New Divergence Model in Intuitionistic Fuzzy Sets for Decision Making

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Abstract. For the first time, Atanassov developed Intuitionistic fuzzy sets (IFSs) as an extended form of fuzzy sets (FSs), which has degree of membership, non-membership and hesitancy. Divergence is the difference between two IFSs and it is a particular case of dissimilarity and Intuitionistic Fuzzy-distance (IF-distance). It is a modern way of decision making and can differentiate qualities of two sets. The importance of divergence measure is the most concern in different areas including segmentation of images and decision making. We suggested a novel and simple divergence between two IFSs which rank the MCDM (Multicriteria Decision Making) systems easily. Particular features of our proposed divergence are monotonicity (it fulfills monotonic properties) and involve simple calculations. So far best divergence has been calculated by Jensen-Renyi Divergence but it failed to some pattern sets. Our proposed divergence holds on those sets, this leads toward better decision making.

Finally, we utilize our proposed divergence in practical problems related to hotel management involving complex MCDM. Numerical simulations reveal that our proposed method is effective, applicable and well suited in managing different real life issues.

Keywords: Intuitionistic Fuzzy Sets, Multicriteria Decision Making, Linguistic Variables, Likert Scale.

1. INTRODUCTION

In real life, fuzzy mathematics plays an important role to tackle difficult situations. MCDM methods were designed to treat complex decision making problems [6]. Scientists and researchers have taken great care to calculate and classify alternatives across all the different scientific areas.

There are many methods that have been developed to resolve such decision making problems, but Intuitionistic Fuzzy Divergence is currently one of the modern method in complex situations.

To deal uncertainty, classical theory is used in the literature, but these classical theories are only valid for accurate data. Within certain restrictions, the decision makers give decisions about uncertainties. So there must always been a factor of hesitancy. That is why these decisions are not considered as ideal. To tackle these type of issues Zadeh proposed fuzzy set theory in 1965 [3]. This theory has received much attention because of its capacity of dealing uncertainty in term of the membership functions. Atanassov [4] suggested the idea of IFSs which are extensions of FSs having the degrees of non-memberships as well. Since, IFSs have numerous applications in various fields to handle the uncertainty existed in data, that's why their analysis is very useful as compare with the classical crisps analysis.

The distances, similarities, and divergences are widely used to solve the complex decision making issues from last couple of years. For this [7] proposed the axiomatic entropy measure for IFSs. After the [8] entropy measures, [9] gave the extensions in the environment of IFSs. [10] proposed similarities of IFSs statistically. Furthermore, [11] proposed the series of similarities for IFSs and used them tackle multi-attribute decision making problems. [12] suggested distance measures to measure two IFSs. [9] proposed the idea of divergences from FSs to IFSs respectively. [13] discussed the extended version of [9] divergence by including the concept of middle values of the membership degree of IFSs and [14] presents the generalization of intuitionistic fuzzy divergence (IF-divergence) which is an extended form of [13] divergence.

Statistical divergences are usually applied while determining the differences of two probability distributions in the theory of probability [15, 16] and Kulback-Liebler divergence is one of the common theoretic divergences. A very useful divergence, known as Jensen Shahnon divergence (JSD) and its modified version is known as Jensen-Renyi Divergence. On the basis of Intuitionistic fuzzy sets, this divergence is known as Intuitionistic Fuzzy Jensen- Renyi Divergence (IF-JRD) which is more effective and give accurate results specially in case of MCDM. The only flaw is the lengthy calculations.

Now we are going to propose a divergence which will be able to give more precise results as well as very simple for calculations. The model will be mansion at the end of this literature review. The mathematical models of above literature given below

Most common Shannon entropy [27] is defined as: $H(P) = \sum_{i=1}^{n} p_i log p_i$ where P represents probability distribution. A generalization of Shannon entropy is proposed by [27].

Renyi's entropy of order a is written as: $H_a(P) = \frac{1}{1-a}\log\left(\sum_{i=1}^{n} p_i^a\right)$where $a > 0, a \neq 1$.

Jensen Shannon divergence (JSD) (lin, J, 1991) is defined as:

$$JS_\lambda(P,Q) = H(\lambda_1 P + \lambda_2 Q) - \lambda_1 H(P) - \lambda_2 H(Q)$$

Where λ_1, λ_2 are weights of two probability distributions P and Q respectively.

Intuitionistic Fuzzy Jensen- Renyi Divergence (IFJRD), May 17, 2013 is given by

$$\begin{aligned} JR_{\lambda,a}(A,B) = \frac{1}{n(1-a)}\Big[\sum\{&(\lambda_1 u_A(x_i) + \lambda_2 u_B(x_i))^a + (\lambda_1 v_A(x_i) + \lambda_2 v_B(x_i))^a \\ &+ (\lambda_1(1-u_A(x_i) - v_A(x_i)) + \lambda_2(1-u_B(x_i) - v_A(x_i)))^a \\ &- \lambda_1 log\{(u_A(x_i)^a + v_A(x_i)^a + (1-u_A(x_i) - v_A(x_i)^a\} \\ &- \lambda_2 log\{(u_B(x_i)^a + v_B(x_i)^a + (1-u_B(x_i) - v_B(x_i)^a\}\}\Big] \end{aligned}$$

Kulback and Leiber [22] presented the directed divergences of two distributions P and Q as:

$$D\left(\frac{P}{Q}\right) = \sum_{i=1}^{n} P_i ln\frac{P_i}{q_i}$$

The concept of divergence from probability to FS theory was presented by [5] as:

$$D(A/B) = \frac{1}{n}\sum_{i=1}^{n}\left[u_A(x_i)\log\left(\frac{u_A(x_i)}{u_B(x_i)}\right) + (1-u_A(x_i))log\left(\frac{1-u_A(x_i)}{1-u_B(x_i)}\right)\right]$$

This measure gives a result of infinity whenever zero u_B tends to 0 or 1, so because of this we get inaccurate results. To tackle this issue, [18] presented a modified measure as:

$$D(A/B) = \frac{1}{n}\sum_{i=1}^{n}\left[u_A(x_i)\log\left(\frac{2u_A(x_i)}{u_A(x_i)+u_B(x_i)}\right) + (1-u_A(x_i))log\left(\frac{2(1-u_A(x_i))}{2-u_A(x_i)-u_B(x_i)}\right)\right]$$

In 2007, Vlachos and Sergiadis [35] forwarded the notion of divergences from FS to IFSs and proposed a measure of IF-divergence of set B relative to set A by

$$D(A/B) = \frac{1}{n}\sum_{i=1}^{n}\left[u_A(x_i)\log\left(\frac{2u_A(x_i)}{u_A(x_i)+u_B(x_i)}\right) + v_A(x_i)log\left(\frac{2v_A(x_i)}{v_A(x_i)+v_B(x_i)}\right)\right]$$

After that Wei and Ye [13] gave an extension of Vlachos and Sergiadis's IF-divergence as:

$$D(A/B) = \frac{1}{n}\sum_{i=1}^{n}\left[u_A(x_i)\log\left(\frac{2u_A(x_i)}{u_A(x_i)+u_B(x_i)}\right) + v_A(x_i)log\left(\frac{2v_A(x_i)}{v_A(x_i)+v_B(x_i)}\right) + \pi_A(x_i)log\left(\frac{2\pi_A(x_i)}{\pi_A(x_i)+\pi_B(x_i)}\right)\right].$$

Later on Verma and Sharma [1] presented an important version of Wei and Ye [13] as $D(A/B) = \frac{1}{n}\sum_{i=1}^{n}\left[u_A(x_i)\log\left(\frac{u_A(x_i)}{\lambda u_A(x_i)+(1-\lambda)u_B(x_i)}\right) + v_A(x_i)log\left(\frac{v_A(x_i)}{\lambda v_A(x_i)+(1-\lambda)v_B(x_i)}\right) + \pi_A(x_i)log\left(\frac{\pi_A(x_i)}{\lambda\pi_A(x_i)+(1-\lambda)\pi_B(x_i)}\right)\right]$.

Now we are going to propose the following simple divergence model which gives better results.

$$D_n(A_j, A^*) = \frac{1}{2|X|}\sum_{i=1}^{n}(|u_{Aj} - u_{A^*}| + |V_{Aj} - V_{A^*}| + |\pi_{AJ} - \pi_{A^*}|)^n$$

2. PRELIMINARIES

Concept of Fuzzy Sets, IFSs and IF-Divergence

In this section, we discussed some fundamentals of FSs, IFSs and IF-divergences.

Definition 1 [3] Consider X be a universal set and P is fuzzy set in X defined as:

P= $\{x, u_{p(x)}; x \in X\}$

Where, $u_{p(x)}: X \to [0,1]$ is the grade of membership of x in P .If $u_{p(x)}$=1, then x entirely belongs to fuzzy set P. Any x ∈ X meet the condition $0 \le u_{A(x)} \le 1$.

Definition 2 [4] Consider X be a finite universe of discourse. Then P be an Intuitionistic Fuzzy set defined as:

P= $\{x, u_{p(x)}, v_{p(x)}; x \in X\}$

Here $u_{p(x)}$ and $v_{p(x)}$ indicates membership and non-membership number respectively.

$u_{p(x)}: X \to [0,1]$, $v_{p(x)}: X \to [0,1]$ having condition $0 \leq u_{p(x)} + v_{P(x)} \leq 1$ for all x $\in X$

Hesitancy is given by $\pi_{p(x)} = 1 - u_{P(x)} - v_{p(x)}$.

Definition 3. Mathematically divergence can be defined as for universal set τ. A map D: P $(\tau) \times$ P $(\tau) \to R$ is said to be divergence iff the following conditions satisfies:

(1) $D(A, B) = D(B, A)$

(2) D(A, A)=0

(3) max(D(AUC, BUC), D(A∩C, B∩ C)) $\leq D(A, B)$ for all $C \in P(\tau)$

2.1. Properties of our proposed Intuitionistic Fuzzy Divergence

Theorem: For $A, B \in IFS(x), D_{IF}(A, B)$ $satisfying\ the\ following\ properties.$

(I) $D_{IF}(A, B) \geq 0$, $with\ equality\ if\ and\ only\ if A = B.$

(II) $0 \leq D_{IF}(A, B) \leq 1.$

$it\ is\ obivious\ that$ our proposed divergence is generalization of hamming distance for $n = 1.$ **or**

$D_{IF}(A, B)$ is a convex function , it attains its minimum value if $A = B$ and attains maximum value for the degenerate cases.

$A = (1,0,0)$ $B = (0,1,0)$ or

$A = (0,1,0)$ $B = (1,0,0)$ or $A = (0,0,1)$ $B = (0,1,0)$

This gives $0 \leq D_{IF}(A, B) \leq 1$

(III) For three IFS A,B,C in X and A $\subseteq B \subseteq C$,

$D_{IF}(A, B) \leq D_{IF}(A, C)$, $D_{IF}(B, C) \leq D_{IF}(A, C)$

Proof: For $A, B \in IFS(x)$, $\|A - B\| \leq \|A - C\|$ and

$\|B - C\| \leq \|A - C\|$, if A$\subseteq B \subseteq C$

Thus $D_{IF}(A, B) \leq D_{IF}(A, C)$ $D_{IF}(B, C) \leq D_{IF}(A, C)$

(IV) $D_{IF}(A \cap B, B) = D_{IF}(A, A \cup B) \leq D_{IF}(A, B)$

Proof:

Case-I: Considering that A$\subseteq B$

$$D_{IF}(A\cap B, B) = \frac{1}{2n}\sum_{i=1}^{n}[|u_{A\cap B}(x) - u_B(x)| + |v_{A\cap B}(x) - v_B| + |\pi_{A\cap B}(x) - \pi_B(x)|]^n$$

$$= \frac{1}{2n}\sum_{i=1}^{n}[|u_A(x) - u_B(x)| + |v_A(x) - v_B| + |\pi_A(x) - \pi_B(x)|]^n$$

$$= D_{IF}(A, B) \quad (1)$$

$$D_{IF}(A, A\cup B) = \frac{1}{2n}\sum_{i=1}^{n}[|u_A(x) - u_{A\cup B}(x)| + |v_A(x) - v_{A\cup B}| + |\pi_A(x) - \pi_{A\cup B}(x)|]^n$$

$$= \frac{1}{2n}\sum_{i=1}^{n}[|u_A(x) - u_B(x)| + |v_A(x) - v_B| + |\pi_A(x) - \pi_B(x)|]^n$$

$$= D_{IF}(A, B) \quad (2)$$

From (1) and (2) we proved that

$$D_{IF}(A\cap B, B) = D_{IF}(A, A\cup B) = D_{IF}(A, B)$$

Case-II: If B⊆ A or A=B then $D_{IF}(A\cap B, B) = D_{IF}(A, A\cup B)$ and $D_{IF}(A\cap B, B) = 0$

So in this case $D_{IF}(A\cap B, B) < D_{IF}(A, B)$.

So in general $D_{IF}(A\cap B, B) = D_{IF}(A, A\cup B) \le D_{IF}(A, B)$.

2.2. Comparison in pattern sets

These sets have been taken from the paper 'Similarity measure of IFS based on Hausdorff distance by [37].

$A_1 = \{(x_1, 0.1, 0.1)(x_2, 0.5, 0.1)(x_3, 0.1, 0.9)\}$, $A_2 = \{(x_1, 0.5, 0.5), (x_2, 0.7, 0.3), (x_3, 0.0, 0.8)\}$,

$A_3 = \{(x_1, 0.7, 0.2), (x_2, 0.1, 0.8), (x_3, 0.4, 0.4)\}$, $B = \{(x_1, 0.4, 0.4), (x_2, 0.6, 0.2), (x_3, 0.0, 0.8)\}$.

$$D(A, B) = \frac{1}{2|X|}\sum_{i=1}^{n}\{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|\}^n$$

$$D(A_1, B) = 1/6\{(.3+.3+.6)^2 + (.1+.1+.2)^2 + (.1+.1+.2)^2\}$$

$$D(A_1, B) = 1/6\{(1.2)^2 + (0.4)^2 + (0.4)^2\}$$

$$D(A_1, B) = 0.294,\ S(A_1, B) = 0.706.$$

$$D(A_2, B) = 0.0534,\ S(A_2, B) = 0.946.$$

$$D(A_3, B) = 0.388,\ S(A_3, B) = 0.6116.$$

2.3. Comparison of our proposed Divergence with Jensen-Renyi Divergence

Example 1. Consider $A_1 = \{(x_1, 0.1, 0.1, 0.8), (x_2, 0.5, 0.1, 0.4), (x_3, 0.1, 0.9, 0)\}$,

$B = \{(x_1, 0.4, 0.4), (x_2, 0.6, 0.2), (x_3, 0.0, 0.8)\}$

Calculation by already existed Jensen-Renyi Divergence when α=0.5 , λ_1=λ_1=0.5 is

$JS_{\lambda,a}(A_1, B)$= 2.8668.

And calculation by our proposed Divergence will be

$D(A_1, B)$= 0.294.

Example 2. Consider $A_2 = \{(x_1, 0.5, 0.5), (x_2, 0.7, 0.3), (x_3, 0.0, 0.8)\}$,

$$B = \{(x_1, 0.4, 0.4), (x_2, 0.6, 0.2), (x_3, 0.0, 0.8)\}$$

$JS_{\lambda,a}(A_1, B)$= 2.726 , $D(A_1, B)$= 0.0534

From the above examples we concluded that our proposed divergence holds on pattern sets while Jensen-Renyi Divergence does not.

3. DECISION MAKING BY IF-DIVERGENCE

We introduced the following algorithm to solve the Multi Criteria Decision Making problem by using our proposed divergence.

Step 1: Let us consider a decision problems involving a set of options $A = \{A_1, A_2, \dots, A_n\}$ are taken under the criterias $C = \{C_1, C_2, \dots, C_n\}$. The decision matrix $(d_{ij})_{n\times m}$ represents the set of alternative and set of criteria.

$$(d_{ij})_{n\times m}=\begin{bmatrix} d_{11}\ d_{12}\ \dots\dots..d_{1m} \\ d_{21}\ d_{22}\ \dots...\dots d_{2m} \\ \dots\dots\dots\dots\dots\dots\dots\dots.. \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ d_{n1}\ d_{n2}\ \dots\dots..d_{nm} \end{bmatrix}$$

where $d_{ij} = (u_{ij}, v_{ij}, \pi_{ij})$,

u_{ij} represents the degree with which alternative A_j satisfy the attribute C_i,

v_{ij}, represents the degree with which alternative A_j does not satisfy the attribute C_i, and

π_{ij} represents the degree of hesitancy of alternative A_j to its attribute C_i.

Step 2: Normalized the Intuitionistic Fuzzy Matrix $R = (r_{ij})_{n\times m}$

$$r_{ij}(u_{ij}, v_{ij}, \pi_{ij}) = \begin{cases} d_{ij}\ for\ benifit\ attribute\ C_i \\ (d_{ij})^c\ for\ cost\ attribute\ C_i \end{cases}$$

Step 3: Find the ideal solution $A^* = \{(u_{1*}, v_{1*}, \pi_{1*}), (u_{2*}, v_{2*}, \pi_{2*}), \dots, (u_{m*}, v_{m*}, \pi_{m*})$.

Where $i = 1,2,\dots,n$, $(u_{*i}, v_{*i}, \pi_{*i}) = \{{}^{max}{}_j u_{ij}, {}^{min}{}_j v_{ij}, 1 - {}^{max}{}_j u_{ij} - {}^{min}{}_j v_{ij}\}$

Step 4: Calculate $D_n(A_j, A^*)$ as:

$$D_n(A_j, A^*) = \frac{1}{2|X|}\sum_{i=1}^{n}(|u_{Aj} - u_{A^*}| + |V_{Aj} - V_{A^*}| + |\pi_{AJ} - \pi_{A^*}|)^n$$

where $n = 1, 2, 3, \dots, n$.

Step 5: Ranking of the alternatives $A_{j\ ,j=1,\dots m}$ in accordance with the value of $D_n(A_j, A^*)$ and select the best alternative denoted by A_k with smallest $D_n(A_j, A^*)$. As the value of n increases, the divergence between the sets decreases and tends to zero which make decision making easy. In this study, we compared six Hotels of Hunza, Gilgit-Baltistan which is very famous for its natural beauty with lush green lands and snow caped mountains. Respondents were asked to state the level of satisfaction with likert scale. The attributes were grouped in three main categories which are Reception Hall, Restaurant, and Guest Room. Each main category is divided into further subcategories as shown in the Table 1.

Example 4. (Selection of best hotels for tourists)

Gilgit-Baltistan is situated in north of Pakistan having high altitude snow-caped mountains like K.2, Rakaposhi and Nanga Parbat and considered as a hub of tourism because of its natural beauty. Millions of people from all over the world are flowing towards these mountains to see the real and natural beauty of this area. Hunza is a district of Gilgit-Baltistan, located in front of Rakaposhi mountain where many hotels available for tourists. Our study consists on a thorough survey of most popular hotels across the district Hunza in order to choose the best option among many hotels. We have taken restaurant, guest room and reception hall in order to find alternatives of the criteria's.

Some rating scales are used in this research as:

Rating Scale of linguistic variables

1- Very Dissatisfied
2- Dissatisfied
3- Neutral
4- Satisfied
5- Very Satisfied

The above scales are used to transform the linguistic variables into intuitionistic fuzzy numbers by using our intuition.

Table 1. Linguistic variables for criteria's

Main Category	Sub Category
C_1 Reception Hall	C_{11} Enivironment and decoration of reception hall C_{12} Courtesy of attendants C_{13} Speed of reception C_{11} Service initiative of attendants
C_2 Guest Room	C_{21} Decoration of room C_{22} Safety of room C_{23} Room facilities C_{24} Room rent

C_3 *Restourent*	C_{31}	Environments of restaurants
	C_{32}	Taste and variety of food
	C_{33}	Cleanses of restaurant table ware
	C_{34}	Food cost

Table 2. Intuitionistic Fuzzy normalized decision matrices for Reception Hall

	c_{11}	c_{12}	c_{13}	c_{14}
H_1	(0.55, 0.25, 0.2)	(0.475, 0.35, 0.175)	(0.575, 0.25, 0.175)	(0.7, 0.25, 0.05)
H_2	(0.65, 0.2, 0.15)	(0.525, 0.33, 0.142)	(0.75, 0.1, 0.15)	(0.575, 0.375, 0.05)
H_3	(0.775,0.05,0.175)	(0.725,0.05,0.225)	(0.675,0.2,0.125)	(0.7,0.1,0.2)
H_4	(0.875,0,0125)	(0.844,0.063,0.094)	(0.906,0,0.094)	(0.813,0.125,0.062)
H_5	(0.95,0,0.05)	(0.85,0,0.15)	(0.6,0.25,0.15)	(0.45.0.40,0.15)
H_6	(0.708,0.083,0.209)	(0.583,0.333,0.o84)	(0.708,0.167,0.125)	(0.542,0.333,o.125)

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Table 3. IF Normalized Decision Matrix for Guest Room

	C_{11}	C_{12}	C_{13}	C_{14}
H_1	(0.45, 0.325, 0.23)	(0.6, 0.325, 0.075)	(0.5, 0.325, 0.175)	(0.225, 0.625, 0.15)
H_2	(0.675,0.15,0,175)	(0.85,0.1,0.05)	(0.65,0.2,0.15)	(0.6,0.15,0.25)
H_3	(0.775,0.05,0.175)	(0.925,0,0.75)	(0.825,0,0.175)	(0.575,0.325,0.1)
H_4	(0.781,0.188,0.0315)	(0.781,0.186,0.0315)	(0.844,0.625,0.0935)	(0.656,0.281,0.063)
H_5	(0.75,0.1,0.15)	(0.8,0.15,0.05)	(0.85,0,0.15)	(0.5,0.4,0.1)
H_6	(0.708,0.167.0.125)	(0.708,0.083,0.208)	(0.458,0.417,0.125)	(0.458,0.333,0.208)

Table 4. IFS Normalized Decision Matrix for Restaurant

	C_{11}	C_{12}	C_{13}	C_{14}
H_1	(0.475,0.35,0.175)	(0.55,0.3,0.15)	(0.5,0.375,0.125)	(0.625,0.22,0.15)
H_2	(0.75,0.1,0.15)	(0.575,0.35,0.075)	(0.6,0.225,0.175)	(0.6,0.15,0.25)
H_3	(0.8,0.05,0.15)	(0.75,0.075,0.175)	(0.825,0.1,0.075)	(0.575,0.325,0.1)

H_4	(0.969,0,0.031)	(0.0875,0,0.125)	(0.875,0.0625,0.6625)	(0.656,0.281,0.063)
H_5	(0,45,0.35,0.200	(0.8,0.1,0.1)	(0.6,0.25,0.15)	(0.5,0.4,0.1)
H_6	(0.667,0.25,0.083)	(0.458,0.417,0.125)	(0.5,0.375,0.125)	(0.458,0.33,0.208)

Table 5. Normalized Intuitionistic Fuzzy decision matrix = $(r_{ij})_{mn}$ for Reception Hall

	c_{11}	c_{12}	c_{13}	c_{14}
H_1	(0.55, 0.25, 0.2)	(0.475, 0.35, 0.175)	(0.575, 0.25, 0.175)	(0.7, 0.25, 0.05)
H_2	(0.65, 0.2, 0.15)	(0.525, 0.33, 0.142)	(0.75, 0.1, 0.15)	(0.575, 0.375, 0.05)
H_3	(0.775,0.05,0.175)	(0.725,0.05,0.225)	(0.675,0.2,0.125)	(0.7,0.1,0.2)
H_4	(0.875,0,0125)	(0.844,0.063,0.094)	(0.906,0,0.094)	(0.813,0.125,0.062)
H_5	(0.95,0,0.05)	(0.85,0,0.15)	(0.6,0.25,0.15)	(0.45.0.40,0.15)
H_6	(0.708,0.083,0.209)	(0.583,0.333,0.084)	(0.708,0.167,0.125)	(0.542,0.333,0.125)

Table 6. Normalized IFS decision matrix for Gest Room

	C_{21}	C_{22}	C_{23}	C_{24}
H_1	(0.45, 0.325, 0.23)	(0.6, 0.325, 0.075)	(0.5, 0.325, 0.175)	(0.225, 0.625, 0.15)
H_2	(0.675,0.15,0,175)	(0.85,0.1,0.05)	(0.65,0.2,0.15)	(0.6,0.15,0.25)
H_3	(0.775,0.05,0.175)	(0.925,0,0.75)	(0.825,0,0.175)	(0.575,0.325,0.1)
H_4	(0.781,0.188,0.0315)	(0.781,0.186,0.0315)	(0.844,0.625,0.0935)	(0.656,0.281,0.063)
H_5	(0.75,0.1,0.15)	(0.8,0.15,0.05)	(0.85,0,0.15)	(0.5,0.4,0.1)
H_6	(0.708,0.167.0.125)	(0.708,0.083,0.208)	(0.458,0.417,0.125)	(0.458,0.333,0.208)

Table 7. Normalized IFS decision matrix for Restaurant

	C_{31}	C_{32}	C_{33}	C_{34}
H_1	(0.475,0.35,0.175)	(0.55,0.3,0.15)	(0.5,0.375,0.125)	(0.225,0.625,0.15)
H_2	(0.75,0.1,0.15)	(0.575,0.35,0.075)	(0.6,0.225,0.175)	(0.15,0.6,0.25)
H_3	(0.8,0.05,0.15)	(0.75,0.075,0.175)	(0.825,0.1,0.075)	(0.325,0.575,0.1)

H_4	(0.969,0,0.031)	(0.0875,0,0.125)	(0.875,0.0625,0.6625)	(0.281,0.656,0.063)
H_5	(0,45,0.35,0.200	(0.8,0.1,0.1)	(0.6,0.25,0.15)	(0.4,0.5,0.1)
H_6	(0.667,0.25,0.083)	(0.458,0.417,0.125)	(0.5,0.375,0.125)	(0.333,0.458,0.208)

Ideal Solution for Reception Hall

$$H^* = \{C_{11}(0.95,0,0.05), C_{12,}(0.85,0,0.15), C_{13}, (0.906,0,0.094), C_{14,}(0.813,0.1,0.087)\}$$

Ideal Solution for Gest Room

$$H^* = \{C_{21}, (0.781,0.05,0.169), C_{22}, (0.925,0,0.075), C_{23}, (0.85,0,0.15), C_{24}, (0.4,0.458,0.142)\}$$

Ideal Solution for restaurant

$$H^* = \{C_{31}, (0.969,0.0.031), C_{32}, (0.8,0,0.2), C_{33}, (0.875,0.1,0,025), C_{34}, (0.4,0.458,0.142)\}$$

Divergence for Reception Hall

	H_1	H_2	H_3	H_4	H_5	H_6
n=1	**0.29587**	**0.1265**	**0.037**	**0.21669**	**0.0605**	**0.20425**
n=2	**0.1849**	**0.0356**	**0.00717**	**0.1057**	**0.0133**	**0.1194**
n=3	**0.1196**	**0.1119**	**0.0016**	**0.0576**	**0.00357**	**0.0845**

Divergence measure for Guest Room

	H_1	H_2	H_3	H_4	H_5	H_6
n=1	**0.31375**	**0.2735**	**0.145**	**0.0406**	**0.16725**	**0.261**
n=2	**0.2160**	**0.1553**	**0.046**	**0.005**	**0.1127**	**0.141**
n=3	**0.156**	**0.092**	**0.016**	**0.0006**	**0.076**	**0.079**

Divergence measure for Restaurant

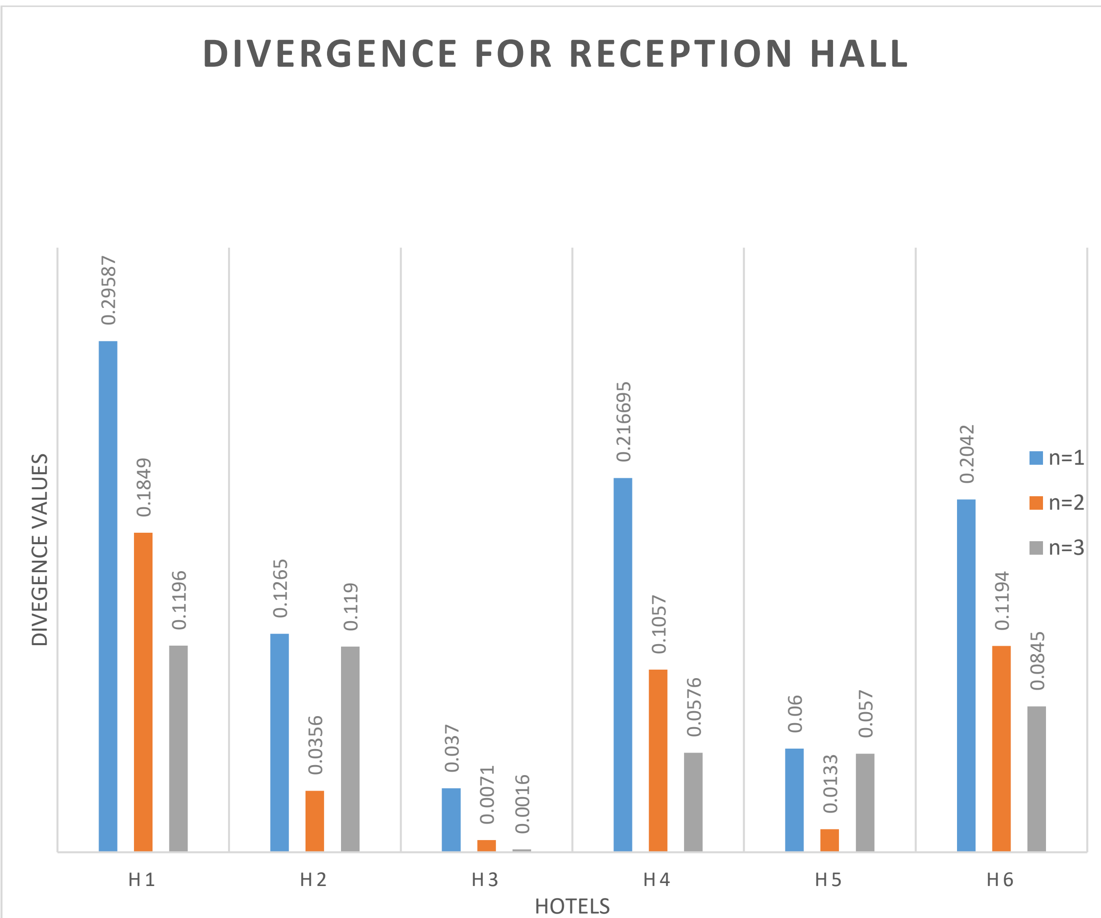
	H_1	H_2	H_3	H_4	H_5	H_6
n=1	**0.336**	**0.2735**	**0.10275**	**0.2323**	**0.234**	**0.2901**
n=2	**0.2526**	**0.1543**	**0.025**	**0.1541**	**0.178**	**0.205**
n=3	**0.2056**	**0.0898**	**0.007**	**0.107**	**0.1616**	**0.1531**

Ranking for Reception Hall

$$H_3 \succ H_5 \succ H_2 \succ H_4 \succ H_6 \succ H_1$$

Graphically, it will be shown as:

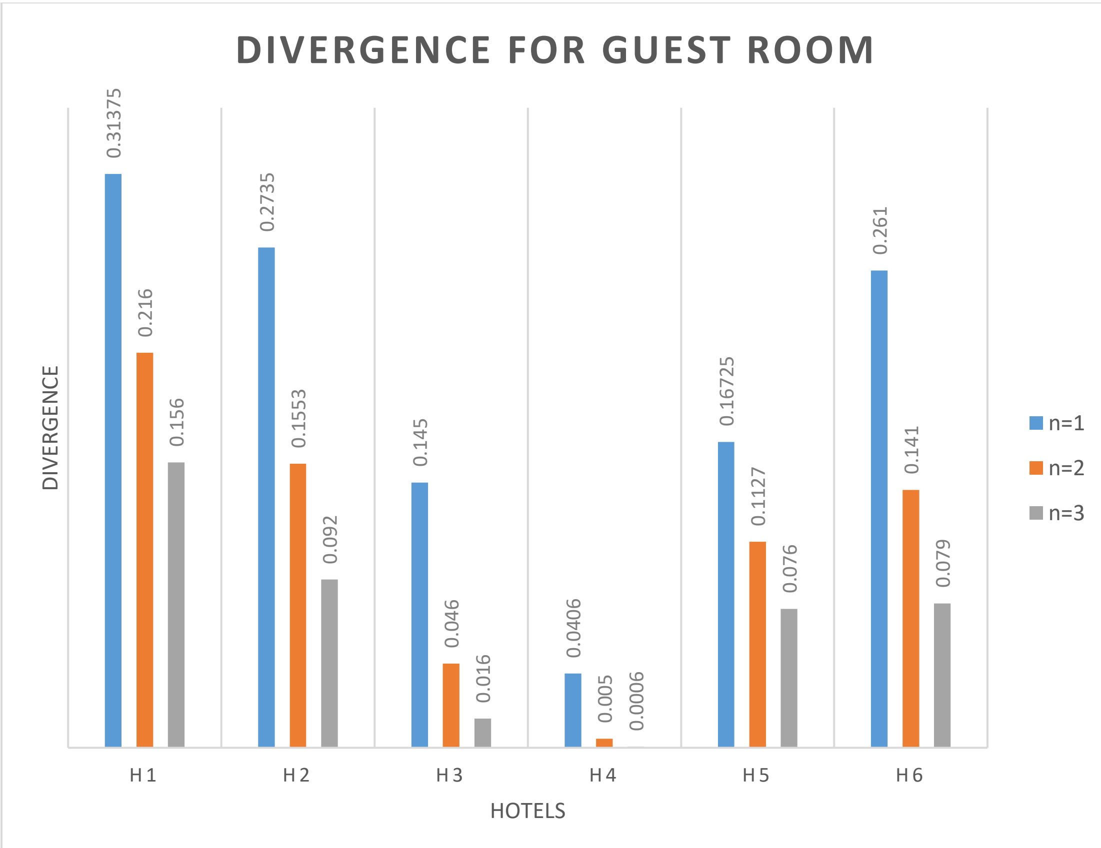
Figure 1. Divergence measure for Reception Hall



Ranking for Guest Room

$$H_4 \succ H_3 \succ H_5 \succ H_6 \succ H_2 \succ H_1$$

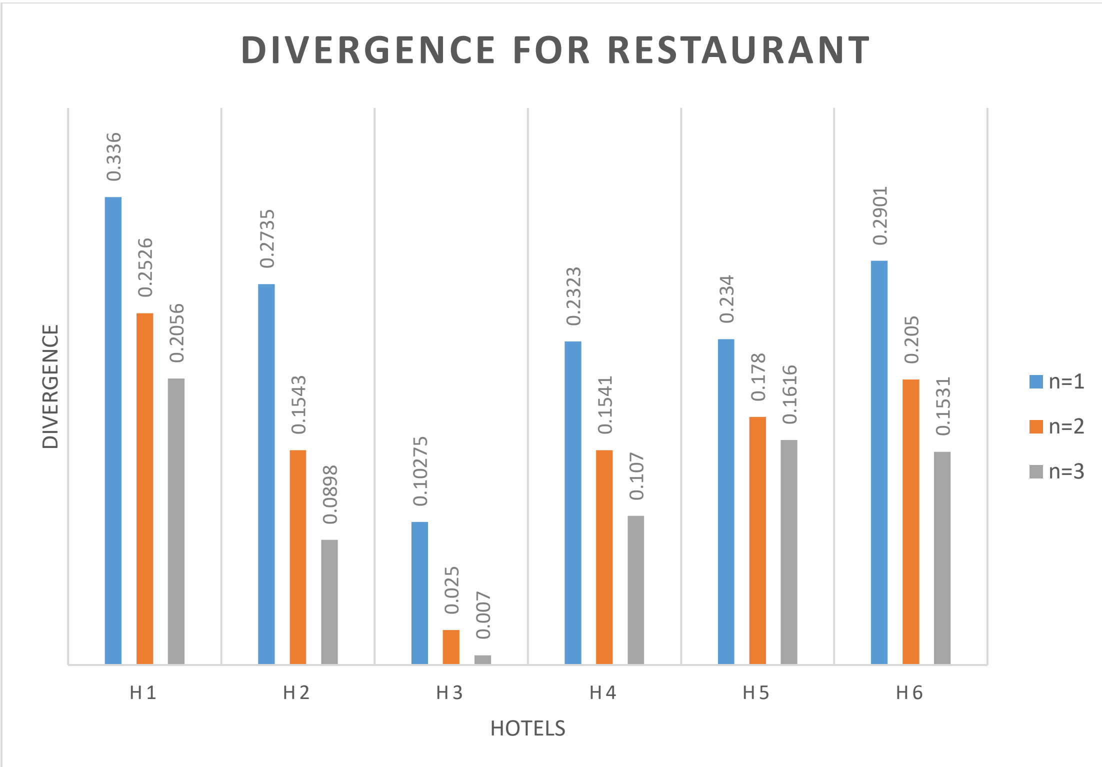
Figure 2. Divergence measure for Guest Room



Ranking for Restaurant

$$H_3 \succ H_2 \succ H_4 \succ H_6 \succ H_5 \succ H_1$$

Figure 3. Divergence measure for Restaurent



CONCLUSION

In this paper we extended the concept of hamming distance and proposed a new divergence model in Intuitionistic Fuzzy Sets. This model satisfies the axioms of divergence measure between IFSs. Our model applied to a numerical problem which gives better results as compared with the already existed well-known Jensen-Renyi Divergence. The important contribution of this paper is conversion of criteria's into sub-criteria's and holds good on pattern sets while most of the divergence models cannot holds. Another feature of our proposed method is that it avoids lengthy calculations and makes them simple. We solved a daily life problem by taking a survey of hotels of Hunza valley of Pakistan. The results attest the validity of our suggested method.

In addition, our future research will also focus on Pandemic COVID-19, this will help in order to rank different patients with respect to their level.

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