Stability analysis of time dependent Convection-Diffusion equation using low storage Explicit R-K method

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ABSTRACT

Finite Volume method is mostly used in CFD to evaluate the PDEs which become the form of algebraic equations. These types of equations are solved by matrix techniques or by numerical methods. In this work we have used Tri-Diagonal Matrix Algorithm to get solution of 2D Convection –Diffusion Equation. Which is capable for mesh problems. It applies line by line mesh nodes. We have worked with time dependent problem so, for this purpose researchers are used LSRK method to decrease the time step but our work is not to deal with step size. Our work is dependent upon stability analysis of solution. For this motive we have used central scheme and peclet number at different value of gamma. Stability of solution in this work dependent upon peclet number. When peclet number goes greater than 2 then our solution become unstable.

Keywords: Convection and Diffusion Equations, Central Scheme,Low storage Explicit RK method, Stability of Finite Volume Method, Tri-Diagonal Matrix Algorithm.

INTRODUCTION

The computational fluid dynamics has huge application. In CFD we mostly analysis the phenomena of a system like heat transfer, fluid flow and related some natural phenomena. It works with three elements in numerical which are, algorithm, convergence, consistency and stability we have ultimate choice to get more detailed in its result. Almost its work done by computer to make some code related to our problem. It is solved many which are involved in chemistry and physics with any lab experiment that's why, researchers used CFD. It gives us many facilities like, whenever we solve physical problem then we have need experimental lab which required some equipment with a large number of previous data but in CFD we just use an algorithm that is easily make on computer and run it to get all information about problem.[1][2]

ERK schemes are normally used for time integration of large-scale spatial discretization of partial differential equations. High-order RK methods are subjected to vast number of order conditions but they tend to have good stability properties compared to multi-step methods, as for example Adams20 Bashforth numerical schemes [3]. However, when a discontinuous solution occurs which regularlyarises hyperbolic solution by conservation laws, as for example Euler equations, linear stability theory is not sufficient and strong stability preserving (SSP) methods are required [4] [5]. We limit the analysis to the compressible Navier-Stokes equations and Burgers Equations at low storage. Either stability or accuracy may constrain the temporal integration step, thus whenever we study CDE's over time depended by numerically and apply ERK method approach [6], for each consciousness of random coefficient firstly we convert our problem into a system of equations which consist a deterministic CDEs then we used RK methodto discretize our problems due to its stability. In numerical problems, we usually apply low-rank variants to decrease memory and computational time computational time. It has been shown in the numerical by some software.R.C. Mittal et al [7] This research paper basically depend upon two dimensional unsteady convection diffusion equation in which author uses a numerical scheme to solve unsteady transport equation. It has a rectangular domain with uniform mesh grid.Peter Hansbo et al [8] in this paper author's work is based on the stabilization. It is reliable on the faces of the background grid whichoffers control over the jump in the normal gradient.

Randolph E.Bank et al [9] this is paper that is depend upon time dependent PDEs which convert into diffusion equation. It is basically a FEM which represent a simulation of any physical phenomenon and Engineers use it to reduced it in numerical scheme and then analyze the accuracy as well as stability.SashikumaarGanesan et al [12] this is paper that is depend upon transient convection diffusion equation which also involve time-depend problems. Sometimes we use Streamline Upwind Petrov–Galerkin (SUPG) to find out numerical solution of convection dominated problems.Hande Fendoğlu et al [14] this paper that is depend upon numerical solution of transient CDEs in 2D. Author convert (TCDR) equation into exponential transforming by some well-known approaches like, DRBEM (Domain reciprocity boundary element method), DBEM (Domain boundary element method), and MHD (Magneto-hydro-dynamics).Muhammad Saqib et al [15] in this paper, author work on Nonhomogeneous convection-diffusion equation in 2D. In which use take a convection diffusion and convert it to another equation by RDTM (reduced differential transformer method) to find out the analytical solution. It is recent work in the world. For finding numerical solutions have been developed to solve these types of convection-diffusion problems.MostafaAbbaszadeh et al [16] in this paper, author analysis of mixed finite method. It is finite element method which solve PDEs numerically.For finding better accuracy and good stability. Arafat Hussain et al [17] this paper is depend upon the second-order convection diffusion equation and author works on interface flux approximations by the numerically scheme along with finite elements method that is known as "Finite volume method" (FVM). This method is useful for time depend problems to find out the numerical simulation of the "Non-Newtonian fluid" with unsteady-state convection diffusion.Gerardo Tinoco-Guerrero et al [18] This paper is also depend upon Advection-diffusion equation in which author work on stability of these type of equation so, for this author uses "Generalized finite Difference" method that works on non-rectangular and highly irregular region using convex. H. Hernández et al [19]the author represent that partial differential equation. It can be convection, diffusion and their reaction term arise naturally in many branches of engineering and social science. These equations have very few analytical solution in some rare cases which has generally lack of interest. If some of them be responsible forcomputational, physical and mathematical valuable insights. Many problems have non-linearity and intricate boundary conditions which make complicate to the real problem. These problems also occur irregular geometries and heterogeneity in space. Somewhere, it makes transport coefficientscomplicate to the solution more where analytical solution fails.

2.1 Finite Volume Method

This method is mostly used in CFD to evaluate the PDEs which form a system into an algebraic equation [24]. In this method, we compute the values at distinct nodes of the control volume with geometrically. The finite volume method works on each small volume surrounding a point on a mesh node. Here we use divergence theorem that helps to convert divergence term into full surface.We evaluate as a flow at the surface of each FV. In which the inflow and outflow always at the adjacent volume. The governing equation is used for its conservative form in the FVM methodology for cracking any CFD problem [25].

2.2 Grid generation

In grid generation method we select a small area by dividing it into discrete control volume in FVM. The edges of the control volumes are locatedhalfway between neighboring nodes. Consequently, control volume or cell enclosed every node [26]. The geometrically representation ofFVM for two-dimensional diffusion problems is shown in fig: 2.1. The methodology used in deriving discretized equations in the one-dimensional. It is a case in which we can easily protracted to two-dimensional problems. To explain the procedure let us consider the two-dimensional steady state diffusion equation given below.

$$u\frac{dT}{dx}+v\frac{dT}{dy}+\frac{d}{dx}(\Gamma\frac{dT}{dx})+\frac{d}{dy}(\Gamma\frac{dT}{dy})S_T=0 \quad (1)$$

A portion of the two-dimensional grid used for the discretization is shown in Figure 2.1

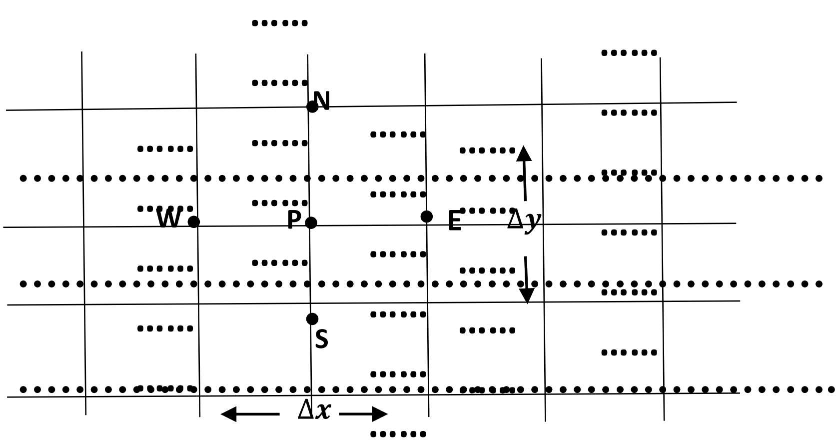


Figure 2.1: Domain Discretization

In above figure, we can observe that P has two neighbors nodes east (E) and west (W) which are right to left respectively. In addition, P has also two other neighbors nodes north (N) and south (S) which are up and down respectively. The same notation as in the one-dimensional analysis is used for faces and cell dimensions. The equation (1) can be written over a control

$$\int_{\Delta v}\frac{d}{dx}\left(\Gamma\frac{dT}{dx}\right)dx.dy+\int_{\Delta v}\frac{d}{dy}\left(\Gamma\frac{dT}{dy}\right)dx.dy+\int S_T dV=0 \quad (2)$$

Where $A_e=A_w=\Delta y$ and $A_n=A_s=\Delta x$,

$$\left[\Gamma_e A_e\left(\frac{\partial T}{\partial x}\right)_e-\Gamma_w A_w\left(\frac{\partial T}{\partial x}\right)_w\right]+\left[\Gamma_n A_n\left(\frac{\partial T}{\partial y}\right)_n-\Gamma_s A_s\left(\frac{\partial T}{\partial y}\right)_s\right]+\bar{S}\Delta V=0 \quad (3)$$

This equation shows that how control volume balances of the generation in temperature (T). CV also fluxes though its cell faces. It is also can be written for the flux by control volume faces.

$$\Gamma_e A_e\left(\frac{T_E-T_P}{\delta X_{PE}}\right)-\Gamma_w A_w\left(\frac{T_P-T_W}{\delta X_{WP}}\right)+\Gamma_n A_n\left(\frac{T_N-T_P}{\delta y_{PN}}\right)-\Gamma_s A_s\left(\frac{T_P-T_S}{\delta y_{SP}}\right)+\bar{S}\Delta V=0 \quad (4)$$

For linearized form, we will take $\bar{S}\Delta V=S_u+S_pT_p$, then it becomes.

$$\left(\frac{\Gamma_w A_w}{\delta X_{WP}}+\frac{\Gamma_e A_e}{\delta X_{PE}}+\frac{\Gamma_s A_s}{\delta y_{SP}}+\frac{\Gamma_n A_n}{\delta y_{PN}}\right)T_P=$$

$$\left(\frac{\Gamma_w A_w}{\delta X_{WP}}\right) T_w + \left(\frac{\Gamma_e A_e}{\delta X_{PE}}\right) T_E + \left(\frac{T_P - T_S}{\delta y_{SP}}\right) T_S \left(\frac{T_N - T_P}{\delta y_{PN}}\right) T_N + S_u \quad (5)$$

Equation (5) is for interior nodes which is known as general discretized equation.

$$a_P T_P = a_w T_W + a_E T_E + a_S T_S + a_N T_N + S_u \quad (6)$$

Where

a_W	a_E	a_S	a_N	a_P
$\frac{\Gamma_w A_w}{\delta X_{WP}}$	$\frac{\Gamma_e A_e}{\delta X_{PE}}$	$\frac{\Gamma_s A_s}{\delta y_{SP}}$	$\frac{\Gamma_n A_n}{\delta y_{PN}}$	$a_w + a_E + a_s + a_N - S_P$

In others words,

$$a_p T_p = \sum a_{np} T_{np} + S_u \quad (7)$$

Where, $a_p = \sum a_{np} - S_p$ (8)

Where sigmashows summation over all neighboring nodes (np) that represent the value of T at neighboring cells. S_u *and* S_P Represents the linearity source term. The boundary condition can be modify by discretized equation. It is known as temperature or flux at a node whenever we divide grid into sub-domain.First we select coefficients to zero on boundary side. The flux crossing the boundary occurs source termS_u *and* S_P.

2.3 The tri-diagonal matrix algorithm

We consider a system of equation in the form of tri-diagonal.

$p_1 = k_1$ (2.3a)

$-b_2 p_1 + D_2 p_2 - a_2 p_3 = k_2$ (2.3b)

$-b_3 p_2 + D_3 p_3 - a_3 p_4 = k_3$ (2.3c)

$-b_4 p_3 + D_4 p_4 - a_4 p_5 = k_4$ (2.3d)

=

$$-b_n p_{n-1} + D_n p_n - a_n p_{n+1} = k_n \quad (2.3n)$$

$$p_{n+1} = k_{n+1} \quad (2.3\ n+1)$$

Where, p_1 and p_{n+1} both are called boundary values. The single equation can be written as.

$$-b_j p_{j-1} + D_j p_j - a_j p_j = k_j \quad (2.3)$$

We can write equation (2.1b-n) as.

$$p_2 = \frac{a_2}{D_2} p_3 + \frac{b_2}{D_2} p_1 + \frac{k_2}{D_2} \quad (2.3.1a)$$

$$p_3 = \frac{a_3}{D_3} p_4 + \frac{b_3}{D_3} p_2 + \frac{k_3}{D_3} \quad (2.3.2b)$$

$$p_4 = \frac{a_4}{D_4} p_5 + \frac{b_4}{D_4} p_3 + \frac{k_4}{D_4} \quad (2.3.3c)$$

. . . .

$$p_{\mathrm{n}} = \frac{a_n}{D_n} a_{n+1} + \frac{b_n}{D_n} p_{n-1} + \frac{k_n}{D_n}$$

The above equation can be solved by forward and backward Gauss elimination method and process start by removing p_2 from equation (2.3.2b) by substitution from equation (2.3.1a) to give

$$p_3 = \left(\frac{a_3}{D_3 - b_3 \frac{a_2}{D_2}} \right) p_4 + \left(\frac{b_3\left(\frac{b_2}{D_2} p_1 + \frac{k_2}{D_2}\right) + k_3}{D_3 - b_3 \frac{a_2}{D_2}} \right) \quad (2.4a)$$

If we assume the equation

$$A_2 = \frac{a_2}{D_2} \text{ And } K'_2 = \frac{b_2}{D_2} p_1 + \frac{k_2}{D_2} \quad (2.4b)$$

Equation (2.4a) can be written as

$$p_3 = \left(\frac{a_3}{D_3 - b_3 A_2} \right) p_4 + \left(\frac{b_3 K'_2 + k_3}{D_3 - b_3 A_2} \right) \quad (2.4c)$$

If we let

$$A_3 = \frac{a_3}{D_3 - b_3 A_2} \text{ And } K'_3 = \frac{b_3 K'_2 + k_3}{D_3 - b_3 A_2}$$

Equation (2.4c) can be re-cast as $p_3 = A_3 p_4 + K'_3$ (2.5)

We apply formula (2.5) to eliminate p_3 from (2.3c). This procedure can be repeat up to the last equation of the set. This creates the onward elimination process.

For the **back-substitution** we use the general form of recurrence relationship (2.5):

$$p_j = A_j p_{j+1} + K'_j \quad (2.6a)$$

Where

$$A_j = \frac{a_j}{D_3 - b_j A_{j-1}} \quad (2.6b)$$

$$K'_j = \frac{b_j K'_{j-1} + k_j}{D_j - b_j A_{j-1}} \quad (2.6c)$$

Here we take j=1 and j = n+1 for boundary points through setting the values for A and$K$':

$A_1 = 0$ And $K'_1 = p_1$

$A_{n+1} = 0$ And $K'_{n+1} = p_{n+1}$

For solving a system of equation, first we convert equation into form of equation (2.2) where a_j, b_j, D_j and K'_j are identified. The values of A_j and K'_j are easily calculated by taken j = 2 and going up to j = n throughusing (2.6b-c). Subsequently, the p called boundary location (n+1).We can also get values for $p_j$in reverse order (p_n, p_{n-1}, p_{n-2}, ..., p_2). By using recurrence formula (2.6a). It is a easiest method which is used in CFD. Here in this work, we use matlab code to get our result.

2.6 Low storage Explicit RK method

This method is also known as an iterative method which is used to find out the approximation solution of ODEs. It is also capable method for differential algebraic equation. It is accustomed discretize in time for finding time dependent PDE. Its accustomed discretize different types of PDEs that would end in a high dimensional ODE or DAE. We have a tendency to use LSRK technique that stores two registers per ODE dimension tht limits the impact of magnified storage necessities. It help to extend of the amount of stages while not increasing storage expense.Let the initial value problem (IVP), we have

$$u'^{(t)} = g\big(t, u(t)\big), \quad u\big(t(0)\big) = u_0, \qquad (2.6.1)$$

Where g and u areused as vector functions. If we are interested to find out the approximating solution of$u(t)$ of the IVP concluded the time interval$t \in [t_0, t_g]$. We divide the interval into sub-intervals which divides P in equally spaced. This sub-intervals will integrate the solution but in RK method it is not required. Therefore, we take approximation points.

$$h = \frac{t_g - t_0}{P}, \qquad (2.6.2)$$

$$t_n = t_o + nh, \text{ n=0, 1… P}, \qquad (2.6.3)$$

Where $h$represents the time step size. For getting approximation solution of $u(t_n)$at $u(t_{n-1})$we have used one step of a general, explicit RK method.

c_1	0			
$\vdots$	$a_{2,1}$	$\ddots$		
$\vdots$	$\vdots$	$\ddots$		
c_s	$a_{s,1}$	$\cdots$	$a_{s,s-1}$	0
	b_1		$\cdots$	b_s

Table 2.3

For numerically solving Equation (2.6.1)

$$K_i = f\big(t_{n-1} + c_i h, u_{n-1} + h \sum_{j=1}^{i-1} a_{ij} K_j\big), \text{ i=1,……,s}, \qquad (2.6.4)$$

$$u_n = u_{n-1} + h \sum_{i=1}^{s} b_i K_i \qquad (2.6.5)$$

Where s shows the number of stages.a_{ij}Representthe intermediate weights at each RK stage.b_jsIsthe concluding stage weights and $c_i$shows the intermediate time levels. We require that

$$c_i = \sum_{j=1}^{s} a_{ij}, \qquad (2.6.6)$$

Therefore equation (2.6.1) is used for RK integration. It is considered as.

$$\hat{u}'(t) = \hat{g}(\hat{u}(t), \quad (2.6.7)$$

$$\hat{u}=\binom{u}{t}, \hat{g}=\binom{g}{t}. \quad (2.6.8)$$

In table: 2.3, it shows how s stages occur through explicit RK by butcher tableau for Equations (2.6.4) and (2.6.5). In this type of problems, we mostly used RK method in fourth order. It is also called four stage method which denoted to RK4. Algorithm 2.1 gives a MATLAB implementation. The RK methodology could be a five-stage methodology with fourth order accuracy. This sort of methodology is taught as an occasional storage specific Runge-Kutta (LSERK). One necessary property is that the incontrovertible fact that every stage is often cypher from the previous stage solely, requiring solely storage for one vector. In this, stability range compares to the other fourth order methods. The algorithmic rule is given by.

$$P^0 = \bar{u}(t)$$

$$i \in [1, \dots, 5]: \begin{cases} k^{(i)} = a_i k^{i-1} + \Delta t L_h(p^{i-1}, t + c_i \Delta t), \\ P^i = P^{i-1} + b_i k^i \end{cases} \quad (2.6.9)$$

$$\bar{u}(t^{n+1}) = P^5,$$

Where $P^i, i = 1, \dots, 5$ are the stage vectors and the coefficients a_i, b_i and c_i are given in Table 2.4.

i	a_i	b_i	c_i
1	0	$\frac{1432997174477}{9575080441755}$	0
2	$-\frac{567301805773}{1357537059087}$	$\frac{5161836677717}{13612068292357}$	$\frac{1432997174477}{9575080441755}$
3	$-\frac{240267990393}{2016746695238}$	$\frac{1720146321549}{2090206949498}$	$\frac{2526269341429}{6820363962896}$
4	$-\frac{3550918686646}{2091501179385}$	$\frac{3134564353537}{4481467310338}$	$\frac{2006345519317}{3224310063776}$

5	$-\frac{1275806237668}{842570457699}$	$\frac{2277821191437}{14882151754819}$	$\frac{2802321613138}{2924317926251}$

Table 2.4: Coefficients of Low-Storage five-stage fourth-order ERK method.

3.1 THE FINITE VOLUME DISCRETIZATION:

Consider the equation (6).

$$a_P T_P = a_W T_W + a_E T_E + a_S T_S + a_N T_N + S_u \quad (1)$$

Where, $a_P = a_W + a_E + a_S + a_N - S_P$

The area $A_e = A_w = D\Delta y, \quad A_n = A_S = D\Delta x$

Where,$a_W = \frac{k}{\Delta x} A_W; \quad a_E = \frac{k}{\Delta x} A_e; \quad a_S = \frac{k}{\Delta y} A_s; \quad a_N = \frac{k}{\Delta y} A_n$

S_u=0 and$S_P = 0$, for all interior points.

For boundary nodes, equation becomes.

For fixed value T $\quad S_u = \frac{2kA}{\Delta} T$ and $S_p = -\frac{2kA}{\Delta}$

4.1 Test case 1:

In test case 1, we take numbers of points in each direction is 51 so the numbers of intervals will be 50. In this case we see the flow of temperature horizontally which is depend upon diffusion parameter.The stability of solution is depended on the value on peclet number. If peclet number is less than 2, it gives stable solution. Here is the result of peclet number after taking some values of diffusion parameterΓ. As shown in below.

Table: 4.1Different peclet numbers with fixed step size h = 0.02 and V=U=Rho=1

S. No	V	U	Rho	N	h	Pe
1	1	1	1	50	0.02	0.2000
2	1	1	1	50	0.02	0.4000
3	1	1	1	50	0.02	0.8000
4	1	1	1	50	0.02	1.6000
5	1	1	1	50	0.02	3.2000

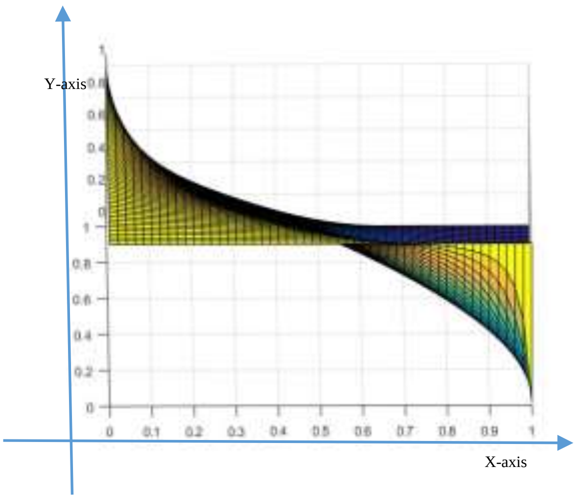


Figure 4.3a: Stable solution, $\Gamma = 0.1$

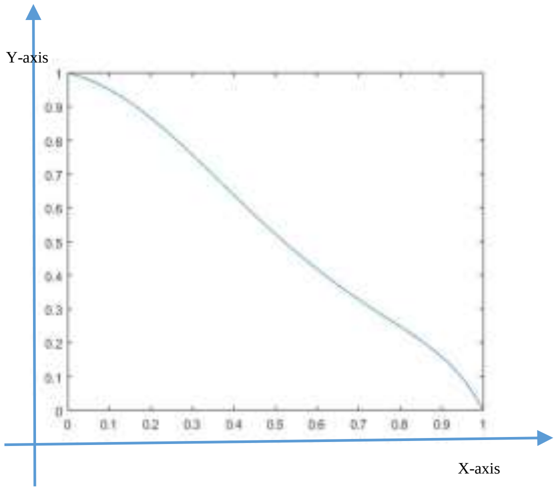


Figure 4.3b: Plate of solution T

In Fig: 4.3a we consider the velocity field $\vec{V}(u, v)$. it seemed how diffusion of the temperature T is high at$\Gamma = 0.1$ by horizontally in first figure and in fig: 4.3b, it is clear the behavior of diffusion which is an effected by the value of gamma. If take large value then it gives us high diffusion because it increased the peclet number but the solution become stable.

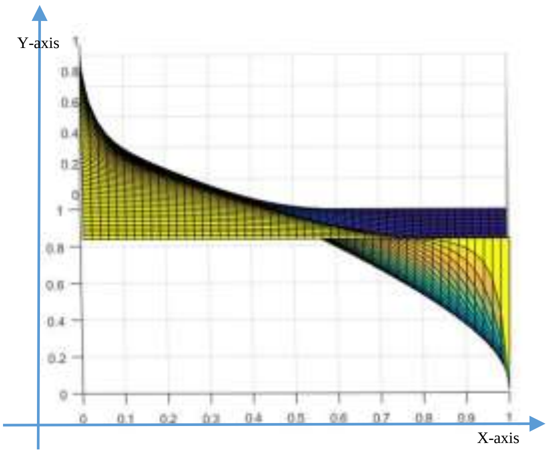


Figure 4.4a: Stable solution, $\Gamma = 0.05$

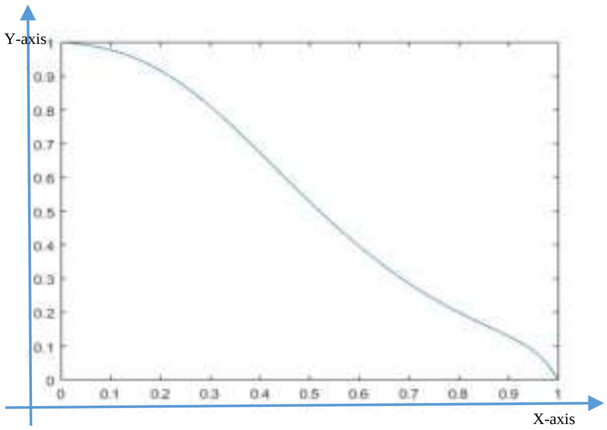


Figure 4.4b Plate of solution T

In Fig: 4.4a we consider the velocity field$\vec{V}(u, v)$. It seemed how diffusion of the temperature T is high at$\Gamma = 0.05$ by horizontally in first figure and in fig: 4.4b, it is clear the behavior of diffusion which is an effected by the value of gamma. If take large value then it gives us high diffusion because it increased the peclet number.

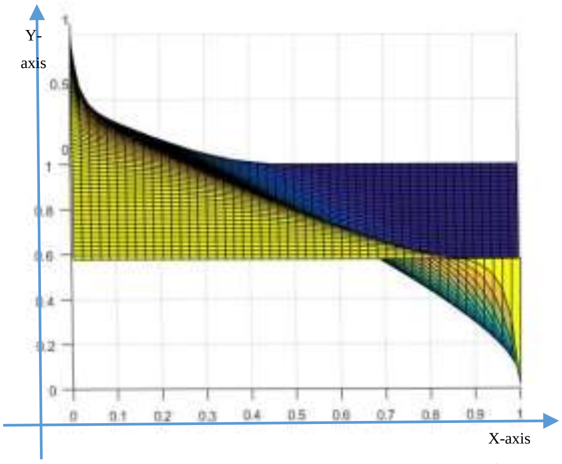


Figure 4.5a: Stable solution, $\Gamma = 0.025$

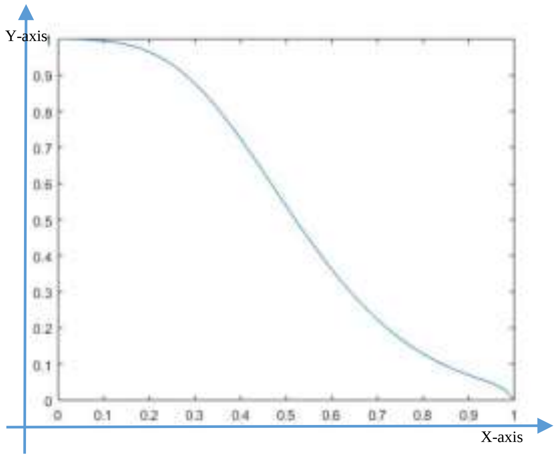


Figure 4.5b Plate of solution T

In Fig: 4.5a we consider the velocity field $\vec{V}(u, v)$. It seemed how diffusion of the temperature T is high at$\Gamma = 0.025$ which gives us less diffusion with peclet number less than 2 but solution is still stable which is shown in fig: 4.5b by smooth curved.

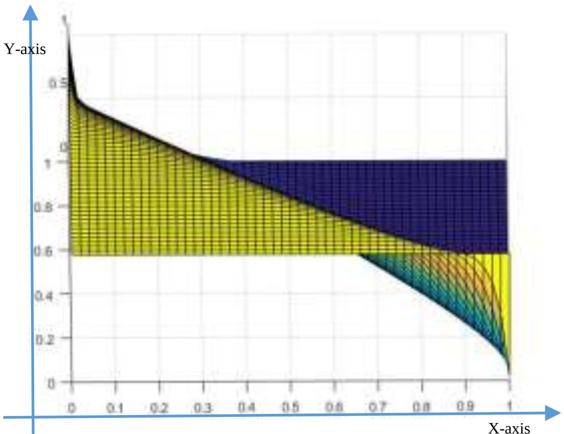


Figure 4.6a: Stable solution, $\Gamma = 0.0125$

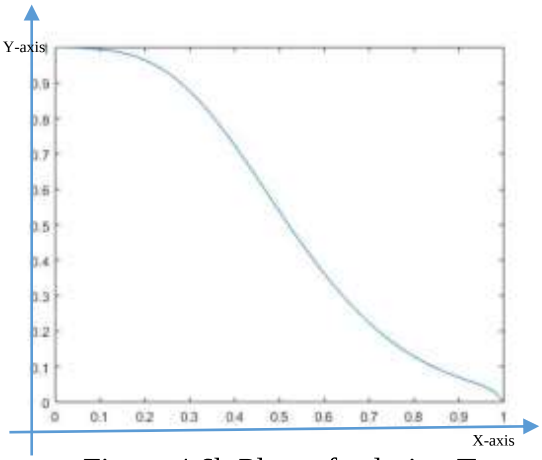


Figure 4.6b Plate of solution T

In Fig: 4.6a we consider the velocity field $\vec{V}(u, v)$. It seemed how diffusion of the temperature T at$\Gamma = 0.0125$ by horizontally in first figure and in fig: 4.6b, it is clear the behavior of diffusion which is an effected by the value of gamma which gives stable solution but if we take$\Gamma > 0.0125$ then our solution become unstable because it increased peclet number more than 2. Shown in fig: 4.7a & 4.7b below.

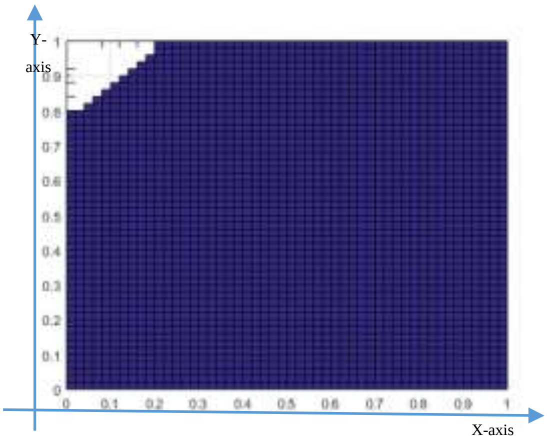


Figure 4.7a: Unstable solution,$\Gamma > 0.0125$

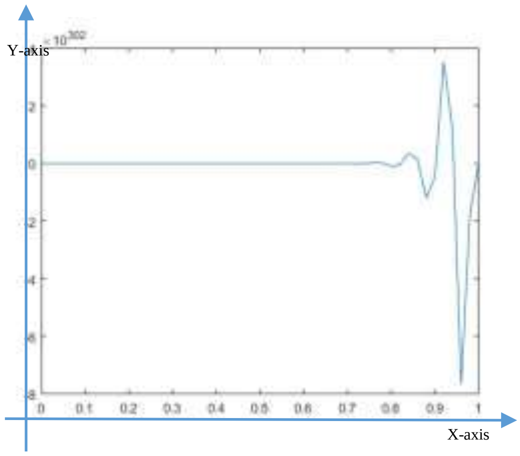


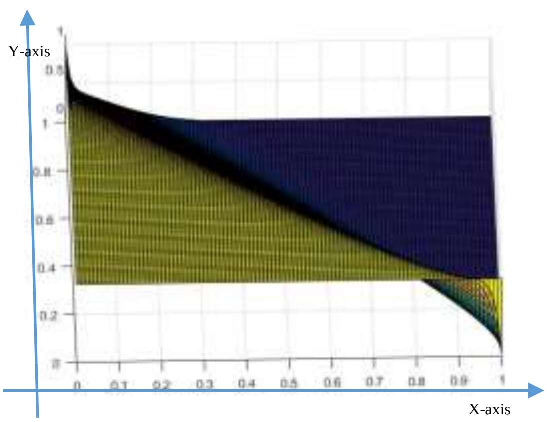
Figure 4.7b Plate of solution T

4.2 Test case 2:

In test case 1, we take numbers of points in each direction is 101 so the numbers of intervals will be 100. In this case we see the flow of temperature horizontally which is depend upon diffusion parameter.The stability of solution is depended on the value on peclet number. If peclet number is less than 2, it gives stable solution. Here is the result of peclet number after taking some values of diffusion parameterΓ. As shown in below.

Table: 4.2 Different peclet numbers with fixed step size h = 0.01 and V=U=Rho=1

S. No	v	u	Rho	N	h	Pe
1	1	1	1	100	0.01	1.000
2	1	1	1	100	0.01	2.000
3	1	1	1	100	0.01	4.000



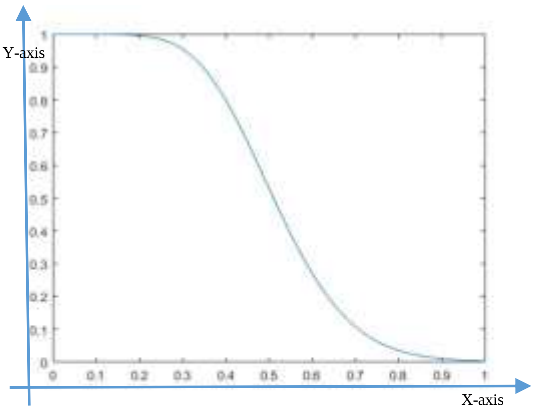
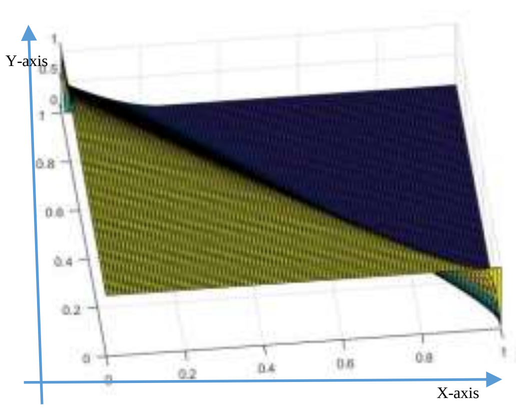


Figure 4.8a: Stable solution, $\Gamma = 0.01$ Figure 4.8b

In Fig: 4.8a we consider the velocity field $\vec{V}(u, v)$. It seemed how diffusion of the temperature T is high at$\Gamma = 0.01$ which gives us less diffusion with peclet number less than 2 and solution is stable which is shown in fig: 4.8b by smoothly curved



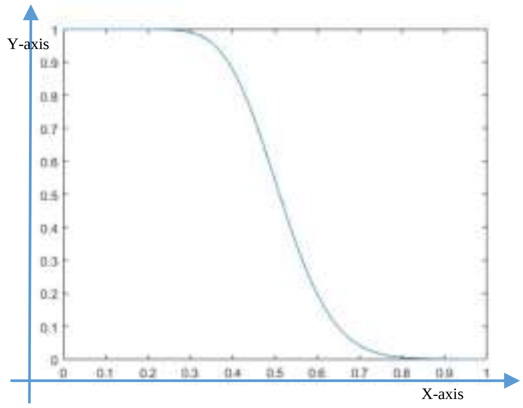
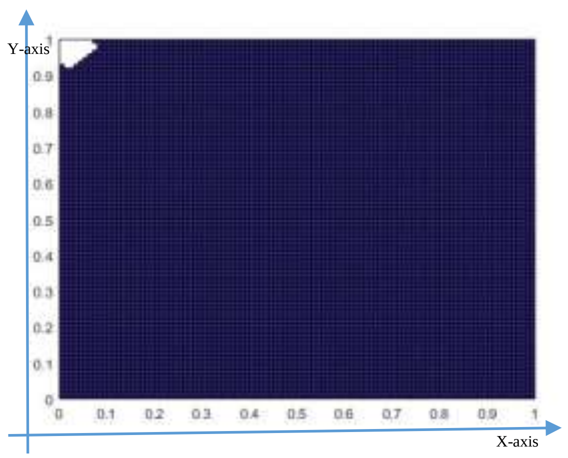


Figure 4.9a: Stable solution, $\Gamma = 0.005$ Figure 4.9b Plate of solution T

In Fig: 4.9a we consider the velocity field $\vec{V}(u, v)$. It appeared how diffusion of the temperature T at$\Gamma = 0.005$ by horizontally in first figure and in fig: 4.9b, it is clear the behavior of diffusion which is an effected by the value of gamma which gives stable solution but if we take$\Gamma > 0.005$ then our solution become unstable because it increased peclet number more than 2. Shown in fig: 4.10a & 4.10b below.



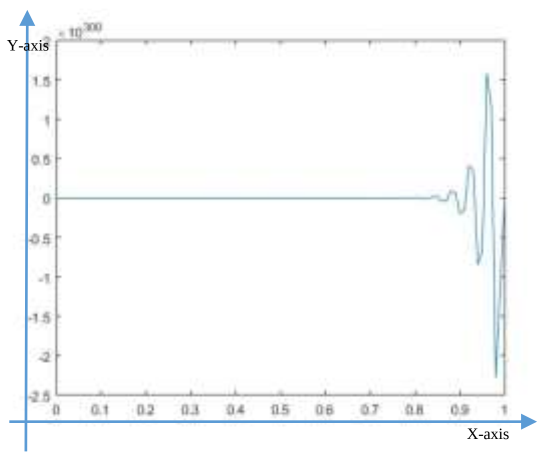


Figure 4.10a: Stable solution, $\Gamma > 0.005$ Figure 4.10b Plate of solution T

4.3 Test case 3:

In test case 1, we take numbers of points in each direction is 201 so the numbers of intervals will be 200. In this case we see the flow of temperature horizontally which is depend upon diffusion parameter.The stability of solution is depended on the value on peclet number. If peclet number is less than 2, it gives stable solution. Here is the result of peclet number after taking some values of diffusion parameterΓ. As shown in below.

Table: 4.3 Different peclet numbers with fixed step size h = 0.01 and V=U=Rho=1

S. No	v	u	Rho	n	h	Pe
1	1	1	1	200	0.01	2.000
2	1	1	1	200	0.01	4.000

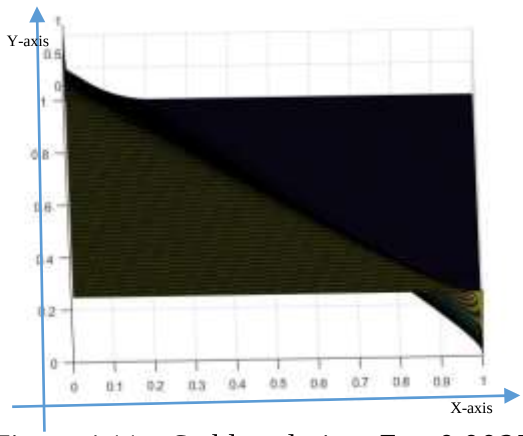


Figure 4.11a: Stable solution, $\Gamma = 0.0025$

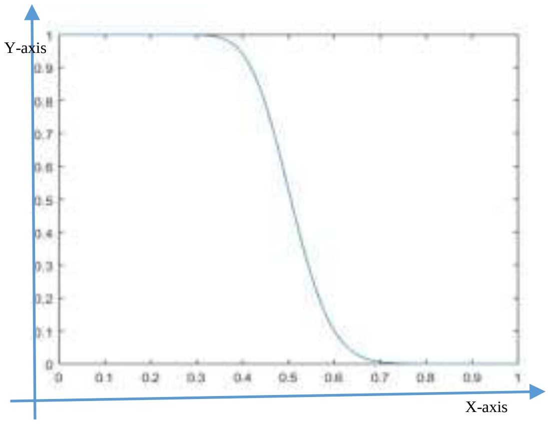


Figure 4.11b Plate of solution T

In Fig: 4.11a we consider the velocity field $\vec{V}(u, v)$. It seemed how diffusion of the temperature T at$\Gamma = 0.0025$ by horizontally in first figure and in fig: 4.11b, it is clear the behavior of diffusion which is an effected by the value of gamma which gives stable solution but if we take$\Gamma > 0.0025$ then our solution become unstable because it increased peclet number more than 2. Shown in fig: 4.11a & 4.11b below

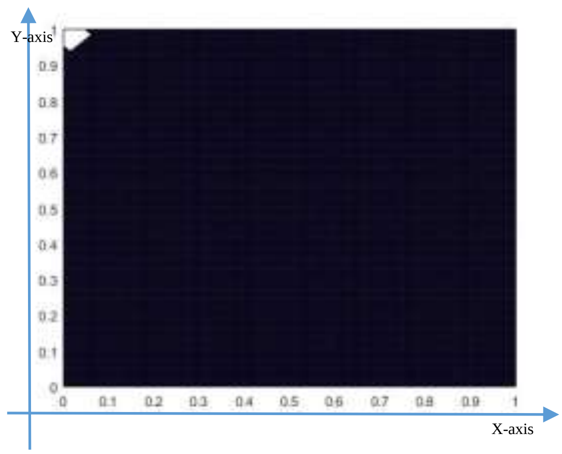


Figure 4.12a: Unstable solution, $\Gamma > 0.002$

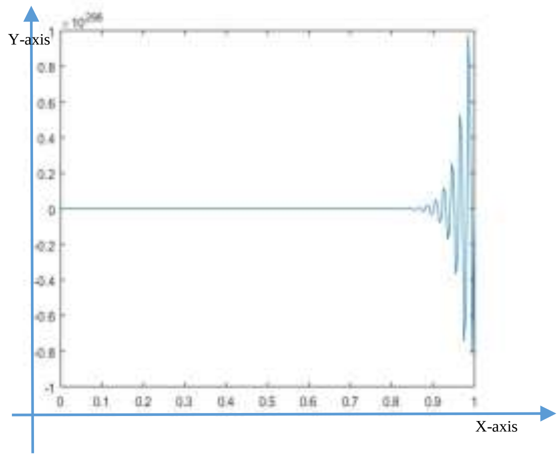


Figure 4.12b Plate of solution T

5. Conclusion

For stability analysis, we took different value of diffusion parameter in our case study and see that how convection is depend upon peclet number. If peclet number is high then diffusion will be decreased and solution becomes unstable when peclet number goes larger than 2.

Highly convective dominate flow lead unstable in the solution due to central scheme**.**

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