilde{A}}^{q}\left(x\right),\mu_{\tilde{A}}^{q}\left(x\right)\right): x\in X\right\}.$

The idea to find out the vagueness from a fuzzy set and its negation were introduced very first time by Yager [3]. Here, we apply the parallel notion to compute the uncertainty of q-ROFSs on the basis of distance measure between a q-ROFS $\tilde{A}$ and its complement $\tilde{A}^c$. In ranking alternatives using an algorithm such as TOPSIS, we use distance measure to find the distance between every alternative to positive ideal solution and negative ideal solution respectively. Therefore, we put forward the following distance measure between a q-ROFS and its complement. Let us consider $X=\left\{x_1,x_2,...,x_n\right\}$ be a fixed discrete universe of discourse then the distance between two q-ROFSs $\tilde{A}$ and $\tilde{B}$ is defined as:

$$D_\eta\left(\tilde{A},\tilde{B}\right)=\left[\frac{1}{2n}\left(\sum_{i=1}^{n}\left(\left|\mu_{\tilde{A}}^{q}(x_i)-\mu_{\tilde{B}}^{q}(x_i)\right|^{q}+\left|\nu_{\tilde{A}}^{q}(x_i)-\nu_{\tilde{B}}^{q}(x_i)\right|^{q}+\left|\pi_{\tilde{A}}^{q}(x_i)-\pi_{\tilde{B}}^{q}(x_i)\right|^{q}\right)\right)\right]^{\frac{1}{q}} \quad (1)$$

Usually, in many practical life setting applications and ranking of alternatives weight vector $\tilde{w}$ of the member $x\in X$ is considered. Therefore, we assign weights in (1) and create weighted distance measure for q-ROFSs. Assume that the weight of every element $x_i\in X$ is $\tilde{w}_i\left(i=1,2,3,\ldots,n\right)$ such that $\sum_{i=1}^{n}\tilde{w}_i=1$, where $0\le\tilde{w}_i\le1$, then the generalized weighted Hausdorff distance measure is defined as follows:

$$D_\eta\left(\tilde{A},\tilde{B}\right)=\left[\frac{1}{2}\left(\sum_{i=1}^{n}w_i\left(\left|\mu_{\tilde{A}}^{q}(x_i)-\mu_{\tilde{B}}^{q}(x_i)\right|^{q}+\left|\nu_{\tilde{A}}^{q}(x_i)-\nu_{\tilde{B}}^{q}(x_i)\right|^{q}+\left|\pi_{\tilde{A}}^{q}(x_i)-\pi_{\tilde{B}}^{q}(x_i)\right|^{q}\right)\right)\right]^{\frac{1}{q}} \quad (2)$$

If we replace $\tilde{B}$ by $\tilde{A}^c$ in (1), then (1) reduces to distance between $\tilde{A}$ and its complement $\tilde{A}^c$ as

$$D_\eta\left(\tilde{A},\tilde{A}^c\right)=\left[\frac{1}{n}\left(\sum_{i=1}^{n}\left|\mu_{\tilde{A}}^{q}(x_i)-\nu_{\tilde{A}}^{q}(x_i)\right|^{q}\right)\right]^{\frac{1}{q}} \quad (3)$$

Based on above analysis, we utilize (3) to propose new entropy for q-ROFSs as

$$e_{m1}\left(\tilde{A}\right)=1-D_\eta\left(\tilde{A},\tilde{A}^c\right)=1-\left[\frac{1}{n}\left(\sum_{i=1}^{n}\left|\mu_{\tilde{A}}^{q}(x_i)-\nu_{\tilde{A}}^{q}(x_i)\right|^{q}\right)\right]^{\frac{1}{q}} \quad (4)$$

Theorem 1. Let $X=\{x_1,x_2,\ldots,x_n\}$ be a fixed discrete universe of discourse then the suggested entropy $e_{m1}\left(\tilde{A}\right)$ for q-ROFSs fulfills the axioms $\left(E_1\right)$ to $\left(E_5\right)$ in definition 6.

Proof: We first prove axiom $\left(E_1\right)$. As the distance $D_\eta\left(\tilde{A},\tilde{A}^c\right)$ lies in the unit interval $[0,1]$. So $0\le 1-D_\eta\left(\tilde{A},\tilde{A}^c\right)\le 1$. This shows that the quantity $0\le E\left(\tilde{A}\right)\le 1$, also lies in $[0,1]$. Hence, the axiom $\left(E_1\right)$ is fulfilled. Next, we prove axiom $\left(E_2\right)$, if $\tilde{A}$ is crisp then $\mu_{\tilde{A}}^{q}(x_i)=1$ and $\nu_{\tilde{A}}^{q}(x_i)=0$ or $\mu_{\tilde{A}}^{q}(x_i)=0$ and $\nu_{\tilde{A}}^{q}(x_i)=1$ in both cases $\left[\frac{1}{n}\sum_{i=1}^{n}\left(\left|\mu_{\tilde{A}}^{q}(x_i)-\nu_{\tilde{A}}^{q}(x_i)\right|^{q}\right)\right]^{\frac{1}{q}}=1$. Thus, $e_{m1}\left(\tilde{A}\right)=1-1=0$. Conversely, if $e_{m1}\left(\tilde{A}\right)=0$, implies that $1-D_\eta\left(\tilde{A},\tilde{A}^c\right)=0$ or

$D_\eta\left(\tilde{A},\tilde{A}^c\right)=1$, or $\left[\frac{1}{n}\sum_{i=1}^{n}\left(\left|\mu_{\tilde{A}}^q(x_i)-\nu_{\tilde{A}}^q(x_i)\right|^q\right)\right]^{\frac{1}{q}}=1$ i.e., $\left|\mu_{\tilde{A}}^q(x_i)-\nu_{\tilde{A}}^q(x_i)\right|=1$, which is possible either $\mu_{\tilde{A}}^q(x_i)=1$ and $\nu_{\tilde{A}}^q(x_i)=0$ or $\mu_{\tilde{A}}^q(x_i)=0$ and $\nu_{\tilde{A}}^q(x_i)=1$ which shows that $\tilde{A}$ is crisps set. This proves the axiom (E_2). Now, we give the proof of axiom (E_3), let $\mu_{\tilde{A}}^q(x_i)=\nu_{\tilde{A}}^q(x_i)$ then $\left[\frac{1}{n}\sum_{i=1}^{n}\left(\left|\mu_{\tilde{A}}^q(x_i)-\nu_{\tilde{A}}^q(x_i)\right|^q\right)\right]^{\frac{1}{q}}=0$, so $e_{m1}\left(\tilde{A}\right)=1-0$, $e_{m1}\left(\tilde{A}\right)=1$. Conversely, $e_{m1}\left(\tilde{A}\right)=1$ implies that $1-\left[\frac{1}{n}\sum_{i=1}^{n}\left(\left|\mu_{\tilde{A}}^q(x_i)-\nu_{\tilde{A}}^q(x_i)\right|^q\right)\right]^{\frac{1}{q}}=1$ then $\left[\frac{1}{n}\sum_{i=1}^{n}\left(\left|\mu_{\tilde{A}}^q(x_i)-\nu_{\tilde{A}}^q(x_i)\right|^q\right)\right]^{\frac{1}{q}}=0$, means that

$\left|\mu_{\tilde{A}}^q(x_i)-\nu_{\tilde{A}}^q(x_i)\right|=0$, i.e., $\mu_{\tilde{A}}^q(x_i)=\nu_{\tilde{A}}^q(x_i)$ .Thus, axiom (E_3) is satisfied. Next, we prove axiom (E_4), since we have $\mu_{\tilde{A}}^q(x_i)\le\mu_{\tilde{B}}^q(x_i)$ and $\nu_{\tilde{A}}^q(x_i)\ge\nu_{\tilde{B}}^q(x_i)$ for $\mu_{\tilde{B}}^q(x_i)\le\nu_{\tilde{B}}^q(x_i)$, which implies that $\mu_{\tilde{A}}^q(x_i)\le\mu_{\tilde{B}}^q(x_i)\le\nu_{\tilde{B}}^q(x_i)\le\nu_{\tilde{A}}^q(x_i)$ thus $\left[\frac{1}{n}\sum_{i=1}^{n}\left(\left|\mu_{\tilde{A}}^q(x_i)-\nu_{\tilde{A}}^q(x_i)\right|^q\right)\right]^{\frac{1}{q}}\ge$ $\left[\frac{1}{n}\sum_{i=1}^{n}\left(\left|\mu_{\tilde{B}}^q(x_i)-\nu_{\tilde{B}}^q(x_i)\right|^q\right)\right]^{\frac{1}{q}}$. Also, $\mu_{\tilde{A}}^q(x_i)\ge\mu_{\tilde{B}}^q(x_i)$ and $\nu_{\tilde{A}}^q(x_i)\le\nu_{\tilde{B}}^q(x_i)$ for $\mu_{\tilde{B}}^q(x_i)\ge\nu_{\tilde{B}}^q(x_i)$, which implies that $\nu_{\tilde{A}}^q(x_i)\le\nu_{\tilde{B}}^q(x_i)\le\mu_{\tilde{B}}^q(x_i)\le\mu_{\tilde{A}}^q(x_i)$, thus

$\left[\frac{1}{n}\sum_{i=1}^{n}\left(\left|\mu_{\tilde{A}}^q(x_i)-\nu_{\tilde{A}}^q(x_i)\right|^q\right)\right]^{\frac{1}{q}}\ge\left[\frac{1}{n}\sum_{i=1}^{n}\left(\left|\mu_{\tilde{B}}^q(x_i)-\nu_{\tilde{B}}^q(x_i)\right|^q\right)\right]^{\frac{1}{q}}$. From the above inequalities, $D_\eta\left(\tilde{A},\tilde{A}^c\right)\ge D_\eta\left(\tilde{B},\tilde{B}^c\right)$, i.e., $1-D_\eta\left(\tilde{A},\tilde{A}^c\right)\le 1-D_\eta\left(\tilde{B},\tilde{B}^c\right)$ or $e_{m1}\left(\tilde{A}\right)\le e_{m1}\left(\tilde{B}\right)$. Finally, we give the proof of axiom (E_5), we have $e_{m1}\left(\tilde{A}\right)=1-\left[\frac{1}{n}\sum_{i=1}^{n}\left(\left|\mu_{\tilde{A}}^q(x_i)-\nu_{\tilde{A}}^q(x_i)\right|^q\right)\right]^{\frac{1}{q}}$

$$=1-\left[\frac{1}{n}\sum_{i=1}^{n}\left(\left|\nu_{\tilde{A}}^q(x_i)-\mu_{\tilde{A}}^q(x_i)\right|^q\right)\right]^{\frac{1}{q}}=e_{m1}\left(\tilde{A}^c\right).$$

This completes the proof of axiom (E_5). □

We proposed next a very simple and novel way to calculate entropy of a q-ROFS. Let us consider $X=\{x_1,x_2,...,x_n\}$ be a finite universe of discourse and $\tilde{A}$ be a q-ROFS on X then a new entropy measure of $\tilde{A}$ is define as

$$e_{m2}\left(\tilde{A}\right)=1-\frac{2}{n}\sum_{i=1}^{n}\frac{\left|\mu_{\tilde{A}}^{q}(x_i)-\nu_{\tilde{A}}^{q}(x_i)\right|}{1+\left|\mu_{\tilde{A}}^{q}(x_i)-\nu_{\tilde{A}}^{q}(x_i)\right|} \tag{5}$$

Finally, we propose new and intuitive entropy for q-ROFSs based on the quotient of min and max operations. Let us consider $X=\{x_1,x_2,...,x_n\}$ be a fixed and $\tilde{A}$ be a q-ROFS on X, then a new entropy measure of $\tilde{A}$ is define as

$$e_{min/max}\left(\tilde{A}\right)=\frac{1}{n}\sum_{i=1}^{n}\frac{min\left(\mu_{\tilde{A}}^{q}(x_i),\nu_{\tilde{A}}^{q}(x_i),\pi_{\tilde{A}}^{q}(x_i)\right)}{max\left(\mu_{\tilde{A}}^{q}(x_i),\nu_{\tilde{A}}^{q}(x_i),\pi_{\tilde{A}}^{q}(x_i)\right)} \tag{6}$$

Theorem. 2 Suppose that $X=\{x_1,x_2,...,x_n\}$ be a fixed set, the proposed entropy $e_{min/max}\left(\tilde{A}\right)$ fulfills the axioms $(E_1)-(E_5)$ in Definition 6.

Proof: Proof is analogous to theorem 1. □

Example 1. Consider $\tilde{A}=\{(x,0.6520,0.7854,0.6201)\}$, $\tilde{B}=\{(x,0.8130,0.5916,0.6346)\}$ and $\tilde{C}=\{(x,0.1990,0.9930,0.2351)\}$ are three q-ROFSs in the singleton universe of discourse $X=\{x\}$ then the entropy measure for these q-ROFSs using proposed entropies (4) to (6) as shown in table 1 for $q=3$.

Table 1. Entropy measures of three q-ROFSs.

q-ROFSs	e_{m1}	e_{m2}	$e_{min/max}$
$\tilde{A}$	0.7927	0.6566	0.4922
$\tilde{B}$	0.6697	0.5034	0.3853
$\tilde{C}$	0.0287	0.0146	0.0080

Table 1. shows the numerical analysis of entropy measures of three different q-ROFSs using suggested entropy measures e_{m1}, e_{m2} and $e_{min/mmax}$.

Property: Let $\tilde{Q}=\left\{\left(x,\mu_{\tilde{Q}}(x),\upsilon_{\tilde{Q}}(x)\right)\right\}$ be a (q-1) rung orthopair fuzzy set, i.e $\mu_{\tilde{Q}}^{q-1}(x)+\upsilon_{\tilde{Q}}^{q-1}(x)\leq 1$then $\tilde{Q}$ is q-ROFS.

Proof: Since $\tilde{Q}$ is (q-1) rung orthopair fuzzy set so $\mu_{\tilde{Q}}^{q-1}(x)+\upsilon_{\tilde{Q}}^{q-1}(x)\leq 1$ as $q>(q-1)$ thus $\mu^{q}{}_{\tilde{Q}}(x)+\upsilon^{q}{}_{\tilde{Q}}(x)\leq \mu^{q-1}{}_{\tilde{Q}}(x)+\upsilon^{q-1}{}_{\tilde{Q}}(x)\leq 1$ which shows that $\tilde{Q}$ is also q-ROFS.

Proposition:

a. Any IFS is also q-ROFS for $q\geq 1$.

b. Any PFS is also q-ROFS for $q\geq 2$.

4. OF-TOPSIS Based on Suggested Entropy Measures with Applications to MCDM

We extend the TOPSIS technique to MCDM which is based on proposed entropy measures of q-ROFSs. Hwang and Yoon [38] first introduced the TOPSIS method to tackle the problems related to MCDM. Usually, the available information related to daily life settings involving multicriteria decision making processes are mostly inexact or imprecise. So, it is very difficult to come up with intuitively acceptable decision without using any efficient method. Therefore, we introduced Orthopairian TOPSIS (O-TOPSIS) method based on proposed entropy measures (4) to (6) to handle problems containing complex multicriteria decision making process associated to practical life. The q-ROFSs are more powerful tools to handle with the MCDM problems containing uncertainty, vagueness or inexact information with more accuracy. To show practical validity we utilized our suggested entropy measures (4) to (6) in MCDM problems with totally unknown criteria weights for alternatives in q-ROFSs environments.

We start to model the problems in the shape of Orthopairian decision matrix in which it lists a variety of criteria are against each alternative. We consider that there are m alternatives and we wish to evaluate them on n criteria. Assume $A=\{A_1,A_2,A_3,\ldots,A_m\}$ be the finite set of alternatives $A=\{A_1,A_2,A_3,\ldots,A_m\}$ with $i=1,2,\ldots,m$ and assume that the set of criteria for the alternatives be denoted by Q_j, with $j=1,2,\ldots,n$. Our objective is to pick the most excellent alternative among the given set of alternatives. The construction steps for the OF-TOPSIS based on suggested entropy measures (4) to (6) are given as:

Step 1: Construction of q-rung Orthopair fuzzy Decision matrix

Suppose that the alternatives A_i acting on criteria Q_j is denoted by q-ROF value $\delta_{ij} = (\mu_{ij}, \nu_{ij}, \pi_{ij})_q$ $i = 1,2,3,\ldots,m$ and $j = 1,2,3,\ldots,n$ where μ_{ij} indicates the degree of membership supports , ν_{ij} denotes the non-membership grades and π_{ij} stand for the degree of indeterminacy against the alternatives A_i to the criteria Q_j with the condition that $0 \le \mu_{ij}{}^q, \nu_{ij}{}^q, \pi_{ij}{}^q \le 1$ and $\mu_{ij}{}^q + \nu_{ij}{}^q + \pi_{ij}{}^q = 1$. The q-rung Orthopair fuzzy decision matrix (q-ROFDM) is represented by

$$\tilde{D} = (\delta_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} Q_1 & Q_2 & \cdots & Q_n \end{array} \\ \begin{bmatrix} (\mu_{11}, \nu_{11}, \pi_{11})_q & (\mu_{12}, \nu_{12}, \pi_{12})_q & \cdots & (\mu_{1n}, \nu_{1n}, \pi_{1n})_q \\ (\mu_{21}, \nu_{21}, \pi_{21})_q & (\mu_{22}, \nu_{22}, \pi_{22})_q & \cdots & (\mu_{2n}, \nu_{2n}, \pi_{2n})_q \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{m1}, \nu_{m1}, \pi_{m1})_q & (\mu_{m2}, \nu_{m2}, \pi_{m2})_q & \cdots & (\mu_{mn}, \nu_{mn}, \pi_{mn})_q \end{bmatrix} \end{array}$$

Step 2: Determination of weights criteria

In this step, we assign weights to each criterion which can be compute by several ways. Assume that all the weights to criteria are completely unknown then we can obtain the weights criteria w_j, with $j = 1,2,\ldots,n$ using the proposed entropies as

$$\tilde{w}_j = \frac{\tilde{E}_j}{\sum_{j=1}^{n} \tilde{E}_j}$$

where $\tilde{E}_j = \frac{1}{m}\sum_{i=1}^{m} \delta_{ij}$ with the condition that $0 \le \tilde{w}_j \le 1$ provided that $\sum_{j=1}^{n} \tilde{w}_j = 1$.

Step 3: Construction of q-rung Orthopair positive ideal solution (q-ROPIS) and q-rung orthopair Negative ideal solution (q-RONIS)

In TOPSIS, it is very significant to determine q-ROPIS and q-RONIS since the evaluation criteria can be categorized into two categories, benefit and non-benefit criteria.

Let B_1 and B_2 be the sets of benefit and non-benefit criteria in the criteria Q_j with the principal of TOPSIS we define q-ROPIS and q-RONIS respectively as:

$A^{+}=\left\{\left\langle Q_j,\left(\mu_j^{+},\nu_j^{+},\pi_j^{+}\right)\right\rangle\right\}$ where $\left(\mu_j^{+},\nu_j^{+},\pi_j^{+}\right)=(1,0,0)$, $j\in B_1$;

$A^{-}=\left\{\left\langle Q_j,\left(\mu_j^{-},\nu_j^{-},\pi_j^{-}\right)\right\rangle\right\}$ where $\left(\mu_j^{-},\nu_j^{-},\pi_j^{-}\right)=(0,1,0)$, $j\in B_2$.

Step 4: Distance from each alternatives A_i to q-ROPIS and q-RONIS

In this step, we utilize (2), the weighted distance between two q-ROFSs to calculate the distance from each alternative A_i to q-ROPIS and q-RONIS, respectively.

$$D^{+}\left(A_i\right)=\left[\frac{1}{2}\sum_{j=1}^{n}w_j\left(\left|1-\mu_{ij}^{q}\right|^{q}+\left|\nu_{ij}^{q}\right|^{q}+\left|\pi_{ij}^{q}\right|^{q}\right)\right]^{\frac{1}{q}} \text{ and}$$

$$D^{-}\left(A_i\right)=\left[\frac{1}{2}\sum_{j=1}^{n}w_j\left(\left|\mu^{q}{}_{ij}\right|^{q}+\left|1-\nu^{q}{}_{ij}\right|^{q}+\left|\pi^{q}{}_{ij}\right|^{q}\right)\right]^{\frac{1}{q}},\ i=1,2,3,\ldots,m \text{ and } j=1,2,3,\ldots,n.$$

Step 5: Computation of relative closeness

In this step, the relative closeness degree $R(A_i)$ of each alternative A_i with respect to

q-ROPIS and q-RONIS, respectively is calculated as

$$\vec{R}\left(\tilde{A}_i\right)=\frac{D^{-}\left(\tilde{A}_i\right)}{D^{+}\left(\tilde{A}_i\right)+D^{-}\left(\tilde{A}_i\right)}$$

The relative closeness degree is utilized to rank the alternative in preference order that is either in ascending or descending order with respect to the relative closeness degrees. The alternative with maximum relative closeness degree is considered as the best alternative among all other alternatives.

4.1. Application to multicriteria decision making

In this subsection, we utilized our suggested entropy measures (4)-(6) to a problem related to practical life settings involving complex MCDM process. Multicriteria decision making is an effective process for selecting a best alternative among the finite set of alternatives under a set of criteria. It is commonly practice in our daily life situation where the available information is often fuzzy or vague. The newly developed q-ROFSs by Yager [28] become a strong tool to deal decision making problems containing inexplicit and incomplete

information with high precision and accuracy. We show the authenticity and practical applicability of our proposed methods (4)-(6) in the following examples.
Example 2. (Application in Monkey Pox)

Monkey Pox is a zoonotic viral disease which can spread from animals to human beings (WHO Report, May, 2022), and is a newly originated disease which is gradually spreading all over the country and creates an alarming situation. PMA (Pakistan Medical Association) suggested various medicines for the treatment of this pandemic. Suppose that there are three available medicines: (A_1) medicine *M1*, (A_2) medicine *M2* and (A_3) medicine *M3*. To choose the best medicine, we need to consult with the medical experts and physicians, and their opinions are denoted by the three criteria as:

(Q_1) Highly effective (Q_2) Easily Access and (Q_3) Side effects

For the selection of best alternative, we apply our proposed entropy measures (4)-(6). First, we construct the Q-ROFDM as follows:

Table 2. q-rung orthopair fuzzy decision matrix

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} Q_1 & Q_2 & Q_3 \end{array} \\ \begin{bmatrix} (0.6252, 0.7855, 0.6201)_q & (0.5520, 0.9152, 0.4026)_q & (0.4320, 0.9324, 0.7740)_q \\ (0.4320, 0.9324, 0.7740)_q & (0.7310, 0.8143, 0.4111)_q & (0.7310, 0.8143, 0.4111)_q \\ (0.1190, 0.9930, 0.2351)_q & (0.4265, 0.9715, 0.1765)_q & (0.0265, 0.9999, 0.0669)_q \end{bmatrix} \end{array}$$

Table 2, reflects the decision matrix in which rows denotes the alternatives and the columns denotes the criteria for each alternative.

Table 3. Entropy measures and weights of criteria

Entropies	$\tilde{w}_1$	$\tilde{w}_2$	$\tilde{w}_3$
e_{m1}	0.4097	0.4594	0.1308
e_{m2}	0.4798	0.4407	0.0795
$e_{min/max}$	0.7148	0.1826	0.1026

In Table 3, we calculate the weights of each criteria using the three proposed entropy measures (4)-(6).

Table 4. Distance for each alternatives

e_{m1}	$D^{+}(\tilde{A}_i)$	$D^{-}(\tilde{A}_i)$	e_{m2}	$D^{+}(\tilde{A}_i)$	$D^{-}(\tilde{A}_i)$	$e_{min/max}$	$D^{+}(\tilde{A}_i)$	$D^{-}(\tilde{A}_i)$
$\tilde{A}_1$	0.7518	0.3416	$\tilde{A}_1$	0.7364	0.3570	$\tilde{A}_1$	0.6989	0.3989
$\tilde{A}_2$	0.6132	0.5577	$\tilde{A}_2$	0.5704	0.5804	$\tilde{A}_2$	0.5553	0.6333
$\tilde{A}_3$	0.9584	0.0623	$\tilde{A}_3$	0.9589	0.0615	$\tilde{A}_3$	0.9758	0.0463

Table 4, represents the distance of each alternative to q-ROPIS and q-RONIS respectively.

Table 5. Degree of relative closeness

e_{m1}	$\vec{R}(\tilde{A}_i)$	e_{m2}	$\vec{R}(\tilde{A}_i)$	$e_{min/max}$	$\vec{R}(\tilde{A}_i)$
$\tilde{A}_1$	0.3124	$\tilde{A}_1$	0.3265	$\tilde{A}_1$	0.3634
$\tilde{A}_2$	0.4763	$\tilde{A}_2$	0.5043	$\tilde{A}_2$	0.5328
$\tilde{A}_3$	0.0610	$\tilde{A}_3$	0.0603	$\tilde{A}_3$	0.0453

Table 5, shows the degree of relative closeness which is the ratio of q-ROPIS to the sum of q-ROPIS and q-RONIS.

Table 6. Ranking of alternatives

Entropy	Ranking	Best Alternative
e_{m1}	$\tilde{A}_2 \succ \tilde{A}_1 \succ \tilde{A}_3$	$\tilde{A}_2$
e_{m2}	$\tilde{A}_2 \succ \tilde{A}_1 \succ \tilde{A}_3$	$\tilde{A}_2$
$e_{min/max}$	$\tilde{A}_2 \succ \tilde{A}_1 \succ \tilde{A}_3$	$\tilde{A}_2$

Table 6, exhibit the ranking of alternatives based on degree of closeness which arranged in decreasing order. It is clearly shown in table 6, there is no conflict in ranking the alternatives using suggested entropy measures (4)-(6). Therefore, the numerical simulations in table 6, shows the consentient in choosing the best alternative $\tilde{A}_2$ among the available alternatives by utilizing the proposed entropy measures (4)-(6).

4.2 Application to MCDM and Covid-19

This subsection is dedicated to apply the proposed entropy measures (4) - (6) in distribution of Covid-19 patients in different level of medical treatment centres.

Example 3: The rapid increase of corona virus in the world more and more people rushing to medical centers for treatment even they have ordinary fever, which causes difficulty in testing Covid-19 which build pressure on medical staff. Therefore, in present case study analysis we focus on the distributing patients with Covid-19 disease in appropriate medical centers, to overcome the pressure the experts of medical team categorized the medical centers into four classes:

i. Class A medical center: This type of medical centers is reserved for the most severe patients having Covid-19 symptoms.

ii. Class B medical centers**:** Those patients should be treated in Class B medical centers having dry cough with Covid-19 symptoms.

iii. Class C medical centers**:** Those patients should be treated in Class C medical centers with normal condition with Covid-19 symptoms.

iv. Class D medical centers**:** Those patients should be treated in Class D medical centers having high fever Covid-19 symptoms.

The specific statement about medical diagnosis problem is described as:

To reduce the pressure, four patients, denoted by $P_i\left(i=1,2,3,4\right)$ who are possibly infected with Covid-19 disease and need to be diagnosed and distributed into above classes of medical centres.

The four patients are diagnosed from the following four symptoms (attributes) of Covid-19:

S_1: High fever, S_2: Dry cough, S_3: difficulty in breathing / shortness of breath, S_4: Sore throat.

The weight criteria of attributes are totally unknown and suppose that the medical expert team gives the evaluation values for the four patients with respect to the symptoms by means of q-ROFNs.

Table 7. q-rung Orthopair fuzzy decision matrix

$$\begin{array}{c} \\ \tilde{P}_1 \\ \tilde{P}_2 \\ \tilde{P}_3 \\ \tilde{P}_4 \end{array}\begin{array}{c} \begin{array}{cccc} \quad\tilde{S}_1 \quad & \quad\tilde{S}_2\quad & \quad\tilde{S}_3\quad & \quad\tilde{S}_4\quad \end{array} \\ \begin{bmatrix} (0.9,0.3,0.6249) & (0.8,0.7,0.5254) & (0.5,0.8,0.7133) & (0.6,0.3,0.9114) \\ (0.8,0.7,0.5254) & (0.9,0.2,0.6407) & (0.8,0.1,0.7868) & (0.5,0.3,0.9465) \\ (0.8,0.5,0.7133) & (0.6,0.8,0.6479) & (0.8,0.7,0.5254) & (0.6,0.4,0.9863) \\ (0.7,0.2,0.8658) & (0.8,0.2,0.7830) & (0.8,0.4,0.7513) & (0.8,0.7,0.5254) \end{bmatrix} \end{array}$$

Table 7, shows the decision matrix which is consist of the grades given by the decision makers to each alternative in the form of degree memberships, non-memberships and the degree of indeterminacy.

Table 8. Entropy measures and weights of criteria

Entropies	ω_1	ω_2	ω_3	ω_4
e_{m1}	0.2131	0.2009	0.2403	0.3456
e_{m2}	0.2179	0.2084	0.2217	0.3521
$e_{min/max}$	0.2627	0.2955	0.2643	0.1775

Table 8, reflects the weight criteria and ranking of weights of each alternative.

Table 9. Distance from each alternatives A_i to q-ROPIS and q-RONIS respectively.

e_{m1}	$D^+(\tilde{P}_i)$	$D^-(\tilde{P}_i)$	e_{m2}	$D^+(\tilde{P}_i)$	$D^-(\tilde{P}_i)$	$e_{min/max}$	$D^+(\tilde{P}_i)$	$D^-(\tilde{P}_i)$
$\tilde{P}_1$	0.6545	0.7700	$\tilde{P}_1$	0.6504	0.7750	$\tilde{P}_1$	0.6055	0.7326
$\tilde{P}_2$	0.6209	0.8281	$\tilde{P}_2$	0.6226	0.8276	$\tilde{P}_2$	0.5374	0.8166
$\tilde{P}_3$	0.6288	0.7140	$\tilde{P}_3$	0.6322	0.7157	$\tilde{P}_3$	0.5970	0.6664
$\tilde{P}_4$	0.5103	0.7688	$\tilde{P}_4$	0.5114	0.7686	$\tilde{P}_4$	0.5269	0.8407

Table 9, represents the distance of each alternative to q-ROPIS and q-RONIS respectively.

Table 10. Degree of relative closeness

e_{m1}	$\vec{R}(\tilde{P}_i)$	e_{m2}	$\vec{R}(\tilde{P}_i)$	$e_{min/max}$	$\vec{R}(\tilde{P}_i)$
$\tilde{P}_1$	0.5405	$\tilde{P}_1$	0.5437	$\tilde{P}_1$	0.5475
$\tilde{P}_2$	0.5715	$\tilde{P}_2$	0.5707	$\tilde{P}_2$	0.6031
$\tilde{P}_3$	0.5317	$\tilde{P}_3$	0.5310	$\tilde{P}_3$	0.5275
$\tilde{P}_4$	0.6010	$\tilde{P}_4$	0.6005	$\tilde{P}_4$	0.6147

Table 10, shows the degree of relative closeness of each alternative. Despite, slightly different degree of relative closeness for each alternative there is no conflict in final ranking of alternatives under suggested entropy measures (4) to (6) as shown in the following table 11.

Table 11. Ranking of Alternatives.

Entropy	Ranking	Best Alternative
e_{m1}	$\tilde{P}_4 \succ \tilde{P}_2 \succ \tilde{P}_1 \succ \tilde{P}_3$	$\tilde{P}_4$
e_{m2}	$\tilde{P}_4 \succ \tilde{P}_2 \succ \tilde{P}_1 \succ \tilde{P}_3$	$\tilde{P}_4$
$e_{min/max}$	$\tilde{P}_4 \succ \tilde{P}_2 \succ \tilde{P}_1 \succ \tilde{P}_3$	$\tilde{P}_4$

Table 11, exhibits the final ranking of each alternative based on the degree of relative closeness in decreasing order. There is no dissension in final raking of alternative based on degree relative closeness under proposed entropy measures (4) to (6). Our numerical analysis from the Table 11 shows that our proposed entropy measures (4) to (6) unanimously agreed that the condition of patient $\tilde{P}_4$ is most serious so should be treated in class 'A' medical centers. Meanwhile the condition of $\tilde{P}_3$ is normal as compare to the other three patients so should be treated in class C medical centers. $\tilde{P}_1$ should be treated in class D medical centers however $\tilde{P}_2$ should be treated in class B.

Hence, Table 11 reflects the credibility and reliability OF-TOPSIS based on presented entropy measures (4) to (6).

5. Conclusion

In this manuscript, we have suggested new fuzzy entropy measures in the framework of q-ROFSs. We utilized the concept of entropy measures based on q-ROPFSs and introduce the axiomatic definition of entropy measures for q-ROFSs. Several numerical examples are presented which shows the validity and practical ability of our proposed entropy measures. We also introduced the new q-rung Orthopairian fuzzy TOPSIS based on suggested entropy measures to deal with the complex problems connecting multicriteria decision making process related to daily life. Keeping in view the current scenario around the globe, we have also put forwarded the applications of our proposed entropy measures with O-TOPSIS in the context of Covid-19 involving complex multicriteria decision making processes. Numerical simulations show that our propounded methods are practically applicable, valid, and well suited in the environment of newly established q-ROPFSs.

List of Abbreviations

q-ROFSs: q-Rung Orthopair Fuzzy Sets,

IFSs: Intuitionistic Fuzzy Sets

PFSs: Pythagorean Fuzzy Sets

OF-TOPSIS: Orthopairian Fuzzy Technique for ordering preference by similarity to ideal Solution

MCDM: Multicriteria Decision Making

Declarations

Ethics approval and consent to participate

All authors equally participate in this research.

Availability of data and material

All data is included in the manuscript.

Competing interests

The authors declare that there are no competing interests.

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