On Support Highly Irregular Intuitionistic Fuzzy Graphs

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Abstract: In this paper, support highly irregular intuitionistic fuzzy graphs and support highly totally irregular intuitionistic fuzzy graphs are defined. Comparative study between support highly irregular intuitionistic fuzzy graphs and support highly totally irregular intuitionistic fuzzy graphs is done. A necessary and sufficient conditions under which they are equivalent are provided. Also, few properties of support highly irregular intuitionistic fuzzy graphs and support highly totally irregular intuitionistic fuzzy graphs are discussed.

Keywords: Support and total support of a vertex in intuitionistic fuzzy graphs, support highly irregular intuitionistic fuzzy graphs and support highly totally irregular intuitionistic fuzzy graphs.

1 Introduction

Azriel Rosenfeld [8] introduced the concept of fuzzy graphs in 1975. In 1999 Atanassov[2] introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs. Research on the theory of intuitionistic fuzzy sets has been witnessing an exponential growth in Mathematics and its applications. This ranges from traditional Mathematics of Information Science. Atanassovp [3] introduced the intuitionistic fuzzy graph theory in the year 2003.

M. Akram, W. Dudek[1] introduced the regular intuitionistic fuzzy graphs. A. Nagoor Gani and Latha [7] introduced On irregular fuzzy graphs in 2012. N. R. Santhi Maheswari and C. Seker introduced the notion of Neighbourly irregular graphs and semi neighbourly irregular graphs [10], On m – neighbourly irregular Fuzzy graphs [11], On Neighbourly Edge Irregular Fuzzy Graphs[12].

J. Krishnaveni and N. R. Santhi Maheswari introduced support and total support of a vertex in fuzzy graphs, support neighbourly irregular fuzzy graphs and support neighbourly totally irregular fuzzy graphs [5]. J. Krishnaveni and N. R. Santhi Maheswari introduced support strongly irregular fuzzy graphs and support strongly totally irregular fuzzy graphs[6]. In this paper, we introduce support and total support of a vertex in intuitionistic fuzzy graphs, support highly irregular intuitionistic fuzzy graphs and support highly totally irregular intuitionistic fuzzy graphs. Also we discuss some of their properties.

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2 PRELIMINARIES

We present some known definitions and results for ready reference to go through the work presented in this chapter.

Definition 2.0.1. An intuitionistic fuzzy graph with underlying set V is defined to be a pair G = (V,E) where

(i) V = v_1, v_2, ... v_n such that μ_1 : V → [0, 1] and ϑ_1 : V → [0, 1] denote the degree of membership and non membership of the element v_i ∈ V (i = 1,2,3...n) such that 0 ≤ μ_1 (v_i) + ϑ_1 (v_i) ≤ 1

(ii) E ⊆ V × V , where μ_2 : V × V → [0, 1] and ϑ_2 : V × V → [0, 1] are such that μ_2 (v_i, v_j) ≤ min {μ_1 (v_i), μ_1 (v_j)} and ϑ_2 (v_i, v_j) ≤ max {v_1 (v_i), v_1 (v_j)} and 0 ≤ μ_2 (v_i, v_j) + ϑ_2 (v_i, v_j) ≤ 1, for every (v_i, v_j) ∈ E (i,j = 1,2,3..n)

Definition 2.0.2. The degree of a vertex v_i in intuitionistic fuzzy graph is defined by d(v_i) = (dμ_1 (v_i), $d_{\vartheta_1}(v_i)$), where dμ_1 (v_i) = Σμ_2 (v_i, v_j) and dϑ_1 (v_i) =Σ ϑ_2 (v_i, v_j), for v_i, v_j ∈ E and ϑ_2 (v_i, v_j) = 0 and μ_2 (v_i, v_j) = 0 for v_i, v_j ∈ E .

Definition 2.0.3. The total degree of a vertex vi in intuitionistic fuzzy graph is defined by td(v_i) = (tdμ_1 (v_i), t$d_{\vartheta_1}(v_i)$), Where tdμ_1 (v_i) = dμ_1 (v_i)+ μ_1 (v_i) and td_{ϑ_1} (v_i) = d_{ϑ_1} (v_i) + ϑ_1 (v_i).

Definition 2.0.4. A intuitionistic fuzzy graph is said to be regular if all vertices have same degree.

Definition 2.0.5. If all vertices have same total degree then intuitionistic fuzzy graph is said to be totally regular intuitionistic fuzzy graph.

Definition 2.0.6. A intuitionistic fuzzy graph is said to be irregular intuitionistic fuzzy graph, if there is a vertex which is adjacent to the vertices with distinct degrees.

Definition 2.0.7. A intuitionistic fuzzy graph is said to be totally irregular intuitionistic fuzzy graph, if there is a vertex which is adjacent to the vertices with distinct total degrees.

3 Support and Total support of a vertex in Intuitionistic Fuzzy Graphs

Definition 3.0.8. Let G : (V,E) be an intuitionistic fuzzy graph. The support (2-degree) of a vertex v is defined as the sum of the degree of vertices adjacent with v and it is denoted by s(v).

Definition 3.0.9. Let G : (V,E) be an intuitionistic fuzzy graph. The total support of a vertex v in G is denoted by ts(v) and is defined as the sum of the support of the vertex and the degree of membership of that vertex.

Example 3.0.10. Consider the intuitionistic fuzzy graph on G (V, E)

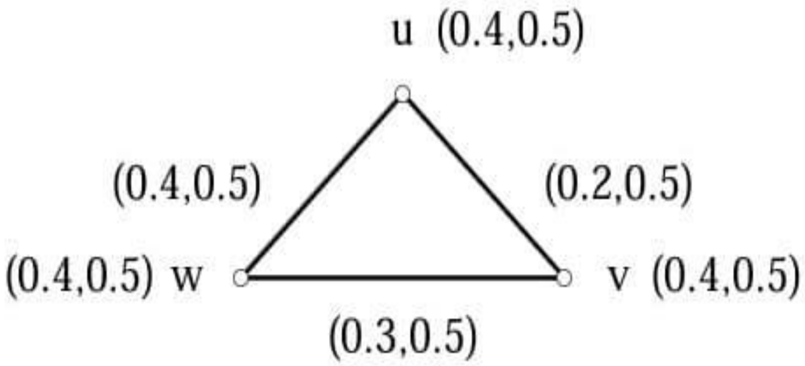

figure.1

From figure.1, Now, we calculate the degrees of the vertices of G are as follows,
d_G (u) = (0.6, 1.0), d_G (v) = (0.5, 1.0), d_G (w) = (0.7, 1.0). Next, we calculate the support of the vertices and total support of the vertices of G are as follows,
s_G (u) = (1.2, 2.0), s_G (v) = (1.3, 2.0), s_G (w) = (1.1, 2.0). Also ts_G (u) = (1.2, 2.0)+(0.4, 0.5) = (1.6, 2.5), ts_G (v) = (1.3, 2.0)+(0.4, 0.5) = (1.7, 2.5),ts_G (w) = (1.1, 2.0) + (0.4, 0.5) = (1.5, 2.5)

4 On Support Highly Irregular Intuitionistic Fuzzy Graphs

Definition 4.0.11. Let G = (V, E) be an intuitionistic fuzzy graph. Then the graph G is said to be support highly irregular intuitionistic fuzzy graph if every vertex of G is adjacent to the vertices have distinct supports.

Definition 4.0.12. Let G = (V, E) be an intuitionistic fuzzy graph. Then the graph G is said to be support highly totally irregular intuitionistic fuzzy graph if every vertex of G is adjacent to the vertices have distinct total supports.

Example 4.0.13. Consider the intuitionistic fuzzy graph G (V, E)

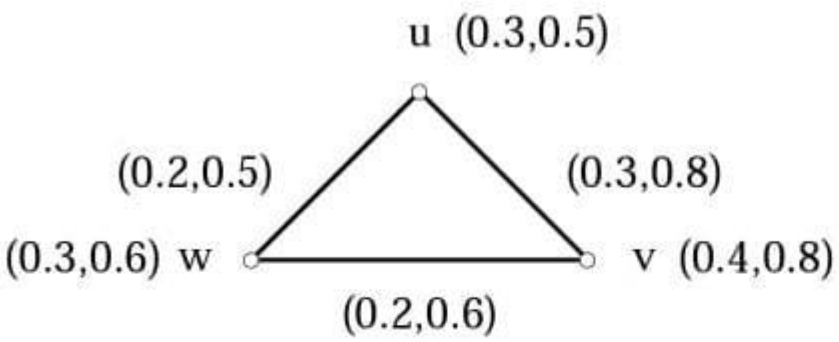

figure.2

From figure.2 Now, we calculate the degrees of the vertices of G are as follows,
d_G (u) = (0.5, 1.3), d_G (v) = (0.5, 1.4), d_G (w) = (0.4, 1.1) . Next, we calculate the support of the vertices of G are as follows s_G (u) = (0.9, 2.5), s_G (v) = (0.9, 2.4), s_G (w) = (1.0, 2.7).
Here, we observe that every vertex of G is adjacent to the vertices have distinct supports. Hence the intuitionistic fuzzy graph G is support highly irregular intuitionistic fuzzy graph. Total supports of the vertices are calculate as follows,
t(s_G (u)) = (1.2, 3.0), t(s_G (v)) = (1.3, 3.2), t(s_G (w)) = (1.3, 3.3). Here every vertex of G is adjacent to the vertices having distinct total supports. Therefore, the intuitionistic fuzzy graph G is Support highly totally irregular intuitionistic fuzzy graph.

Remark 4.0.14. A support highly irregular intuitionistic fuzzy graph need not be a support highly totally irregular intuitionistic fuzzy graph. Example 4.0.15. Consider the intuitionistic fuzzy graph G (V, E)

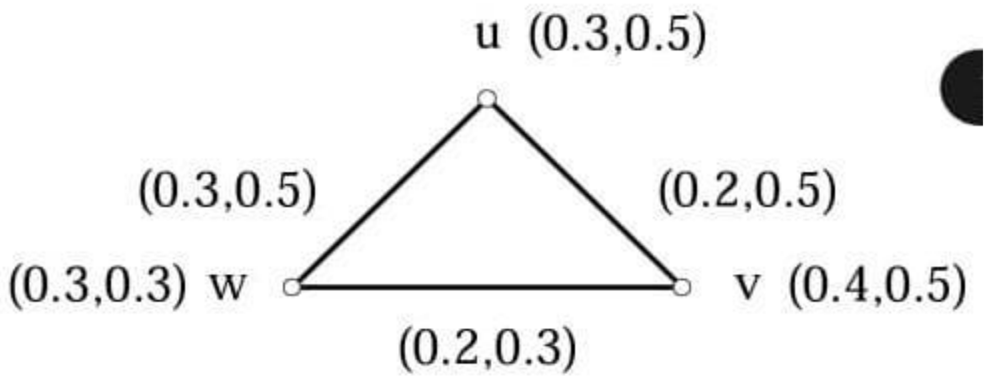

figure.3

From figure.3, the support of the vertices of G are s_G (u) = (0.9, 1.6), s_G (v) = (1.0, 1.8), s_G (w) = (0.9, 1.8). Every vertex of G is adjacent to the vertices of G have distinct supports. So, G is support highly irregular intuitionistic fuzzy graph. The total support of the vertices of G are t(s_G (u)) = (1.2, 2.1), t(s_G (v)) = (1.4, 2.3), t(s_G (w)) = (1.2, 2.1). Here the vertex v is adjacent to the u and w having same support. Therefore, this fuzzy graph is not support highly totally irregular intuitionistic fuzzy graph.

Remark 4.0.16. A support highly totally irregular intuitionistic fuzzy graph need not be a support highly irregular intuitionistic fuzzy graph.

Example 4.0.17. Consider the intuitionistic fuzzy graph G (V, E).

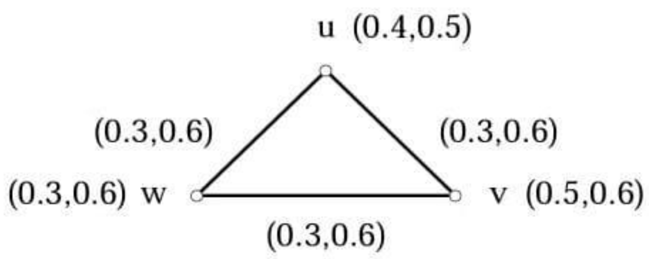

figure.4

From figure.4, the support of the vertices of G are s_G (u) = (1.2, 2.4), s_G (v) = (1.2, 2.4), s_G (w) = (1.2, 2.4). Here, support of all the vertices are same. Hence the intuitionistic fuzzy graph G is not Support Highly Irregular Intuitionistic

Fuzzy Graph. The total support of the vertices of G are t(s_G (u)) = (1.6, 2.9) ,t(s_G (v)) = (1.7, 3.0), t(s_G (w)) = (1.5, 3.0). Here, every vertex of G is adjacent to the vertices having distinct total supports. Therefore, the intuitionistic fuzzy graph G is Support Highly Totally Irregular Intuitionistic Fuzzy Graph.

Theorem 4.0.18. Let G (V, E) be a intuitionistic fuzzy graph. If $\mu_1$and ϑ_1 are constant functions. Then the following are equivalent.

(i) G is a support highly irregular intuitionistic fuzzy graph.

(ii) G is a support highly totally irregular intuitionistic fuzzy graph.

Proof. Assume that μ_1 both and ϑ_1 are constant functions. Let μ_1 (u) = k_1 and ϑ_1 (u) = k_2, for all u ∈ V . Then, (μ_1 (u), ϑ_1 (u)) = (k_1, k_2). Suppose G is support highly irregular intuitionistic fuzzy graph. Then every vertex of G is adjacent to the vertices have distinct supports. Let u1 and u2 be the adjacent vertices of G with s(u_1) = (m_{11}, m_{12}) and s(u_2) = (m_{21}, m_{22}). $m_{11} \neq m_{21}$ and $m_{12} \neq m_{22}$. Suppose G is not support highly totally irregular intuitionistic fuzzy graph, then at least two adjacent vertices have same total supports. Suppose ts(u_1) = ts(u_2) ⇒ s(u_1) + (μ_1 (u_1), ϑ_1 (u_1)) = s(u_2) + (μ_1 (u_2), ϑ_1 (u_2)) ⇒ (m_{11}, m_{12}) + (k_1, k_2) = (m_{21}, m_{22}). + (k_1, k_2) ⇒ (m_{11}, m_{12}) = (m_{21}, m_{22}), which is a contradiction. Hence G is support highly totally irregular intuitionistic fuzzy graph. Thus (i) ⇒ (ii) is proved. Now, suppose G is support highly totally irregular intuitionistic fuzzy graph. Then every vertex of G is adjacent to the vertices have distinct total supports. Let u1 and u2 be the adjacent vertices of G with ts(u_1) ≠ ts(u_2) ⇒ s(u_1) + (μ_1 (u_1), ϑ_1 (u_1)) ≠ s(u_2) + (μ_1 (u_2), ϑ_1 (u_2)) ⇒ (m_{11}, m_{12}) + (k_1, k_2) ≠(m_{21}, m_{22}) + (k_1, k_2) ⇒ (m_{11}, m_{12}) ≠(m_{21}, m_{22}). Thus every vertex of G is adjacent to the vertices have distinct supports. Hence G is a support highly irregular intuitionistic fuzzy graphs. Thus (ii) ⇒ (i) is proved.

Remark 4.0.19. Converse of the above theorem is not true.

Example 4.0.20. Consider the intuitionistic fuzzy graph G (V, E).

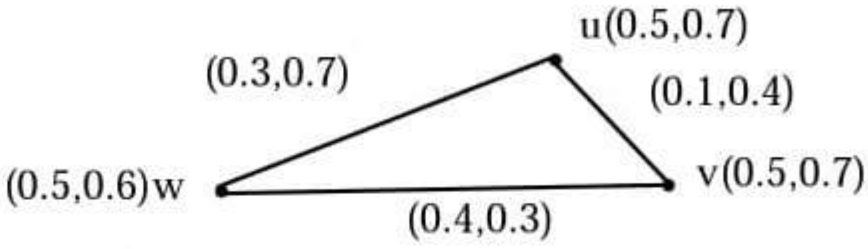

figure.5

From figure.5, the support of the vertices of G are s_G (u) = (1.2, 1.7), s_G (v) = (2.0, 2.1), s_G (w) = (1.8, 1.8). Here, every vertex of G is adjacent to the vertices having distinct supports. Therefore, the graph G is Support highly irregular intuitionistic fuzzy graph. The total support of the vertices of G are t(s_G (u)) = (1.7, 2.4), t(s_G (v)) = (2.5, 2.8), t(s_G (w)) = (2.3, 2.4). Here, every vertex is adjacent to the vertices having distinct total supports. Therefore, the graph G is support highly totally irregular intuitionistic fuzzy graph. But μ_1 and ϑ_1 are constant functions.

Theorem 4.0.21. Let G : (V, E) be an intuitionistic fuzzy graph. If the support of all the vertices of G are distinct, then G is support highly irregular intuitionistic fuzzy graph.

Proof: Assume that the support of all the vertices of G are distinct. Then every vertex of G is adjacent to the vertices have distinct supports. Hence G is support highly irregular intuitionistic fuzzy graph.

Remark 4.0.22. If every support highly irregular intuitionistic fuzzy graph G with membership values of the vertices are same. Then G is support highly totally irregular intuitionistic fuzzy graph.

Theorem 4.0.23. Let G : (V, E) be an intuitionistic fuzzy graph, a cycle of length n and $\mu_2$and $\vartheta_2$are constant functions, then G is not a support highly irregular intuitionistic fuzzy graph.

Proof. Assume that μ_1 and ϑ_1 are constant functions, say μ_1 $(u_1, u_2) = k_1$ and ϑ_1 $(u_1, u_2) = k_2$, i ≠ j, for all u_i, u_j ∈ E. Since G is a cycle of length n, we have s(u_1) = ($4k_1$, $4k_2$), for all u_i ∈ V . Thus s(u_i) is constant for all u_i ∈V . Hence G is not a support highly irregular intuitionistic fuzzy graph.

Theorem 4.0.24. Let G : (V, E) be an intuitionistic fuzzy graph, a cycle of length n and μ_2 and ϑ_2 are constant functions and μ_1 and $\vartheta_1$are distinct, then G is a support highly totally irregular intuitionistic fuzzy graph.

Proof. Assume that μ1 and ϑ_1 are constant functions, say μ_1 (u_1, u_2) = k_1 and ϑ_1 (u_1, u_2) = k_2, i ≠ j, for all u_i, u_j ∈ E. Since G is a cycle of length n, we have s(u_1) = ($4k_1$, $4k_2$), for all u_i ∈V . Also, given μ(u_i) = mi and $\vartheta(u_i)$ = n_i, for all u_i ∈ V . Thus $m_1 \neq m_2 \neq m_3 \neq$........≠ $m_n$and $n_1 \neq n_2 \neq n_3 \neq$........≠ n_n. Now, ts(u_i) = s(u_i) +(μ_1 (u_i), ϑ_1 (u_i)) = ($4k_1$, $4k_2$) + , (m_i, n_i), for i = 1,2,3,....,n. Hence G is support highly totally irregular intuitionistic fuzzy graph.

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