ROBUST UNCERTAINTY ALLEVIATION BY H-INFINITY ANALYSIS AND CONTROL FOR SINGULARLY PERTURBED SYSTEMS WITH DISTURBANCES

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Abstract- The central point of this paper is the problem of robust stability and robust H_∞ control design for a class of continuous-time singularly perturbed systems with time varying norm bounded uncertainties in all systems matrices. By using the fixed point principle, a sufficient condition to guarantee that the given system is in the standard from is given. Secondly the two time scale technique is applied to decompose the system into slow and fast subsystems. Based on the slow and fast subsystems, the problem of H_∞ robust uncertainty alleviation with stability and control is solved via the notion of generalized quadratic stability and stabilization with H_∞ norm bound for all sufficiently small values of the perturbation parameter. Necessary and sufficient conditions for generalized quadratic stability and stabilizability with a prescribed H_∞ performance level are derived. Our result which has not been discussed in earlier reports can be regarded as extensions of existing results on H_∞ control and robust stabilization.

Index Terms- SingularlyPerturbed Systems (SPSs), Linear Matrix Inequality (LMI), Generalized Quadratic Stability (GQS), Robust Analysis and H_∞ Control

I. INTRODUCTION

Singularly perturbed systems (SPSs) have been an emerging topic that attracted many researchersdue to their applications in engineering such as aircraft and racket systems, power systems and nuclear reactors, see e.g. [14] and[5]. The traditional method applied to SPSs is the singular perturbation method or reduction technique which provides an egress in the case of singularity leading to a prospective ill-defined problem**Error! Reference source not found.**. The survey on the progress of SPSs and their applications can be found in**Error! Reference source not found.**, [11] and the references therein. In recent years, the H_∞ control for SPSs is a problem of recurring interest. Mostly, it is known that the solution to this problem for linear time invariant system involves solving a pair of indefinite algebraic Riccati equations; see e.g. [15]. In the meantime, the Riccati equation approach to the quadratic stabilization of uncertain linear systems has been considered in numerous papers; see e.g. [8], [12] and [14].

Recently, interest has grown for the problem of robust H_∞ control for uncertain singularly perturbed systems with parameter uncertainty. The goal is to design a controller such that both robust stability and prescribed H_∞ performance level are satisfied.ByRiccati equations approach, [1]investigates robust disturbance attenuation for a class of singularly perturbed linear systems with norm-bounded parameter uncertainties in both state and output equations where a composite linear controller is designed such that both robust stability and a prescribed H-infinity performance for the full-order system are achieved, irrespective of the uncertainties.

In [6] by solving two independent Lyapunov equations, a control law is designed for singularly perturbed systems with nonlinear uncertainties and robust stabilization is achieved for all admissible parameter uncertainties. Regardless of how important they are, we agree that these methods are complex and difficult for application.

Very recently, the relationship between H_∞ control and the robust stabilization for a class of linear systems has been established in [8]. Also based on the reduction technique, the linear matrix inequality (LMI)has been used to solve different kinds of singularly perturbed systems. For example, in [4]a unified H_∞ approach is established by solving a set of Riccati equations; in[7] a control law is designed to make the system asymptotically stable under prescribed performance level and conservative when $\varepsilon \to 0$. Furthermore the LMI technique for H_∞ control problem has been developed in [13], where the H_∞ controller is given in terms of the solution of LMIs. It is quite relevant pointing out that the reduction technique is not adopted in these results where the singular perturbation parameter is viewed as a static scalar or the results are simply restricted to discrete time.

In this work, attention is focused on the robust uncertainty alleviation by the H_∞ approach for a class singularly perturbed system with time varying norm bounded parameter uncertainties in the state matrix, the input control matrix and the controlled output which is usually assumed to be zero in many cases. The approach adopted here relies on the notion of generalized quadratic stability and stabilization with an H_∞ norm bound which was introduced in [16]. First, by using the reduction technique, a necessary and sufficient LMI conditions are given for the performance analysis which alleviate not only the ill-conditioned problem but also guarantee the generalized quadratic stability with H_∞ norm bound property to the corresponding slow and fast subsystems. Based on this result, a unified LMI condition is presented to maintain the full order system in standard form and generalized quadratically stable with a prescribed H_∞ performance level irrespective of the uncertainties, provided that ε is sufficiently small. Secondly, if the nominal system is unstable, then a robust H_∞ controller is designed such that the resulting closed-loop system is generalized quadratically stabilizable with a prescribed H_∞ performance level irrespective of the uncertainties, provided that ε is sufficiently small.

Finally a new condition on searching the upper bound ε^* is proposed and explicitly estimated in a workable computational way. Note that this upper bound is not prescribed and fewer matrices variables are used, while such requirement is needed in [9] and [10]. Thus the effectiveness of the proposed method is clearly shown.

The notation used in this paper is fairly standard. $P > 0$ means that the matrix is symmetric and positive definite; $\|*\|$ stands for the Euclidean vector norm or the induced Euclidean matrix norm; '$*$' in a symmetric block matrices denotes the entry implied by symmetry; '$\neq$'in a matrix denotes the entry will not be used in the subsequent discussions; $L_2[0,\infty)$ stands for the space of square integrable vector functions over the interval $[0,\infty)$; $\|*\|_2$ denotes the L_2 vector norm.

The rest of the paper is organized s follows. Section2 gives the formulation. The performance analysis and control design are respectively given in Section 3 and Section 4. Section 5 gives the example to show effectiveness of the proposed method. Finally, the conclusion is drawn in Section 6

II. PROBLEM STATEMENT

In this brief we are interested in linear uncertain singularly perturbed systems with disturbance described by :

$$E_\varepsilon x(t) = (A + \Delta A(t))x(t) + B_w w(t) + (B_u + \Delta B_u(t))u(t), \quad (1)$$

$$z(t) = (C + \Delta C)x(t) + D_w w(t), \quad (2)$$

where $x = (x_1^T, x_2^T)^T \in R^n$ $(x_1 \in R^{n_1}, x_2 \in R^{n_2}, n = n_1 + n_2)$ is the state space , $u(t) \in R^n$ is the control input; $w \in R^m$ is the exogenous disturbance inputwhichbelongto $L_2[0,\infty)$; $\varepsilon > 0$ is the perturbation parameter which is small and positive butmay be unknown; $y(t) \in R^m$ is the output of the system, $E_\varepsilon = \begin{pmatrix} I & O \\ O & \varepsilon I \end{pmatrix}$, $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$, $B_w = \begin{pmatrix} B_{w1} \\ B_{w12} \end{pmatrix}$, $B_u = \begin{pmatrix} B_{u1} \\ B_{u2} \end{pmatrix}$, $C = (C_1 \quad C_2)$, $H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$, $E = (E_1 \quad E_2)$ and E_3 are constant matrices with appropriate dimensions; $\Delta A(t) = \begin{pmatrix} \Delta A_{11}(t) & \Delta A_{12}(t) \\ \Delta A_{21}(t) & \Delta A_{22}(t) \end{pmatrix}$, $\Delta B_u(t) = \begin{pmatrix} \Delta B_{u1}(t) \\ \Delta B_{u2}(t) \end{pmatrix}$ are time varying uncertainties satisfying the matching conditions

$$\begin{bmatrix}\Delta A & \Delta B_u & \Delta C\end{bmatrix} = \begin{bmatrix}HF(t)E & HF(t)E_3 & H_3F(t)E\end{bmatrix}, \quad (3)$$

where H_3 is constant matrix and $F(t)$ an unknown time-varying matrix satisfying

$$F^T(t)F(t) \le I,\ t > 0 .(4)$$

The H_∞ control problem studied in this paper can be described as follows: given a singularly perturbed system (1)-(2) and scalar $\lambda > 0$, design a state feedback controller in the following form

$$u(t) = Kx(t), \quad (5)$$

where $K = \begin{pmatrix}K_1 & K_2\end{pmatrix}$ is the control gain to be determined, such that the resulting closed-loop system satisfies the following requirements simultaneously: there exists $\varepsilon^* > 0$ such that

1) the resulting closed-loop system is *generalized quadratically stable* (GQS) for any $\varepsilon \in (0, \varepsilon^*]$;

2) under zero-initial condition $x(0) = 0$, the performance measurement

$$\int_0^\infty y^T(\tau)y(\tau)d\tau \le \gamma^2 \int_0^\infty w^T(\tau)w(\tau)d\tau$$

is satisfied for any nonzero $w(t) \in L_2[0,\infty)$.

The slow subsystem is obtained by setting $\varepsilon = 0$. Let $\begin{pmatrix}x_1 & x_2\end{pmatrix}\big|_{\varepsilon=0} = \begin{pmatrix}x_s & \bar{x}_2\end{pmatrix} = \bar{x}$, then system (1)-(2) became (6a-6b)

$$E_0\dot{\bar{x}}(t) = (A + \Delta A(t))\bar{x}(t) + B_w w_s(t)) \quad (6a)$$

$$y_s(t) = (C + \Delta C)\bar{x}(t) + D_w w_s(t) \quad (6b)$$

During the fast transient, the slow variables are assumed to be constant $(x_s = cst = 0)$. The fast variables represent the gap between the original value x_2 and the solution $\bar{x}_2 = \varphi = x_2\big|_{\varepsilon=0}$ i.e. $x_f = x_2 - \bar{x}_2$. Let introduce the following time scale $t = \varepsilon\,\tau$, then we have the following subsystem

$$\dot{x}_f(t) = (A_{22} + \Delta A_{22})x_f(\tau) + B_{w2}w_f(\tau) \quad (7a)$$

$$y_f(\tau) = (C_2 + \Delta C_2)x_f(\tau) + D_w w_f(\tau) \quad (7b)$$

where $w_f = w - w_s$ and $\Delta C_2 = H_3F(t)E_2$.

Lemma Error! Reference source not found.

If the uncertain SPSs in (1)-(2) is GQS with an H_∞-norm less than γ, then the system is robustly stabilizable with an H_∞-norm less than γ over the horizon $[0,\infty)$.

III. ROBUST H_∞ UNCERTAINTY ALLEVIATION

In this section, based on the reduced technique, we will provide sufficient condition such that the full order system (1) is GQS with an H_∞ norm less than γ, irrespective with uncertainty and uniformly in $\varepsilon > 0$ which is sufficiently small.

First we have Theorem1 and 2 for the slow and fast subsystems respectively.

Theorem 1

If there exist a scalar $\sigma > 0$, $\rho > 0$, $\mu > 0$ matrices P_{21}, P_{22} and symmetric positive definite matrix P_{11} such that the following LMI holds

$$\begin{pmatrix} A^TP + P^TA + C^TC + \sigma E^TE & P^TH & E^T & C^T & H_3^TD_w & P^TB_w + C^TD_w \\ * & -\sigma I & 0 & 0 & 0 & 0 \\ * & * & -\alpha I & 0 & 0 & 0 \\ * & * & * & -\rho I & 0 & 0 \\ * & * & * & * & -\mu I & 0 \\ * & * & * & * & * & -\gamma^2 I + D_w^TD_w \end{pmatrix} < 0, (8)$$

then the slow subsystems (6a-6b) is GQS with H_∞-norm less than γ.

Proof

The slow subsystem is well-defined. In fact, from condition (8) we have

$$\begin{pmatrix} A^TP + P^TA + \sigma E^TE & P^TH \\ * & -\sigma I \end{pmatrix} < 0,$$

which characterise the standard model.

Step 1: Selection of Storage function

Choose a storage function as

$$S_0(x_s(t)) = x_s^TP_{11}x_s$$

Since $\bar{x} = \begin{pmatrix}x_s^T & \bar{x}_2^T\end{pmatrix}^T$, then

$$x_s^TP_{11}x_s = \begin{pmatrix}x_s^T & \bar{x}_2^T\end{pmatrix}\begin{pmatrix}P_{11} & O \\ O & O\end{pmatrix}\begin{pmatrix}x_s \\ \bar{x}_2\end{pmatrix}$$

$$= \begin{pmatrix}x_s^T & \bar{x}_2^T\end{pmatrix}\begin{pmatrix}I & O \\ O & O\end{pmatrix}\begin{pmatrix}P_{11} & O \\ P_{21} & P_{22}\end{pmatrix}\begin{pmatrix}x_s \\ \bar{x}_2\end{pmatrix}$$

$$= \bar{x}^TE_0^TP\bar{x}.$$

where $E_0 = E_\varepsilon\big|_{\varepsilon=0}$. it is obvious that $S_0(x_s) > 0$ and for $x_s(0) = 0$ we have $S_0(0) = 0$.

Step 2: Derivation along the trajectories of (6)

$$\dot{S}_0(x_s) = \dot{\bar{x}}^TE_0^TP\bar{x} + \bar{x}^TE_0^TP\dot{\bar{x}} = (E_0\dot{\bar{x}})^TP\bar{x} + \bar{x}^TP^T(E_0\dot{\bar{x}})$$

$$= [(A + \Delta A)\bar{x} + B_w w_s]^TP\bar{x} + \bar{x}^TP^T[(A + \Delta A)\bar{x} + B_w w_s]$$

Noting that $\Delta A = HFE$, there exists $\sigma > 0$ such that

$$\Delta A^TP + P^T\Delta A \le \sigma E^TE + \sigma^{-1}P^THH^TP,$$

which implies

$$\dot{S}_0(x_s) \le \bar{x}^T(A^TP + P^TA + \sigma E^TE + \sigma^{-1}P^THH^TP)\bar{x} + 2\bar{x}^TP^TB_w w_s \quad (9)$$

Step 3 : The slow subsystems is subjected to the H_∞ -norm less than γ

Define the performance measurement as

$$J_s(t) = \int_0^t \left[(y_s^T(\tau)y_s(\tau) - \gamma^2 w_s^T(t)w_s(t) \right] d\tau \quad (10)$$

Then it is obvious that

$$J_s(t) = \int_0^t \left[(y_s^T(\tau)y_s(\tau) - \gamma^2 w_s^T(t)w_s(t) + \dot{S}_0(x_s(\tau)) \right] d\tau + S_0(x_s(0)) - S_0(x_s(t))$$

Substituting in the above equality $\dot{S}_0(x_s)$ obtained in (9) and y_s by (6b) it yields

$$J_s(t) \le \int_0^t \left\{ \begin{matrix} [(C+\Delta C)\bar{x} + D_w w_s]^T[(C+\Delta C)\bar{x} + D_w w_s] - \gamma^{-2}w_s^T w_s \\ +\bar{x}^T(A^TP + P^TA + \sigma E^TE + \sigma^{-1}P^THH^TP)\bar{x} + 2\bar{x}^TP^TB_w w_s \end{matrix} \right\} d\tau + S_0(x_s(0))$$

$$= \int_0^t \{ \bar{x}^T[A^TP + P^TA + \sigma E^TE + C^TC + C^T\Delta C + \Delta C^TC + \Delta C^T\Delta C + \sigma^{-1}P^THH^TP + 2\bar{x}^T[P^TB_w + (C+\Delta C)^TD_w]w_s + w_s^T(D_w^TD_w - \gamma^{-2})w_s \} d\tau + S_0(x_s(0))$$

$$J_s(t) \le \int_0^t \begin{pmatrix} \bar{x}^T & w_s^T \end{pmatrix} (\Phi_s + \Phi_{\Delta C}) \begin{pmatrix} \bar{x} \\ w_s \end{pmatrix} d\tau + S_1(x_s(0)), \quad (11)$$

where

$$\Phi_{\Delta C} = \begin{pmatrix} C^T\Delta C + \Delta C^TC + \Delta C^T\Delta C & \Delta C^TD_w \\ D_w^T\Delta C & 0 \end{pmatrix}, (12)$$

$$\Phi_s = \begin{pmatrix} A^TP + P^TA + C^TC + \sigma E^TE & P^TH & P^TB_w + C^TD_w \\ * & -\sigma I & 0 \\ * & * & -\gamma^2 I + D_w^TD_w \end{pmatrix}, (13)$$

Step 4: Alleviation of Uncertainties in $\Phi_{\Delta C}$

The alleviation of uncertainties in $\Phi_{\Delta C}$ in mandatory and it is only after this step, a readable sufficient criterion can be proposed to guarantee that the slow subsystems is GQS with an H_∞ -norm less than γ over the horizon $[0, \infty)$.

Using $\Delta C = H_3FE$

$$\Phi_{\Delta C} = \begin{pmatrix} C^TH_3FE + E^TF^TH_3^TC + E^TF^TH_3^TH_3FE & E^TF^TH_3^TD_w \\ D_w^TH_3FE & 0 \end{pmatrix}$$

$$= \begin{pmatrix} C^TH_3FE & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} E^TF^TH_3^TC & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} E^TF^TH_3^TH_3FE & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ D_w^TH_3FE & 0 \end{pmatrix} + \begin{pmatrix} 0 & E^TF^TH_3^TD_w \\ 0 & 0 \end{pmatrix}, \quad (14)$$

Or

$$\begin{pmatrix} C^TH_3FE & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} C^TH_3 \\ 0 \end{pmatrix} F(t) \begin{pmatrix} E & 0 \end{pmatrix},$$

$$\begin{pmatrix} E^TF^TH_3^TC & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} E^T \\ 0 \end{pmatrix} F^T(t) \begin{pmatrix} H_3^TC & 0 \end{pmatrix} = \left[\begin{pmatrix} C^TH_3 \\ 0 \end{pmatrix} F(t) \begin{pmatrix} E & 0 \end{pmatrix} \right]^T,$$

$$\begin{pmatrix} E^TF^TH_3^TH_3FE & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} E^T \\ 0 \end{pmatrix} F^TH_3^TH_3F \begin{pmatrix} E & 0 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 0 \\ D_w^TH_3FE & 0 \end{pmatrix} = \begin{pmatrix} 0 \\ D_w^TH_3 \end{pmatrix} F(t) \begin{pmatrix} E & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & E^TF^TH_3^TD_w \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} E^T \\ 0 \end{pmatrix} F^T(t) \begin{pmatrix} 0 & H_3^TD_w \end{pmatrix} = \left[\begin{pmatrix} 0 \\ D_w^TH_3 \end{pmatrix} F(t) \begin{pmatrix} E & 0 \end{pmatrix} \right]^T$$

Based on the above transformations and using the Lemma in **Error! Reference source not found.**, there exists $\rho > 0$ and $\mu > 0$ such that

$$\begin{pmatrix} C^TH_3 \\ 0 \end{pmatrix} F(t) \begin{pmatrix} E & 0 \end{pmatrix} + \left[\begin{pmatrix} C^TH_3 \\ 0 \end{pmatrix} F(t) \begin{pmatrix} E & 0 \end{pmatrix} \right]^T \le \rho^{-1} \begin{pmatrix} C^TH_3 \\ 0 \end{pmatrix} \begin{pmatrix} C^TH_3 \\ 0 \end{pmatrix}^T + \rho \begin{pmatrix} E^T \\ 0 \end{pmatrix} \begin{pmatrix} E^T \\ 0 \end{pmatrix}^T, \quad (15)$$

$$\begin{pmatrix} 0 \\ D_w^TH_3 \end{pmatrix} F(t) \begin{pmatrix} E & 0 \end{pmatrix} + \left[\begin{pmatrix} 0 \\ D_w^TH_3 \end{pmatrix} F(t) \begin{pmatrix} E & 0 \end{pmatrix} \right]^T \le \mu \begin{pmatrix} E^T \\ 0 \end{pmatrix} \begin{pmatrix} E^T \\ 0 \end{pmatrix}^T + \mu^{-1} \begin{pmatrix} 0 \\ D_w^TH_3 \end{pmatrix} \begin{pmatrix} 0 \\ D_w^TH_3 \end{pmatrix}^T, (16)$$

$$\begin{pmatrix} E^T \\ 0 \end{pmatrix} F^T H_3^T H_3 F \begin{pmatrix} E & 0 \end{pmatrix} \le \lambda \begin{pmatrix} E^T \\ 0 \end{pmatrix} \begin{pmatrix} E^T \\ 0 \end{pmatrix}^T, (17)$$

where $\lambda = \lambda_{\max}(H_3^T H_3)$.

Substituting (15), (16), and (17) into (14) it yields

$$\Phi_{\Delta C} \le \begin{pmatrix} (\lambda+\rho+\mu)E^T E + \rho^{-1} C^T H_3 H_3^T C & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & \mu^{-1} D_w^T H_3 H_3^T \end{pmatrix}, \quad (18)$$

Letting $\lambda+\rho+\mu=\alpha^{-1}$ and by using the Schur's complement principle on both terms of inequality (18) we can have

$$\begin{pmatrix} (\lambda+\rho+\mu)E^T E + \rho^{-1} C^T H_3 H_3^T C & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \alpha^{-1} E^T E & C^T H_3 \\ 0 & -\rho I \end{pmatrix} = \begin{pmatrix} 0 & E^T & C^T H_3 \\ 0 & -\alpha I & 0 \\ H_3^T C & 0 & -\rho I \end{pmatrix} (19)$$

$$\mu^{-1} D_w^T H_3 H_3^T = \begin{pmatrix} 0 & H_3^T D_w \\ D_w^T H_3 & -\mu I \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & H_3^T D_w \\ 0 & 0 & 0 \\ D_w^T H_3 & 0 & -\mu I \end{pmatrix}. \quad (20)$$

Putting (18) and (19) in (17) it yields

$$\Phi_{\Delta C} \le \begin{pmatrix} 0 & E^T & C^T H_3 \\ 0 & -\alpha I & 0 \\ H_3^T C & 0 & -\rho I \end{pmatrix} + \begin{pmatrix} 0 & 0 & H_3^T D_w \\ 0 & 0 & 0 \\ D_w^T H_3 & 0 & -\mu I \end{pmatrix} =$$

$$\begin{pmatrix} 0 & E^T & C^T H_3 & H_3^T D_w \\ * & -\alpha I & 0 & 0 \\ * & * & -\rho I & 0 \\ * & * & * & -\mu I \end{pmatrix}. \quad (21)$$

Step 5 Sufficient Condition for GQS and H_∞ norm less than γ

The condition $J_s(t)<0$ holds if in addition to the zero initial condition ($S_0(0)=0$) and letting $t\to\infty$, we have

$$\Phi_s + \Phi_{\Delta C} < 0. \quad (22)$$

If (22) holds, then the slow subsystems is GQ Stable with an H_∞ performance level γ over the horizon $[0,\infty)$ which implies that the slow subsystems is robustly stable by Lemma **Error! Reference source not found.**. The inequality (22) is defined as a unified LMI as follow

$$\Phi_s + \Phi_{\Delta C} =$$

$$\begin{pmatrix} A^T P + P^T A + C^T C + \sigma E^T E & P^T H & E^T & C^T H_3 & H_3^T D_w & P^T B_w + C^T D_w \\ * & -\sigma I & 0 & 0 & 0 & 0 \\ * & * & -\alpha I & 0 & 0 & 0 \\ * & * & * & -\rho I & 0 & 0 \\ * & * & * & * & -\mu I & 0 \\ * & * & * & * & v & -\gamma^2 I + D_w^T D_w \end{pmatrix}$$

which correspond to LMI (8). This completes the proof of Theorem 1.

Theorem 2

If there exist a scalar $\sigma>0$, $\rho>0$, $\mu>0$ and matrix $P_{22}=P_{22}^T>0$ such that the following LMI holds

$$\begin{pmatrix} A_{22}^T P_{22} + P_{22}^T A_{22} + C_2^T C_2 + \sigma E_2^T E_2 & P_{22}^T H_2 & E_2^T & C_2^T & H_3^T D_w & P_{22}^T B_w + C_2^T D_w \\ * & -\sigma I & 0 & 0 & 0 & 0 \\ * & * & -\alpha I & 0 & 0 & 0 \\ * & * & * & -\rho I & 0 & 0 \\ * & * & * & * & -\mu I & 0 \\ * & * & * & * & * & -\gamma^2 I + D_w^T D_w \end{pmatrix} < 0$$

(23)

then the fast subsystem (7a)-(7b) is Generalized Quadratically Stable with H_∞ norm less than γ over the horizon $[0,\infty)$.

Proof

Step 1: Selection of Storage Function

Suppose $P_{22}=P_{22}^T>0$, then $S_1(x_f(t))$ is our storage function defined as

$$S_1(x_f(t)) = x_f^T P_{22} x_f, \quad (24)$$

It is clear that $S_1(x_f(t))>0$ and $x_f(0)=0$, $S_1(0)=0$.

Step 2: Derivation along the trajectories of (7a)-(7b)

$$\dot{S}_1(x_f) = \dot{x}_f^T P_{22} x_f + x_f^T P_{22} \dot{x}_f$$
$$= [(A_{22}+\Delta A_{22})x_f + B_{w2} w_f]^T P_{22} x_f$$
$$+ x_f^T P_{22} [(A+\Delta A)x_f + B_{w2} w_f]$$

Since $\Delta A_{22} = H_2 F E_2$, there exists $\sigma>0$ such that

$$\Delta A_{22}^T P_{22} + P_{22}^T \Delta A \le \sigma E_2^T E_2 + \sigma^{-1} P_{22}^T H_2 H_2^T P_{22}, \quad (25)$$

Step 4 : Performance condition

The performance is characterized by the following measurement

$$J_f(t)=\int_0^t\left[(y_f^T(\tau)y_f(\tau)-\gamma^2 w_f{}^T(t)w_f(t)\right]d\tau\,, \qquad (26)$$

It is clear that

$$J_f(t)=\int_0^t\left[(y_f^T(\tau)y_f(\tau)-\gamma^2 w_f{}^T(t)w_f(t)+\dot{S}_1(x_f(\tau))\right]d\tau$$

$$+S_1(x_f(0))-S_1(x_f(t))\,,$$

Substituting $\dot{S}_1(x_f)$ by (25) and $y_f(t)$ by (7b) it yields

$$J_f(t)\le$$

$$\int_0^t\{[(C_2+\Delta C_2)x_f+D_w w_f]^T[(C_2+\Delta C_2)x_f+D_w w_f]$$

$$-\gamma^{-2}w_f^T w_f+x_f^T(A_{22}^T P_{22}+P_{22}^T A_{22}+\sigma E_2^T E_2$$

$$+\sigma^{-1}P_{22}^T H_2 H_2^T P_{22})x_f$$

$$+2x_f^T P_{22}^T B_{w2}w_f\}d\tau+S_1(x_s(0))$$

$$=\int_0^t\{x_f^T[A_{22}^T P_{22}+P_{22}^T A_{22}+\sigma E_2^T E_2+\sigma^{-1}P_{22}^T H_2 H_2^T P_{22}$$

$$+C_2^T C_2+C_2^T\Delta C_2+\Delta C_2^T C_2+\Delta C_2^T\Delta C_2]x_f$$

$$+2x_f^T[P_{22}^T B_{w2}+(C_2+\Delta C_2)^T D_w]w_f$$

$$+w_f^T(D_w^T D_w-\gamma^{-2})w_f\}d\tau+S_1(x_f(0))$$

$$J_f(t)\le\int_0^t\begin{pmatrix}x_f^T & w_f^T\end{pmatrix}(\Phi_f+\Phi_{\Delta C_2})\begin{pmatrix}x_f\\ w_f\end{pmatrix}d\tau+S_1(x_f(0)) \quad (27)$$

where

$$\Phi_{\Delta C_2}=\begin{pmatrix}C_2^T\Delta C_2+\Delta C_2^T C_2+\Delta C_2^T\Delta C_2 & \Delta C_2^T D_w\\ D_w^T\Delta C_2 & 0\end{pmatrix} \quad (28)$$

$$\Phi_f=\begin{pmatrix}A_{22}^T P_{22}+P_{22}^T A_{22}+C_2^T C_2+\sigma E_2^T E_2+\sigma^{-1}P_{22}^T H_2 H_2^T P_{22} & P_{22}^T B_{w2}+C_2{}^T D_w\\ * & D_w^T D_w-\gamma^{-2}\end{pmatrix} \quad (29)$$

Step 4: Alleviation of uncertainties in $\Phi_{\Delta C_2}$

$$\Delta C=\begin{pmatrix}\Delta C_1 & \Delta C_2\end{pmatrix}=H_3 F\begin{pmatrix}E_1 & E_2\end{pmatrix}=\begin{pmatrix}H_3 F E_1 & H_3 F E_2\end{pmatrix}$$

that implies $\Delta C_2=H_3 F E_2$.

Similar to the proof of Theorem 1 in step 4, the alleviation of uncertainties in this section can be done and we have:

$$\Phi_{\Delta C_2}\le\begin{pmatrix}0 & E_2^T & C_2^T H_3 & H_3^T D_w\\ * & -\alpha I & 0 & 0\\ * & * & -\rho I & 0\\ * & * & * & -\mu I\end{pmatrix}. \quad (30)$$

Step 5: Sufficient Condition for GQS with H_∞ norm less than γ

Using the zero initial condition ($S_1(0)=0$) and by letting $t\to\infty$, we have $J_f(t)<0$ holds if

$$\Phi_f+\Phi_{\Delta C_2}<0. \qquad (31)$$

If (31) holds, then the fast systems is GQS with an H_∞ norm less than γ.

The LMI (31) is a unified Linear Matrix Inequality and correspond exactly to LMI (23). Which complete the proof ofTheor2.

Theorem 3

If the condition of Theorem 1 and Theorem 2 holds, then there exist an $\varepsilon^*>0$ such that the original system (1)-(2) is Generalized Quadratically Stable with an H_∞ norm less than γ for any $\varepsilon\in(0,\varepsilon^*]$.

Proof

Under the conditions of Theorem 1 and 2, it is shown that P_{11} and P_{22} are symmetric and positive definite matrices, then there exists a sufficiently small scalar $\varepsilon_1>0$ such that $P_{11}-P_{21}^T P_{22}^{-1}P_{21}>0$ for $\forall\varepsilon\in(0,\varepsilon_1]$. Thus, by the Schur's complement

$$E_\varepsilon^T P_\varepsilon=P_\varepsilon^T E_\varepsilon=\begin{pmatrix}P_{11} & \varepsilon P_{21}^T\\ \varepsilon P_{21} & \varepsilon P_{22}\end{pmatrix}>0\,,$$

where $P_\varepsilon=\begin{pmatrix}P_{11} & \varepsilon P_{21}^T\\ P_{21} & P_{22}\end{pmatrix}>0,\ \varepsilon\in(0,\varepsilon_1]$.

Define a storage function as follows

$$S(x)=x^T E_\varepsilon^T P_\varepsilon x\,,$$

then for any constant $\sigma>0$, the derivative of $S(x)$ satisfies

$$\dot{S}(x)\le x^T(A^T P_\varepsilon+P_\varepsilon^T A+\sigma E^T E+\sigma^{-1}P_\varepsilon^T H H^T P_\varepsilon)x$$

$$+2x^T P_\varepsilon^T B_w w \quad (32)$$

Define the performance function as follows

$$J(t)=\int_0^t\left[(y^T(\tau)y(\tau)-\gamma^2 w^T(\tau)w(\tau)\right]d\tau\,. \quad (33)$$

Then it is obvious that

$$J(t)=\int_0^t\left[y^T(\tau)y(\tau)-\gamma^2 w^T(\tau)w(\tau)+\dot{S}(x(\tau))\right]d\tau$$
$$+S(x(0))-S(x(t))$$
$$\leq\int_0^t\{x^T[A^TP_\varepsilon+P_\varepsilon^TA+\sigma E^TE+C^TC+C^T\Delta C$$
$$+\Delta C^TC+\Delta C^T\Delta C+\sigma^{-1}P_\varepsilon^THH^TP_\varepsilon]x$$
$$+2x^T[P_\varepsilon^TB_w+(C+\Delta C)^TD_w]w$$
$$+w^T(D_w^TD_w-\gamma^{-2})w\}d\tau+S(x(0))$$
$$=\int_0^t\begin{pmatrix}x^T & w^T\end{pmatrix}(\Phi_\varepsilon+\Phi_{\Delta C})\begin{pmatrix}x\\ w\end{pmatrix}d\tau+S(x(0)),$$

where

$$\Phi_s=\begin{pmatrix}A^TP_\varepsilon+P_\varepsilon^TA+C^TC+\sigma E^TE & P_\varepsilon^TH & P_\varepsilon^TB_w+C^TD_w\\ * & -\sigma I & 0\\ * & * & -\gamma^2I+D_w^TD_w\end{pmatrix}, \tag{34}$$

and $\Phi_{\Delta C}$ is given in (12).

It follow from $P_\varepsilon=P+\varepsilon P_0$ that $\Phi_\varepsilon=\Phi_s+\varepsilon\Phi_0$, where Φ_s is defined in (13) and

$$\Phi_0=\begin{pmatrix}A^TP_0+P_0^TA & P_0^TH & P_0^TB_w\\ * & 0 & 0\\ * & * & 0\end{pmatrix}. \tag{35}$$

The condition $J(t)<0$ is satisfied if in addition to the zero initial condition ($S(x(0))=0$) and letting $t\to\infty$ we have

$$\Phi_s+\varepsilon\Phi_0+\Phi_{\Delta C}<0. \tag{36}$$

If (36) holds, then

$$\Phi_s+\Phi_{\Delta C}<0, \tag{37}$$

can be guaranteed. Therefore there exist a sufficiently small scalar $\varepsilon_2>0$ such that

$$\Phi_s+\varepsilon\Phi_0+\Phi_{\Delta C}<0,$$

for any given $\varepsilon\in(0,\varepsilon_2]$, which implies $J(t)<0$ for $\varepsilon\in(0,\varepsilon_2]$ .Let $\varepsilon^*=\min\{\varepsilon_1,\varepsilon_2\}$. Then we have $E_\varepsilon^TP_\varepsilon>0$ and $J(t)<0$ holds simultaneously for all $\varepsilon\in(0,\varepsilon^*]$. Thus the system is Generalized Quadratically Stable with H_∞ norm less than γ for any $\varepsilon\in(0,\varepsilon^*]$ over the horizon $[0,\infty)$. This complete the proof of Theorem 3.

Theorem 4

If there exist a constant scalar $\lambda>0$, positive definite matrices $\Pi>0$, $P_{11}>0$, P_{22} and P_{21} satisfying the following LMI

$$\Pi<\lambda P_{11},\begin{pmatrix}\Pi & P_{21}^T\\ P_{21} & P_{22}\end{pmatrix}>0,\Phi+\Phi_{\Delta C}<\lambda\Phi_0, \tag{38}$$

where Φ_0, Φ and $\Phi_{\Delta C}$ are defined in (35), (12) and (13) respectively. Then the system (1)-(2) in the standard form and GQS with an H_∞ norm less than γ for any $\varepsilon\in(0,\varepsilon^*]$ over the horizon $[0,\infty)$ and $\varepsilon^*=\lambda^{-1}$

It follows from Theorem 4 that the upper bound ε^* can be obtained by solving the following generalized eigenvalue problem

$$\min \quad \lambda$$
$$\textit{Subject to } (38)$$

which can be solved effectively by applying GEVP solver in LMI control

IV. Robust H_∞ Control of the Closed loop Systems

In many cases, when the unforced system is not robust stable, we include feedback transformation to make the system generalized quadratically stablizable and achieve an H_∞ performance level γ . Therefore, we need to find a linear state feedback controller

$$u(t)=K_1x_1(t)+K_2x_2(t), \tag{39}$$

where $K=(K_1\ K_2)$ is a constant matrix, such that the resulting closed-loop system is GQS with H_∞ norm less than γ .

Substituting the above control law (39) into (1) we obtain the closed-loop system as follows:

$$E_\varepsilon\dot{x}(t)=(A_c+\Delta A_c)x(t)+B_ww(t), \tag{40a}$$
$$y(t)=C_cx(t)+D_ww(t), \tag{40b}$$

where $A_c=A+B_uK$, $\Delta A_c=\Delta A_c+\Delta B_uK$ and $C_c=C+\Delta C$.

Applying Theorem 3 to the closed-loop system (40a)-(40b) we have the following result:

Theorem 5

If there exists constant scalars $\sigma>0$, $\rho>0$, $\mu>0$, a matrix Y and lower matrix triangular

$$X=\begin{pmatrix} X_{11} & 0 \\ X_{21} & X_{22} \end{pmatrix},$$

with $0<X_{11}\in R^{n_1\times n_1}$, $0<X_{22}\in R^{n_2\times n_2}$ satisfying the following LMI

$$\begin{pmatrix} AX+X^TA^T+B_uY+Y^TB_u^T+\sigma HH^T & X^TE^T+Y^TE_3^T & X^TC^T & X^TE^T & X^TC^TH_3 & X^TH_3D_w^T & B_w+X^TC^TD_w \\ * & -\sigma I & 0 & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & -\alpha I & 0 & 0 & 0 \\ * & * & * & * & -\rho I & 0 & 0 \\ * & * & * & * & * & -\mu I & 0 \\ * & * & * & * & * & * & D_w^TD_w-\gamma^2 I \end{pmatrix}<0, \tag{41}$$

then there exists an ε^* such that the resulting closed-loop systems (40a)-(40b) is generalized quadraticallystabilizable with an H_∞ norm less than γ over the horizon $[0,\infty)$ for all $\varepsilon\in(0,\varepsilon^*]$. Moreover, the robust stabilizing state feedback controller can be chosen as

$$u(t)=YX^{-1}x(t), \tag{42}$$

where the control gain is determined by

$$K=YX^{-1}, \tag{43}$$

Proof

Putting (43) into (41) and using the Schur's complement Lemma, we obtain the inequality (41) is equivalent to

$$\begin{pmatrix} (A+B_uK)X+X^T(A+B_uK)^T+X^T[\sigma(E+E_3K)^T(E+E_3K)+\alpha^{-1}E^TE+C^TC] & H & X^TC^TH_3 & X^TH_3D_w^T & B_w+X^TC^TD_w \\ * & -\sigma I & 0 & 0 & 0 \\ * & * & -\rho I & 0 & 0 \\ * & * & * & -\mu I & 0 \\ * & * & * & * & D_w^TD_w-\gamma^2 I \end{pmatrix}<0. \tag{44}$$

Let $X^{-1}=\bar{P}$. Pre and post multiplying (44) by $diag(X^{-T},I,I,I,I)$ and $diag(X^{-1},I,I,I,I)$ respectively, then (44) is equivalent to

$$\begin{pmatrix} \bar{P}^T(A+B_uK)+(A+B_uK)^T\bar{P}+\sigma(E+E_3K)^T(E+E_3K)+\alpha^{-1}E^TE+C^TC & \bar{P}^TH & C^TH_3 & H_3D_w^T & \bar{P}^TB_w+C^TD_w \\ * & -\sigma I & 0 & 0 & 0 \\ * & * & -\rho I & 0 & 0 \\ * & * & * & -\mu I & 0 \\ * & * & * & * & D_w^TD_w-\gamma^2 I \end{pmatrix}<0. \tag{45}$$

Choose a Lyapunov function as follows

$$S(x)=X^TE_\varepsilon^T\bar{P}_\varepsilon X,$$

where

$$\bar{P}_\varepsilon=\bar{P}+\varepsilon\bar{P}_0,\ \bar{P}=\begin{pmatrix} \bar{P}_{11} & 0 \\ \bar{P}_{21} & \bar{P}_{22} \end{pmatrix} \text{and } \bar{P}_0=\begin{pmatrix} 0 & \bar{P}_{21}^T \\ 0 & 0 \end{pmatrix}.$$

Then, the derivation of $S(x)$ along the trajectories of (40a) yields

$$\dot{S}(x)=x^T[(A_c+\Delta A_c)^T\bar{P}_\varepsilon+\bar{P}_\varepsilon^T(A_c+\Delta A_c)]x$$
$$+2x^T\bar{P}_\varepsilon^TB_ww$$

The performance function defined as follows

$$J(t)=\int_0^t\left[(y^T(\tau)y(\tau)-\gamma^2w^T(\tau)w(\tau)\right]d\tau$$

satisfies

$$J(t)\le\int_0^t\begin{pmatrix} x^T & w^T \end{pmatrix}\left(\bar{\Phi}+\varepsilon\bar{\Phi}_0+\bar{\Phi}_{\Delta C}\right)\begin{pmatrix} x \\ w \end{pmatrix}d\tau+S(x(0)) \tag{46}$$

For the above inequality, by using the zero initial condition, we have $J(t)<0$ is equivalent to

$$\bar{\Phi}+\varepsilon\bar{\Phi}_0+\bar{\Phi}_{\Delta C}<0$$

where

$$\bar{\Phi}=\begin{pmatrix} A_c^T\bar{P}+\bar{P}^TA_c+\sigma(E+E_3K)^T(E+E_3K)+\bar{C}^T\bar{C} & \bar{P}^TH & \bar{P}^TB_w+C^TD_w \\ * & -\sigma I & 0 \\ * & * & -\gamma^2I+D_w^TD_w \end{pmatrix}$$

$$\bar{\Phi}_0=\begin{pmatrix} A_c^TP_0+\bar{P}_0^TA_c & \bar{P}_0^TH & \bar{P}_0^TB_w \\ * & 0 & 0 \\ * & * & 0 \end{pmatrix}$$

$$\bar{\Phi}_{\Delta C}=\begin{pmatrix} \bar{C}^T\Delta C+\Delta C^T\bar{C}+\Delta C^T\Delta C & \Delta C^TD_w \\ D_w^T\Delta C & 0 \end{pmatrix}$$

The proof is similar to that of Theorem 3, thus there exists a scalar $\varepsilon^*>0$ such that the closed-loop systems (40a)-(40b) is Generalized Quadratically Stable with H_∞ norm less than γ over the horizon $[0,\infty)$ for all $\varepsilon\in(0,\varepsilon^*]$.Which complete the proof of Theorem 5.

According to Theorem 5, we have the following result which gives the method for solving the upper bound the Generalized Quadratically Stable with H_∞ performance level γ of the closed-loop system.

Theorem 6

After the control gain matrix K has been obtained from (43) and if there exist a constant scalar $\bar{\lambda}>0$, positive matrices $\bar{\Pi}>0$, $\bar{P}_{11}>0$, $\bar{P}_{22}$ and $\bar{P}_{21}$ satisfying the following LMI

$$\bar{\Pi}<\bar{\lambda}\bar{P}_{11},\ \begin{pmatrix} \bar{\Pi} & \bar{P}_{21}^T \\ \bar{P}_{21} & \bar{P}_{22} \end{pmatrix}>0,\ \bar{\Phi}+\bar{\Phi}_{\Delta C}<-\bar{\lambda}\bar{\Phi}_0. \tag{47}$$

Then the resulting closed-loop system (40a)-(40b) is generalized quadratically stable with H_∞ norm less than γ over the horizon $[0,\infty)$ for all $\varepsilon \in (0,\varepsilon^*]$ with $\varepsilon^* = \bar{\lambda}^{-1}$.

V. NUMERICAL EXAMPLE

Consider (1)-(2) with the following parameters

$A=\begin{pmatrix} -0.3417 & 0.3417 \\ 0.2733 & 0.7267 \end{pmatrix}$, $B_u=\begin{pmatrix} 9.0021 \\ 42.7983 \end{pmatrix}$,

$B_w=\begin{pmatrix} 0 \\ 0.2 \end{pmatrix}$, $C=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $D_w=0$, $H=H_3=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$,

$E_3=1$, $E=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Let also consider the exogenous disturbance

$w(t)=\dfrac{1}{t^2+1}$ .Then, by applying Theorem 5 to the above parameters, we find a solution of LMI (41) as follows

$X=\begin{pmatrix} 0.6794 & 0 \\ -0.2085 & 0.9759 \end{pmatrix}$, $Y=\begin{pmatrix} -0.0063 & -0.0171 \end{pmatrix}$,

$\rho=\alpha=\sigma=\mu=0.5$. Thus the control gain can be obtained as $K=YX^{-1}=\begin{pmatrix} -0.0147 & -0.0175 \end{pmatrix}$.

Moreover, the upper bound $\varepsilon^*=0.5204$ of theperturbation parameter is obtained by solving the corresponding GEVP in (38)

VI. CONCLUSION

In this workrobust alleviation of uncertainty for stability is presented by combining the reduction technique, LMI and H_∞ approach. In [1], disturbance attenuation for a class a SPSs has been addressed with complex state transformation for stability achievement. The method present valuable transformation and has achieved notable improvement of the existing results. However, the absence of uncertainty in certain components of the matrices and the complex transformation associated with the results narrows considerably the ability of application for engineers and the system itself is less global.

Some results have been reported in [7] but most of them are limited to discrete case or without uncertainty. As far as we know, solving the problem of robust H_∞ control for SPSs via GQS has not been reported in the literature.

In our work, the established LMIs conditions have discard not only the loss of system performance when $\varepsilon \to 0$ but also guarantee the GQS property for the unforced SPSsregardless of disturbances.

When the unforced system is unstable, we used a feedback transformation to design control strategy to stabilise the closed loop system and made it GQS for all admissible parameter uncertainties.

In contrast with the above works where on one side ε^* is a viewed as simple parameter, the uncertainties are missing in some of the system components for simplicity and, on the other side the perturbation is reduced to a function of time and system state satisfying the Lipchitz principle which do not provide a clear cut response for the system's performance conservation as $\varepsilon \to 0$ .Also the method proposed in this work provides an upper bound of the perturbationparameter that can be estimated and could be eventually improved in our further works. Finally, numerical example is given to illustrate the effectiveness of the proposed method

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