Heat transfer of Maxwell fluid with Magnetohydrodynamic and porous effect in cylindrical region

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Abstract- In this paper, we inaugurate the exact solutions for rotational and oscillations of Maxwell fluid through a cylindrical region. Maxwell fluid model has abundant significance in engineering and industrial applications Due to its considerable interest in the technical field such as glass-fiber, paper production, application of paint, food processing and biological transportation. This thesis is trying to scrutinize the heat transfer analysis in mixed convection flow of Maxwell fluid over an oscillating cylinder with the impact of magnetic field and constant well temperature. The problem is modelled in term of partial differential eq. with initial and boundary condition. Non-dimensional variables are used to transform the governing problem into dimensionless from. The dimensionless form is fixed via the process of transformation of Laplace and Hankel to determine the proper solution for velocity, shear stress, and temperature. The solution for MHD and porous, Newtonian and Non-Newtonian fluid is also obtained as a special case and some visual descriptions are addressed at the end.

Keywords- Maxwell fluid, oscillating flows, velocity field, shear stresses, Laplace and Hankel transforms.

1. Introduction

Heat transfer in fluid has noteworthy attention due to its wide application in Geothermal process, Petroleum reservoirs, Chemical catalytic reaction, Nuclear waste repository etc. The cylindrical fluid flow is indeed applicable in the food industry, chemistry of oil exploitation, bioengineering, etc [1]. Researchers proposed a number of mathematical models to explain the physical structure of Non-Newtonian fluid, subcategorized by differential form fluid or rate type fluid. Most of the researchers have keen interest to study in rate type fluid because they incorporate in both the elastic and viscous effect together. Maxwell model used to investigate the rheological effect, that is the first simplest model of rate type fluid [2-7]

In 1867 James clerk Maxwell recommended this model. Initially, to demonstrate the elastic and viscous reaction of air, the Maxwell fluid model was introduced [3, 4, 6, 7]. However, it was frequently used in various viscoelastic fluid. Furthermore, Friedrich's researches were executed in this direction [8]. Haitao and Mingya [9] studied fractional Maxwell model in channel.

Maxwell model took the first place in the list of Non-Newtonian fluid [4]. The flow characteristic of Non-Newtonian fluid pays a significant role in industry and engineering, further more exact solution of viscous fluid problem is also great importance in literature Jamil et al [10]. An unstable flow of generalized Maxwell fluid between two cylinders was investigated. In other exploration Jamil et al [11] observed helices of fractional Maxwell fluid. Jamil [12] evaluated slip effect on oscillating fractional Maxwell fluid. Zheng et al [13] for oscillatory and continuously accelerated plate motion, the exact solution for generalized Maxwell fluid was achieved. Zeng el al [14] the same fluid model for heat mass transfer in the hyperbolic $sine$ accelerating plate was further investigated. Fetecau and Fetecau [15] a new detailed approach to the Maxwell fluid flow in an infinite plate was studied. Another research they conclude exact solution by mean of Fourier sin transformation for an incompressible Maxwell fluid [16]. Jamil et al [17], Vieru and Rauf [18] Vieru and zafar [19] and Khan et al [20] they focused on heat transfer analysis, in the field of mixed convection flow over an oscillating vertical layer. Madeeha Tahir, M. A. Imran, N. Raza, M. Abdullah, Maryam Aleem [21] The heat transfer effect of Maxwell fluid over an oscillating vertical plate was observed and investigated that temperature can be enhanced for increasing the fractional parameter α while velocity and shear stress can be increased by decreasing the value of fractional parameter $\alpha,$ with a new definition of fractional caputo fabrizio derivatives. Corina et al [22] gave a major experiment on Maxwell fluid flow by the use of integral transformation in the account of continuously accelerating plate and also found the velocity and Shear Tension. For Maxwell fluid with and without MHD effect and porous medium, Khan [23] implemented $sine$ and $cosine$ oscillation. He observed the steady and transient solution for velocity and Shear Stress. Nadeem et al [24] proposed the fractional solution to intermittent unidirectional flows of a viscoelastic Maxwell type fluid with MHD effect on the regulation of non-linear partial diff equation was also calculated. K. Q. Zhu, K. X. Hu, D. Yang New developments in fluid mechanics Analysis using Heaviside Operational Calculus for Fractional Element of Viscoelastic Fluids [25]. W. Akhtar, M. Jamil, was working on a Maxwell fluid's axial Couette flow due to shear stress depending on longitudinal time [26]. S. Wang, M. Xu, evaluated analytical solutions with fractional derivatives on unsteady Couette flow of generalized Maxwell fluid [27]. Z. Zhang, exploring oscillatory convection in a viscoelastic fluid-saturated porous cylinder [28].

Convectional energy transfer is highly necessary and happens in a number of physical conditions [29]. In contrast to the other two forms of convection (free, induced, and mixed), mixed convection has gained less study. Mixed convection occurs as induced and free convections occur simultaneously. This effect is most frequently found in channel flow as a result of channel wall heating or cooling. Mixed convection energy transfer is investigated in a variety of physical conditions with a variety of boundary constraints. Fan et al. [30] investigated energy transfer in a horizontal channel filled with Nano fluids due to mixed convection. M Sheikholeslami, MM Bhatti [31] investigated forced convection of nanofluid in presence of constant magnetic field considering shape effects of nanoparticles. The buoyancy force is responsible for free convection in mixed convection energy transfer, according to Aaiza et al. [32], and at least one of the two non-homogeneous boundary conditions on velocity or external pressure differential results in forced convection. We find Kumari et al. [33], Tiwari and Das [34], Chamkha et al. [35], Sheikhzadeh et al. [36], Prasad et al. [37], Hasnain et al. [38], and Ganapathirao et al. [39] among the relevant studies on mixed convection energy transfer. However, the majority of these energy transfer experiments were limited to basic geometrical configurations.

The aim of this paper is to determine the precise Maxwell fluid heat transfer solution with an MHD and porous effect over a cylindrical area oscillation where the motion of the fluid is longitudinal. We determine the exact transitional solution of the standardized Maxwell fluid model for *sine* and *cosine* oscillation with MHD and porous effect of velocity as $u_c(r,t)$ and $u_s(r,t)$, shear stress as $\tau_c(r,t)$ and $\tau_s(r,t)$ and temperature as $T(r,t)$. The generated solution can satisfy all initial and boundary conditions. As special cases we determine both Newtonian and Non newtonian solutions of Maxwell fluid with and without MHD for velocity field, shear stress and temperature. Also we get Newtonian and Non newtonian fluid solutions of Maxwell fluid with and without porous effect for velocity field, shear stress and temperature. Because there has been no research in this domain so we can use preceding methodology to determine the both Newtonian and Non newtonian behavior of Maxwell fluid model in simplest form and can obtain simplest solutions with some graphical representation.

2. Governing equation and its formulation:

Imagine a Maxwell fluid in-compressible over an oscillating vertical cylinder. Initially, both the fluid and the cylinder are at rest with a constant temperature of T_∞, when $t = 0$. Rotation and vertical oscillation begin with any boundary conditions between the fluid and the cylinder's wall at the time $t = 0^+$. Temperature is raised to the constant value T_w. Under the consideration as shown in Figure (1), a well define Maxwell fluid model in term of the partial differential equations is defined as,

$$(1+\lambda\frac{\partial}{\partial t})\frac{\partial u}{\partial t} = \nu(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r})u - \frac{\sigma\beta_\circ^2}{\rho}(1+\lambda\frac{\partial}{\partial t})u - \frac{\mu\phi}{\kappa}u + (1+\lambda\frac{\partial}{\partial t})g\beta_1(T-T_\infty). \quad (1)$$

$$(1+\lambda\frac{\partial}{\partial t})\tau = \mu\frac{\partial u}{\partial r}. \quad (2)$$

$$C_p(\frac{\partial T}{\partial t}) = \kappa(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r})T. \quad (3)$$

$$u(r,0)=0,\quad T(r,0)=T_\infty; r=(0,r_\circ), t>0, \quad (4)$$

$$u(R,t)=U_\circ H(t)\cos(\omega t),\quad or \quad u(R,t)=U_\circ sin(\omega t), \quad (5)$$

$$T(R,t)=T_\omega. \quad (6)$$

Eq. (2-6) are the initial and boundary conditions. After the implementation of some parameters of dimensionless quantities into Eq. (1-3) (For simplicity, the sign is discarded.), we obtain

$$(1+\lambda\frac{\partial}{\partial t})\frac{\partial u}{\partial t} = (\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r})u - M(1+\lambda\frac{\partial}{\partial t})u - \psi u + G_r(1+\lambda\frac{\partial}{\partial t})T. \quad (7)$$

$$(1+\lambda\frac{\partial}{\partial t})\tau = \frac{\partial u}{\partial r}. \quad (8)$$

$$(\frac{\partial T}{\partial t}) = \frac{1}{P_r}(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r})T. \quad (9)$$

Parameters of dimensionless quantities;

$$u^* = \frac{u}{U_\circ},\quad t^* = \frac{t\nu}{R_\circ^2},\quad T^* = \frac{T-T_\infty}{T_\omega - T_\infty},\quad r^* = \frac{r}{R_\circ},\quad \lambda^* = \frac{\lambda\nu}{R_\circ},$$
$$\tau^* = \frac{R_\circ\tau}{\mu U_\circ},\quad \omega^* = \frac{\omega R_\circ^2}{\nu},$$

$$G_r = \frac{R_\circ^2 g\beta_1(T_\omega - T_\infty)}{\nu U_\circ},\quad M = \frac{\sigma\beta_\circ^2 R_\circ^2}{\rho\nu},\quad \psi = \frac{\mu\phi R_\circ^2}{\nu},\quad P_r = \frac{\rho C_p\nu}{\kappa}.$$

Here, $G_r =$ Grashoff number, $M =$ Magnetic effect, $\psi =$ porous effect, $P_r =$ prandtl number.

Similarly, by applying some parameters of dimensionless quantities into Eq. (4-6), we get.

$$u(r,0)=0,\quad T(r,0)=T_\infty; r=(0,1), t>0, \quad (10)$$

$$u(1,t)=cos(\omega t),\quad or \quad u(1,t)=sin(\omega t), \quad (11)$$

$$T(1,t)=1. \quad (12)$$

3. Transformation of partial differential equations:

3.1 Integral transforms:

The $g(y)$ function defined in $c \le y \le d$ is a general integral transform and is denoted by $G(x)$ and defined by,

$$G(x) = \int_c^d K(y,x)g(y)\,dy,$$

here, $K(y,x)$ is the transformation's integral kernel. There are too many integral transformations to solve various equations like Laplace transform, finite Hankel transform, Fourier *sine* transform etc.

3.2 Laplace transform:

The Laplace transform is a function, usually described as,

$$\mathcal{L}\{g(t)\} = \bar{g}(s) = \int_0^\infty e^{-st}g(t)dt = \lim_{t\to\infty}\int_0^t e^{-st}g(t)dt \qquad Re\ s > 0,$$

while the inverse Laplace transform $L^{-1}g(s)$ is given by,

$$\mathcal{L}-1\{g(s)\} = g(t) = \int_{x-\iota\infty}^{x+\iota\infty} e^{st}g(s)ds,\ \ x>0, s>0,$$

3.3 Inverse Laplace transformation:

The inverse Laplace transform $\mathcal{L}^{-1}\{\bar{g}(s)\}$ is defined as,

$$\mathcal{L}^{-1}\{\bar{g}(s)\} = g(t) = \frac{1}{2\pi i}\int_{x-i\infty}^{x+i\infty} e^{st}\bar{g}(s)\,ds; \qquad x>0.$$

3.4 Finite Hankel transform:

If $g(r)$ is defined in $0 \le r \le R$, then the nth order Hankel transformation of the function $g(r)$ is represented as,

$$H_n\{g(r)\} = \tilde{g}_n(r_i) = \int_0^R rg(r)J_n(r\,r_i)dr,$$

here, $J_n(\bullet)$ would be the first component of Bessel feature throughout the nth order and $r_i = (0 < r_1 < r_2 < \ldots\ldots r_n)$ is perhaps the positive root of the $J_n(R_n) = 0$ model.

3.5 Inverse Finite Hankel transform:

The inverse finite Hankel transform is defined as,

$$H_n^{-1}[\tilde{g}_n(r_i)] = g(r) = \frac{2}{R^2}\sum_{i=1}^{\infty}\frac{J_n(r\,r_i)}{J_{n+1}^2(R\,r_i)}\tilde{g}_n(r_i).$$

3.6 Mixed Laplace and Finite Hankel transform:

In current years, mixed Laplace and finite Hankel transformation of the order $(n = 0, 1)$ are used to solve partial differential equations involving fractional calculus cylindrical polar coordinates [16]. one of them use in the paper define below,

$$\int_0^R r\left(\frac{\partial^2\bar{u}_H(r,q)}{\partial r^2} + \frac{1}{r}\frac{\partial\bar{u}(r,q)}{\partial r}\right)J_\circ(r\,r_i)dr = R\,r_n\,J_1(R\,r_n)\bar{u}(R,q) - r_n^2\bar{u}_H(r_n,q).$$

4. Analytical solution of temperature, velocity and share stress:

4.1 *Temperature computation:*

Taking Laplace transformation of Eq. (9) and Eq. (12) and using initial condition of Eq. (10), we get.

$$q\bar{T}(r,q) = \frac{1}{P_r}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\bar{T}(r,q), \tag{13}$$

$$\bar{T}(1,q) = \frac{1}{q}. \tag{14}$$

Taking Hankel transformation in Eq. (13), we get.

$$q\bar{T}_H(r_n,q) = \frac{1}{P_r}[r_nJ_1(r_n)\bar{T}(1,q) - r_n^2\bar{T}_H(r_n,q)]. \tag{15}$$

Simplify Eq. (15), we get.

$$\bar{T}_H(r_n,q) = \frac{r_nJ_1(r_n)}{q(P_rq + r_n^2)}. \tag{16}$$

To satisfy the boundary condition add and subtract Eq.(16) by $\frac{J_1(r_n)}{r_nq}$, we get.

$$\bar{T}_H(r_n,q) = \frac{J_1(r_n)}{r_nq} - \frac{J_1(r_n)}{r_n(q + \frac{r_n^2}{P_r})}. \tag{17}$$

Taking Laplace inverse transformation to Eq. (17), we get.

$$T_H(r_n,t) = \frac{J_1(r_n)}{r_n} - \frac{J_1(r_n)}{r_n}\exp(\frac{-r_n^2t}{P_r}). \tag{18}$$

Taking Hankel inverse transformation in Eq. (18), we get.

$$T(r,t) = 1 - 2\sum_{n=1}^{\infty}\exp(\frac{-r_n^2t}{P_r})\frac{J_\circ(r\,r_n)}{r_nJ_1(r_n)}. \tag{19}$$

Hence, the above equation is the exact solution of temperature.

4.2 *Velocity calculation: (for* ***cosine*** *oscillation.)*

In order to find exact solution of velocity field, we take Laplace transformation in Eq. (7), we get.

$$(1+M\lambda+\lambda q)q\bar{u}=(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r})\bar{u}-(M+\psi)\bar{u}+G_r(1+\lambda q)\bar{T}. \quad (20)$$

Taking Hankel transformation in Eq. (20) and applying Laplace transformation in Eq. (11), we get.

$$(q+M\lambda q+\lambda q^2+M+\psi+r_n^2)q\bar{u}_H=\frac{r_nJ_1(r_n)q}{q^2+\omega^2}+G_r(1+\lambda q)\bar{T}_H, \quad (21)$$

here, $\bar{T}_H(r_n,q)=\frac{r_nJ_1(r_n)}{q(P_rq+r_n^2)}$.. Put in Eq. (21), we get.

$$\bar{u}_H(r_n,q)=\frac{r_nJ_1(r_n)[q^2(P_rq+r_n^2)+G_r(1+\lambda q)(q^2+\omega^2)]}{q(q^2+\omega^2)(P_rq+r_n^2)(q+\lambda qM+\lambda q^2+M+\psi+r_n^2)}. \quad (22)$$

Add and Subtract by $\frac{qJ_1(r_n)}{r_n(q^2+\omega^2)}$ in Eq. (22), we get.

$$\bar{u}_H(r_n,q)=\frac{qJ_1(r_n)}{r_n(q^2+\omega^2)}+\frac{J_1(r_n)\begin{Bmatrix}r_n^2q^2(P_rq+r_n^2)+r_n^2G_r(1+\lambda q)(q^2+\omega^2)\\-\lambda q^2(qP_r+r_n^2)(q+\lambda qM+\lambda q^2+M+\psi+r_n^2)\end{Bmatrix}}{\lambda r_nq(q^2+\omega^2)(P_rq+r_n^2)(q+\lambda qM+\lambda q^2+M+\psi+r_n^2)}, \quad (23)$$

Assuming a and b from Eq. (23) as,

$$a=\frac{-(1+M\lambda)+\sqrt{1+M^2\lambda^2-2M\lambda-4\psi\lambda+4r_n^2\lambda}}{2\lambda},$$

$$b=\frac{-(1+M\lambda)-\sqrt{1+M^2\lambda^2-2M\lambda-4\psi\lambda+4r_n^2\lambda}}{2\lambda},$$

we get,

$$\bar{u}_H(r_n,q)=\frac{qJ_1(r_n)}{r_n(q^2+\omega^2)}+\frac{J_1(r_n)\begin{Bmatrix}r_n^2q^2(P_rq+r_n^2)+r_n^2G_r(1+\lambda q)(q^2+\omega^2)\\-\lambda q^2(qP_r+r_n^2)(q-a)(q-b)\end{Bmatrix}}{\lambda r_nq(q^2+\omega^2)(P_rq+r_n^2)(q-a)(q-b)}. \quad (24)$$

Taking Laplace inverse transformation in Eq. (24), we get.

$$u_H(r_n,t)=\frac{J_1(r_n)\cos\omega t}{r_n}+\frac{J_1(r_n)}{\lambda r_n}\left[\frac{G_r}{ab}+\frac{G_rP_r\exp(\frac{-r_n^2t}{P_r})(-P_r+\lambda r_n^2)}{(aP_r+r_n^2)(bP_r+r_n^2)}\right.$$

$$+\frac{r_n^2\exp(at)\{a^3(G_r\lambda+P_r)+a^2(G_r+r_n^2)+aG_r\lambda\omega^2+G_r\omega^2\}}{a(a-b)(aP_r+r_n^2)(a^2+\omega^2)}$$

$$+\frac{r_n^2\exp(bt)\{b^3(G_r\lambda+P_r)+b^2(G_r+r_n^2)+bG_r\lambda\omega^2+G_r\omega^2\}}{b(-a+b)(bP_r+r_n^2)(b^2+\omega^2)}$$

$$\left.+\frac{\{ab(-ab\lambda+r_n^2)-\lambda\omega^2(a^2+b^2+\omega^2)-r_n^2\omega^2\}\cos\omega t-r_n^2\omega(a+b)\sin\omega t}{(b^2+\omega^2)(a^2+\omega^2)}\right]. \quad (25)$$

Taking Hankel inverse transformation in Eq. (25), we get.

$$u_c(r,t)=\cos\omega t+2\sum_{n=1}^{\infty}\frac{J_\circ(rr_n)}{\lambda r_nJ_1(r_n)}\left[\frac{G_r}{ab}+\frac{G_rP_r\exp(\frac{-r_n^2t}{P_r})(-P_r+\lambda r_n^2)}{(aP_r+r_n^2)(bP_r+r_n^2)}\right.$$

$$+\frac{r_n^2\exp(at)\{a^3(G_r\lambda+P_r)+a^2(G_r+r_n^2)+aG_r\lambda\omega^2+G_r\omega^2\}}{a(a-b)(aP_r+r_n^2)(a^2+\omega^2)}$$

$$+\frac{r_n^2\exp(bt)\{b^3(G_r\lambda+P_r)+b^2(G_r+r_n^2)+bG_r\lambda\omega^2+G_r\omega^2\}}{b(-a+b)(bP_r+r_n^2)(b^2+\omega^2)}$$

$$\left.+\frac{\{ab(-ab\lambda+r_n^2)-\lambda\omega^2(a^2+b^2+\omega^2)-r_n^2\omega^2\}\cos\omega t-r_n^2\omega(a+b)\sin\omega t}{(b^2+\omega^2)(a^2+\omega^2)}\right]. \quad (26)$$

Above equation is the exact solution of velocity field for *cosine* oscillation of cylinder with MHD and porous effect.

4.3 *Shear stress calculation: (for* ***cosine*** *oscillation.)*

Taking Laplace transformation on Eq. (8), we get.

$$(1+\lambda q)\bar{\tau}=\frac{\partial\bar{u}}{\partial r}. \quad (27)$$

Take Eq. (22) and consider a and b as,

$$a=\frac{-(1+M\lambda)+\sqrt{1+M^2\lambda^2-2M\lambda-4\psi\lambda+4r_n^2\lambda}}{2\lambda},$$

$$b=\frac{-(1+M\lambda)-\sqrt{1+M^2\lambda^2-2M\lambda-4\psi\lambda+4r_n^2\lambda}}{2\lambda},$$

we get,

$$\bar{u}_H(r_n,q)=\frac{r_n^2J_1(r_n)[r_nq^2(P_rq+r_n^2)+G_r(1+\lambda q)(q^2+\omega^2)]}{q(q^2+\omega^2)(P_rq+r_n^2)(q-a)(q-b)}. \quad (28)$$

Applying inverse Hankel transformation in Eq. (28) and substitute in Eq. (27), we get.

$$(1+\lambda q)\bar{\tau}=\frac{\partial}{\partial r}\left[2\sum_{n=1}^{\infty}\frac{r_nJ_\circ(rr_n)}{J_1(r_n)}\left\{\frac{q^2(P_rq+r_n^2)+G_r(1+\lambda q)(q^2+\omega^2)}{\lambda q(q^2+\omega^2)(P_rq+r_n^2)(q-a)(q-b)}\right\}\right], \quad (29)$$

$$\Rightarrow\bar{\tau}(r,q)=2\sum_{n=1}^{\infty}\frac{r_n^2J_\circ(rr_n)}{J_1(r_n)(1+\lambda q)}\left\{\frac{q^2(P_rq+r_n^2)+G_r(1+\lambda q)(q^2+\omega^2)}{\lambda q(q^2+\omega^2)(P_rq+r_n^2)(q-a)(q-b)}\right\}. \quad (30)$$

Taking Laplace inverse transformation in Eq. (30), we get.

$$\tau_c(r,t) = 2\sum_{n=1}^{\infty}\frac{r_n J_\circ(rr_n)}{\lambda(P_r+r_n)J_1(r_n)}\left[\frac{-G_r}{ab} + \frac{\exp(at)\{a^3(G_r\lambda+P_r)+a^2(G_r+r_n^2)+aG_r\omega^2\lambda+G_r\omega^2\}}{a(a+b)(1+a\lambda)(a^2+\omega^2)}\right.$$

$$+\frac{\exp(-bt)\{b^3(G_r\lambda+P_r)-b^2(G_r+r_n^2)+bG_r\omega^2\lambda-G_r\omega^2\}}{b(a+b)(-1+b\lambda)(b^2+\omega^2)} + \frac{\lambda\exp(-\frac{t}{\lambda})(-P_r+\lambda r_n^2)}{(1+a\lambda)(-1+b\lambda)(1+\lambda^2\omega^2)}$$

$$-\frac{\{(ab+\omega^2)(r_n^2+\lambda p_r\omega^2)+\omega^2(-a+b)(P_r-\lambda r_n^2)\}\cos\omega t}{(a^2+\omega^2)(b^2+\omega^2)(1+\lambda^2\omega^2)}$$

$$\left.+\frac{\{-a\omega(r_n^2+\lambda P_r\omega^2)+b\omega(r_n^2+\lambda\omega^2-r_n^2\omega^2)+ab(P_r\omega-\lambda r_n^2)+P_r\omega^3\}\sin\omega t}{(a^2+\omega^2)(b^2+\omega^2)(1+\lambda^2\omega^2)}\right] \quad (31)$$

Above equation is the Exact solution of shear stress for *cosine* oscillation of cylinder with MHD and porous effect.

4.4 *Velocity calculation: (for **sine** oscillation.)*

By following the above procedure of calculating velocity field equation for *cosine* oscillation, we conclude the exact solution of velocity field for *sine* oscillation of cylinder with MHD and porous effect, given as.

$$u_s(r,t) = \sin\omega t + 2\sum_{n=1}^{\infty}\frac{J_\circ(rr_n)}{\lambda r_n J_1(r_n)}\left[\frac{G_r}{ab} + \frac{G_rP_r\exp(\frac{-r_n^2t}{P_r})(-P_r+\lambda r_r^2)}{(aP_r+r_n^2)(bP_r+r_n^2)}\right.$$

$$+\frac{r_n^2\exp(at)\{a^3(G_r\lambda+P_r)+a^2(G_r+r_n^2)+aG_r\lambda\omega^2+G_r\omega^2\}}{a(a-b)(aP_r+r_n^2)(a^2+\omega^2)}$$

$$+\frac{r_n^2\exp(bt)\{b^3(G_r\lambda+P_r)+b^2(G_r+r_n^2)+bG_r\lambda\omega^2+G_r\omega^2\}}{b(-a+b)(bP_r+r_n^2)(b^2+\omega^2)}$$

$$\left.+\frac{\{ab(-ab\lambda+r_n^2)-\lambda\omega^2(a^2+b^2+\omega^2)-r_n^2\omega^2\}\sin\omega t+r_n^2\omega(a+b)\cos\omega t}{(b^2+\omega^2)(a^2+\omega^2)}\right]. \quad (32)$$

4.5 *Shear stress calculation: (for **sine** oscillation.)*

Similarly, by using above procedure of calculating shear stress equation for *cosine* oscillation, we conclude the permanent solution of shear stress for *sine* oscillation of cylinder with MHD and porous effect as given below.

$$\tau_s(r,t) = 2\sum_{n=1}^{\infty}\frac{r_n J_\circ(rr_n)}{\lambda(P_r+r_n)J_1(r_n)}\left[\frac{G_r}{ab} + \frac{\exp(at)\{a^3G_r\lambda+a^2(G_r+P_r\omega)+a\omega(G_r\lambda\omega+r_n^2)+G_r\omega^2\}}{a(a+b)(1+a\lambda)(a^2+\omega^2)}\right.$$

$$+\frac{\exp(-bt)\{b^3G_r\lambda-b^2(G_r+P_r\omega)+b\omega(G_r\lambda\omega+r_n^2)-G_r\omega^2\}}{b(a+b)(1+b\lambda)(b^2+\omega^2)} + \frac{\exp(-\frac{t}{\lambda})(P_r-\lambda r_n^2)\omega\lambda^2}{(1+a\lambda)(-1+b\lambda)(1+\lambda^2\omega^2)}$$

$$\left.+\frac{\begin{Bmatrix}\{-ab(r_n^2+\lambda P_r\omega^2)-P_r\omega^2(a-b)-r_n^2\omega^2\}\sin\omega t\\ -\{\omega(ab+\omega^2)(P_r-\lambda r_n^2)-\omega(a+b)(r_n^2+\lambda P_r\omega^2)\}\cos\omega t\end{Bmatrix}}{(a^2+\omega^2)(b^2+\omega^2)(1+\lambda^2\omega^2)}\right]. \quad (33)$$

5. Special cases:

5.1 *Maxwell without MHD: ($\boldsymbol{M} \rightarrow \mathbf{0}$)*

5.1.1 *Velocity computation: (for **cosine** oscillation.)*

Taking Eq. (23), by substituting $M \rightarrow 0$ and assuming c and d as,

$$c = \frac{-1+\sqrt{1-4\psi-4r_n^2\lambda}}{2\lambda},$$

$$d = \frac{-1-\sqrt{1-4\psi-4r_n^2\lambda}}{2\lambda}.$$

Eq. (23) become,

$$\overline{u}_H(r_n,q) = \frac{qJ_1(r_n)}{r_n(q^2+\omega^2)} + \frac{J_1(r_n)\begin{Bmatrix}r_n^2q^2(P_rq+r_n^2)+r_n^2G_r(1+\lambda q)(q^2+\omega^2)\\ -q^2(qP_r+r_n^2)(q-c)(q-d)\end{Bmatrix}}{r_nq(q^2+\omega^2)(P_rq+r_n^2)(q-c)(q-d)}. \quad (34)$$

Taking inverse Laplace and Hankel transformation, we get.

$$u_c(r,t) = \cos\omega t + 2\sum_{n=1}^{\infty}\frac{J_\circ(rr_n)}{\lambda r_n J_1(r_n)}\left[\frac{G_r}{cd} + \frac{G_rP_r\exp(\frac{-r_n^2t}{P_r})(-P_r+\lambda r_n^2)}{(cP_r+r_n^2)(dP_r+r_n^2)}\right.$$

$$+\frac{r_n^2\exp(ct)\{c^3(G_r\lambda+P_r)+c^2(G_r+r_n^2)+cG_r\lambda\omega^2+G_r\omega^2\}}{c(c-d)(cP_r+r_n^2)(c^2+\omega^2)}$$

$$+\frac{r_n^2\exp(dt)\{d^3(G_r\lambda+P_r)+d^2(G_r+r_n^2)+dG_r\lambda\omega^2+G_r\omega^2\}}{d(-c+d)(dP_r+r_n^2)(d^2+\omega^2)}$$

$$+\frac{\{cd(-cd\lambda+r_n^2)-\lambda\omega^2(c^2+d^2+\omega^2)-r_n^2\omega^2\}\cos\omega t-r_n^2\omega(c+d)\sin\omega t}{(d^2+\omega^2)(c^2+\omega^2)}\Bigg]. \quad (35)$$

Above equation is the exact solution of velocity field for *cosine* oscillation without MHD effect.

5.1.2 Velocity computation: (for ***sine*** oscillation.)

Assuming $M \rightarrow 0$, we obtained exact solution without MHD effect.

$$u_s(r,t)=\sin\omega t+2\sum_{n=1}^{\infty}\frac{J_\circ(rr_n)}{\lambda r_n J_1(r_n)}\Bigg[\frac{G_r}{cd}+\frac{G_rP_r\exp(\frac{-r_n^2t}{P_r})(-P_r+\lambda r_r^2)}{(cP_r+r_n^2)(dP_r+r_n^2)}$$

$$+\frac{r_n^2\exp(ct)\{c^3(G_r\lambda+P_r)+c^2(G_r+r_n^2)+cG_r\lambda\omega^2+G_r\omega^2\}}{c(c-d)(cP_r+r_n^2)(c^2+\omega^2)}$$

$$+\frac{r_n^2\exp(dt)\{d^3(G_r\lambda+P_r)+d^2(G_r+r_n^2)+dG_r\lambda\omega^2+G_r\omega^2\}}{d(-c+d)(dP_r+r_n^2)(d^2+\omega^2)}$$

$$+\frac{\{cd(-cd\lambda+r_n^2)-\lambda\omega^2(c^2+d^2+\omega^2)-r_n^2\omega^2\}\sin\omega t+r_n^2\omega(c+d)\cos\omega t}{(d^2+\omega^2)(c^2+\omega^2)}\Bigg]. \quad (36)$$

5.1.3 Solution of shear stress: (for ***cosine*** oscillation.)

Taking Eq. (30), by replacing $a = c$ and $b = d$, we get.

$$\bar{\tau}=2\sum_{n=1}^{\infty}\frac{r_n^2J_\circ(rr_n)}{\lambda J_1(r_n)(1+\lambda q)}\left\{\frac{q^2(P_rq+r_n^2)+G_r(1+\lambda q)(q^2+\omega^2)}{\lambda q(q^2+\omega^2)(P_rq+r_n^2)(q-c)(q-d)}\right\} \quad (37)$$

Taking inverse Laplace transformation in Eq. (37), we get.

$$\tau_c(r,t)=2\sum_{n=1}^{\infty}\frac{r_nJ_\circ(rr_n)}{\lambda(P_r+r_n)J_1(r_n)}\Bigg[\frac{-G_r}{cd}+\frac{\exp(ct)\{c^3(G_r\lambda+P_r)+c^2(G_r+r_n^2)+cG_r\omega^2\lambda+G_r\omega^2\}}{c(c+d)(1+c\lambda)(c^2+\omega^2)}$$

$$+\frac{\exp(-dt)\{d^3(G_r\lambda+P_r)-d^2(G_r+r_n^2)+dG_r\omega^2\lambda-G_r\omega^2\}}{d(c+d)(-1+d\lambda)(d^2+\omega^2)}+\frac{\lambda\exp(-\frac{t}{\lambda})(-P_r+\lambda r_n^2)}{(1+c\lambda)(-1+d\lambda)(1+\lambda^2\omega^2)}$$

$$+\frac{\begin{Bmatrix}\{-(cd+\omega^2)(r_n^2+\lambda p_r\omega^2)-\omega^2(-c+d)(P_r-\lambda r_n^2)\}\cos\omega t\\+\{-c\omega(r_n^2+\lambda P_r\omega^2)+d\omega(r_n^2+\lambda\omega^2-r_n^2\omega^2)+cd(P_r\omega-\lambda r_n^2)+P_r\omega^3\}\sin\omega t\end{Bmatrix}}{(c^2+\omega^2)(d^2+\omega^2)(1+\lambda^2\omega^2)}\Bigg]. \quad (38)$$

Above equation is the exact solution of shear stress for *cosine* oscillation without MHD effect.

5.1.4 Shear stress: (for ***sine*** oscillation.)

We conclude exact solution of shear stress for *sine* oscillation without MHD effect as,

$$\tau_s(r,t)=2\sum_{n=1}^{\infty}\frac{r_nJ_\circ(rr_n)}{\lambda(P_r+r_n)J_1(r_n)}\Bigg[\frac{G_r}{cd}+\frac{\exp(ct)\{c^3G_r\lambda+c^2(G_r+P_r\omega)+c\omega(G_r\lambda\omega+r_n^2)+G_r\omega^2\}}{c(c+d)(1+c\lambda)(c^2+\omega^2)}$$

$$+\frac{\exp(-dt)\{d^3G_r\lambda-d^2(G_r+P_r\omega)+d\omega(G_r\lambda\omega+r_n^2)-G_r\omega^2\}}{d(c+d)(1+d\lambda)(d^2+\omega^2)}+\frac{\exp(-\frac{t}{\lambda})(P_r-\lambda r_n^2)\lambda^2\omega}{(1+c\lambda)(-1+d\lambda)(1+\lambda^2\omega^2)}$$

$$+\frac{\begin{Bmatrix}\{-cd(r_n^2+\lambda P_r\omega^2)-P_r\omega^2(c-d)-r_n^2\omega^2\}\sin\omega t\\-\{\omega(cd+\omega^2)(P_r-\lambda r_n^2)-\omega(c+d)(r_n^2+\lambda P_r\omega^2)\}\cos\omega t\end{Bmatrix}}{(c^2+\omega^2)(d^2+\omega^2)(1+\lambda^2\omega^2)}\Bigg]. \quad (39)$$

5.2 Newtonian with MHD and porous: $(\boldsymbol{\lambda \rightarrow 0})$

5.2.1 Velocity: (for ***cosine*** oscillation.)

Taking Eq. (23) and by substituting $\lambda \rightarrow 0$, we obtained.

$$\bar{u}_H=\frac{qJ_1(r_n)}{r_n(q^2+\omega^2)}+\frac{J_1(r_n)\begin{Bmatrix}r_n^2q^2(P_rq+r_n^2)+r_n^2G_r(q^2+\omega^2)\\-q^2(qP_r+r_n^2)(q+M+\psi+r_n^2)\end{Bmatrix}}{r_nq(q^2+\omega^2)(P_rq+r_n^2)(q+M+\phi+r_n^2)}. \quad (40)$$

By replacing $g = M + \psi + r_n^2$ and taking inverse Laplace and Hankel transformation, we get.

$$u_c=\cos\omega t+2\sum_{n=1}^{\infty}\frac{J_\circ(rr_n)}{r_nJ_1(r_n)}\Bigg[\frac{G_r}{g}+\frac{\exp(\frac{-r_n^2t}{P_r})\{G_rP_r^2(r_n^4+P_r^2\omega^2)-r_n^6(P_r^2-r_n^2)\}}{P_r(gP_r-r_n^2)(r_n^4+P_r^2\omega^2)}$$

$$+\frac{r_n^2\exp(-gt)\{G_r(g^2-\omega^2)+g^3(g-P_r)\}}{g(gP_r-r_n^2)(\omega^2+g^2)}+\frac{\{gP_rr_n^2\omega(r_n^2-\omega^2)+r_n^2\omega^3(P_r^2-r_n^2)\}\sin\omega t}{(g^2+\omega^2)(r_n^4+P_r^2\omega^2)}$$

$$+\frac{\{-(g^2+\omega^2)(r_n^4+P_r^2\omega^2)+P_rr_n^2\omega^2(r_n^2+\omega^2)+g^2r_n^2\omega^2(P_r^2-r_n^2)\}\cos\omega t}{(g^2+\omega^2)(r_n^4+P_r^2\omega^2)}\Bigg]. \quad (41)$$

Hence, above equation is the permanent solution of velocity field for *cosine* oscillation. By replacing $\lambda \rightarrow 0$, Maxwell model shows properties of Newtonian fluid with MHD and porous effect.

5.2.2 Velocity: (for *sine* oscillation.)

Similarly, the permanent solution of velocity field for *sine* oscillation is given below.

$$u_c(r,t) = sin\omega t + 2\sum_{n=1}^{\infty}\frac{J_\circ(rr_n)}{r_nJ_1(r_n)}\left[\frac{G_r}{g} - \frac{\exp(\frac{-r_n^2t}{P_r})G_rP_r}{(gP_r+r_n^2)} + \frac{(-g+gr_n^2-\omega^2)\sin\omega t}{(g^2+\omega^2)} + \frac{r_n^2\exp(-gt)\{G_r(g^2+\omega^2)+g\omega(gP_r-r_n^2)\}}{g(gP_r-r_n^2)(g^2+\omega^2)} - \frac{r_n^2\omega\cos\omega t}{(g^2+\omega^2)}\right]. \quad (42)$$

5.2.3 Shear stress: (for *cosine* oscillation.)

Take Eq. (30), by putting $\lambda \to 0$ and taking $g = M + \varphi + r_n^2$, we get.

$$\bar{\tau} = 2\sum_{n=1}^{\infty}\frac{r_n^2J_\circ(rr_n)}{J_1(r_n)}\left\{\frac{q^2(P_rq+r_n^2)+G_r(q^2+\omega^2)}{q(q^2+\omega^2)(P_rq+r_n^2)(q+g)}\right\}. \quad (43)$$

Taking Laplace inverse transformation in Eq. (40), we get.

$$\tau_c(r,t) = 2\sum_{n=1}^{\infty}\frac{r_n^2J_\circ(rr_n)}{J_1(r_n)}\left[\frac{G_r}{gr_n^2} + \frac{\exp(\frac{-r_n^2t}{P_r})G_rP_r}{r_n^2(-gP_r+r_n^2)} + \frac{\exp(-gt)\{G_r(g^2+\omega^2)-g^2(gP_r+r_n^2)\}}{g(gP_r-r_n^2)(g^2+\omega^2)} + \frac{g\cos\omega t+\omega\sin\omega t}{(g^2+\omega^2)}\right]. \quad (44)$$

Hence, above Equation is the permanent solution of shear stress for *cosine* oscillation that shows the Newtonian's fluid properties with MHD and porous effect.

5.2.4 Shear stress: (for *sine* oscillation.)

$$\tau_c(r,t) = 2\sum_{n=1}^{\infty}\frac{r_n^2J_\circ(rr_n)}{J_1(r_n)}\left[\frac{G_r}{gr_n^2} + \frac{\exp(\frac{-r_n^2t}{P_r})G_rP_r}{r_n^2(-gP_r+r_n^2)} + \frac{\exp(-gt)\{G_r(g^2+\omega^2)+g\omega(gP_r-r_n^2)\}}{g(gP_r-r_n^2)(g^2+\omega^2)} + \frac{g\sin\omega t-\omega\cos\omega t}{(g^2+\omega^2)}\right]. \quad (45)$$

Above equation is the permanent solution of shear stress for *sine* oscillation that shows the Newtonian's fluid properties with MHD and porous effect.

5.3 Newtonian without MHD: ($\lambda \to 0$ $M \to 0$)

5.3.1 Velocity: (for *cosine* oscillation.)

Taking Eq. (23) and by assuming $\lambda \to 0$ $M \to 0$, we get.

$$\bar{u}_H = \frac{qJ_1(r_n)}{r_n(q^2+\omega^2)} + \frac{J_1(r_n)\left\{\begin{matrix} r_n^2q^2(P_rq+r_n^2)+r_n^2G_r(q^2+\omega^2) \\ -q^2(qP_r+r_n^2)(q+\psi+r_n^2)\end{matrix}\right\}}{r_nq(q^2+\omega^2)(P_rq+r_n^2)(q+\psi+r_n^2)}. \quad (46)$$

By replacing $h = \psi + r_n^2$ and taking inverse Laplace and Hankel transformation, we get.

$$u_c(r,t) = cos\omega t + 2\sum_{n=1}^{\infty}\frac{J_\circ(rr_n)}{r_nJ_1(r_n)}\left[\frac{G_r}{h} + \frac{\exp(\frac{-r_n^2t}{P_r})\{G_rP_r^2(r_n^4+P_r^2\omega^2)-r_n^6(P_r^2-r_n^2)\}}{P_r(hP_r-r_n^2)(r_n^4+P_r^2\omega^2)} + \frac{r_n^2\exp(-ht)\{G_r(h^2-\omega^2)+h^3(h-P_r)\}}{h(hP_r-r_n^2)(\omega^2+h^2)} + \frac{\{hP_rr_n^2\omega(r_n^2-\omega^2)+r_n^2\omega^3(P_r^2-r_n^2)\}\sin\omega t}{(h^2+\omega^2)(r_n^4+P_r^2\omega^2)} + \frac{\{-(h^2+\omega^2)(r_n^4+P_r^2\omega^2)+P_rr_n^2\omega^2(r_n^2+\omega^2)+h^2r_n^2\omega^2(P_r^2-r_n^2)\}\cos\omega t}{(h^2+\omega^2)(r_n^4+P_r^2\omega^2)}\right]. \quad (47)$$

Hence, above equation is the permanent solution of velocity field for *cosine* oscillation that shows the Newtonian's fluid properties without MHD effect.

5.3.2 Velocity: (for *sine* oscillation.)

Similarly, the permanent solution of velocity field for *sine* oscillation that shows the Newtonian's fluid properties without MHD effect, defined as.

$$u_s(r,t) = sin\omega t + 2\sum_{n=1}^{\infty}\frac{J_\circ(rr_n)}{r_nJ_1(r_n)}\left[\frac{G_r}{h} - \frac{\exp(\frac{-r_n^2t}{P_r})G_rP_r}{+} \frac{r_n^2\exp(-ht)\{G_r(h^2+\omega^2)+h\omega(hP_r-r_n^2)\}}{h(hP_r-r_n^2)(h^2+\omega^2)} - \frac{r_n^2\cos\omega t}{(h^2+\omega^2)} + \frac{(-h+hr_n^2-\omega^2)\sin\omega t}{(h^2+\omega^2)}\right] \quad (48)$$

5.3.3 Shear stress: (for *cosine* oscillation.)

Take Eq. (30), by using $\lambda \to 0$ $M \to 0$ and taking $h = \psi + r_n^2$, we get.

$$\bar{\tau} = 2\sum_{n=1}^{\infty} \frac{r_n^2 J_\circ(rr_n)}{J_1(r_n)} \left\{ \frac{q^2(P_r q + r_n^2) + G_r(q^2+\omega^2)}{q(q^2+\omega^2)(P_r q + r_n^2)(q+h)} \right\}. \quad (49)$$

Taking Laplace inverse transformation in Eq. (2.49), we get.

$$\tau_c(r,t) = 2\sum_{n=1}^{\infty} \frac{r_n^2 J_\circ(rr_n)}{J_1(r_n)} \left[\frac{G_r}{hr_n^2} + \frac{\exp(\frac{-r_n^2 t}{P_r}) G_r P_r}{r_n^2(-hP_r + r_n^2)} + \frac{\exp(-ht)\{G_r(h^2+\omega^2) - h^2(hP_r + r_n^2)\}}{h(hP_r - r_n^2)(h^2+\omega^2)} + \frac{h\cos\omega t + \omega \sin\omega t}{(h^2+\omega^2)} \right]. \quad (50)$$

Hence, above equation is the permanent solution of shear stress for *cosine* oscillation that shows the Newtonian's fluid properties without MHD effect.

5.3.4 Shear stress: (for *sine* oscillation.)

Similarly, the permanent solution of velocity field for *sine* oscillation that shows the Newtonian's fluid properties without MHD effect.

$$\tau_s(r,t) = 2\sum_{n=1}^{\infty} \frac{r_n^2 J_\circ(rr_n)}{J_1(r_n)} \left[\frac{G_r}{hr_n^2} + \frac{\exp(\frac{-r_n^2 t}{P_r}) G_r P_r}{r_n^2(-hP_r + r_n^2)} + \frac{\exp(-ht)\{G_r(h^2+\omega^2) + h\omega(hP_r - r_n^2)\}}{h(hP_r - r_n^2)(h^2+\omega^2)} + \frac{h\sin\omega t - \omega \cos\omega t}{(h^2+\omega^2)} \right]. \quad (51)$$

5.4 Maxwell without porous: ($\psi \to 0$)

5.4.1 Velocity computation: (for *cosine* oscillation.)

Taking Eq. (23), by substituting $\psi \to 0$ and assuming e and f as,

$$e = \frac{-(1+M\lambda) + \sqrt{1 + M^2\lambda^2 - 2M\lambda + 4r_n^2\lambda}}{2\lambda},$$

$$f = \frac{-(1+M\lambda) - \sqrt{1 + M^2\lambda^2 - 2M\lambda + 4r_n^2\lambda}}{2\lambda}.$$

Eq. (23) become,

$$\bar{u}_H(r_n,q) = \frac{qJ_1(r_n)}{r_n(q^2+\omega^2)} + \frac{J_1(r_n)\left\{ \begin{matrix} r_n^2 q^2(P_r q + r_n^2) + r_n^2 G_r(1+\lambda q)(q^2+\omega^2) \\ -q^2(qP_r + r_n^2)(q-e)(q-f) \end{matrix} \right\}}{r_n q(q^2+\omega^2)(P_r q + r_n^2)(q-e)(q-f)}. \quad (52)$$

Taking inverse Laplace and Hankel transformation, we get.

$$u_c(r,t) = \cos\omega t + 2\sum_{n=1}^{\infty} \frac{J_\circ(rr_n)}{\lambda r_n J_1(r_n)} \left[\frac{G_r}{ef} + \frac{G_r P_r \exp(\frac{-r_n^2 t}{P_r})(-P_r + \lambda r_n^2)}{(eP_r + r_n^2)(fP_r + r_n^2)} + \frac{r_n^2 \exp(et)\{e^3(G_r\lambda + P_r) + e^2(G_r + r_n^2) + eG_r\lambda\omega^2 + G_r\omega^2\}}{e(e-f)(eP_r + r_n^2)(e^2+\omega^2)} + \frac{r_n^2 \exp(ft)\{f^3(G_r\lambda + P_r) + f^2(G_r + r_n^2) + fG_r\lambda\omega^2 + G_r\omega^2\}}{f(-e+f)(fP_r + r_n^2)(f^2+\omega^2)} + \frac{\{ef(-ef\lambda + r_n^2) - \lambda\omega^2(e^2+f^2+\omega^2) - r_n^2\omega^2\}\cos\omega t - r_n^2\omega(e+f)\sin\omega t}{(f^2+\omega^2)(e^2+\omega^2)} \right]. \quad (53)$$

Above equation is the exact solution of velocity field without porous effect.

5.4.2 Velocity computation: (for *sine* oscillation.)

Assuming $\psi \to 0$, we obtained exact solution of velocity without porous effect.

$$u_s(r,t) = \sin\omega t + 2\sum_{n=1}^{\infty} \frac{J_\circ(rr_n)}{\lambda r_n J_1(r_n)} \left[\frac{G_r}{ef} + \frac{G_r P_r \exp(\frac{-r_n^2 t}{P_r})(-P_r + \lambda r_r^2)}{(eP_r + r_n^2)(fP_r + r_n^2)} + \frac{r_n^2 \exp(et)\{e^3(G_r\lambda + P_r) + e^2(G_r + r_n^2) + eG_r\lambda\omega^2 + G_r\omega^2\}}{e(e-f)(eP_r + r_n^2)(e^2+\omega^2)} + \frac{r_n^2 \exp(ft)\{f^3(G_r\lambda + P_r) + f^2(G_r + r_n^2) + fG_r\lambda\omega^2 + G_r\omega^2\}}{f(-e+f)(fP_r + r_n^2)(f^2+\omega^2)} + \frac{\{ef(-ef\lambda + r_n^2) - \lambda\omega^2(e^2+f^2+\omega^2) - r_n^2\omega^2\}\sin\omega t + r_n^2\omega(e+f)\cos\omega t}{(f^2+\omega^2)(e^2+\omega^2)} \right]. \quad (54)$$

5.4.3 Solution of shear stress: (for *cosine* oscillation.)

Taking Eq. (30), by replacing $a = e$ and $b = f$, we get.

$$\bar{\tau} = 2\sum_{n=1}^{\infty} \frac{r_n^2 J_\circ(rr_n)}{\lambda J_1(r_n)(1+\lambda q)} \left\{ \frac{q^2(P_r q + r_n^2) + G_r(1+\lambda q)(q^2+\omega^2)}{\lambda q(q^2+\omega^2)(P_r q + r_n^2)(q-e)(q-f)} \right\}. \quad (55)$$

Taking inverse Laplace transformation, we get.

$$\tau_c(r,t) = 2\sum_{n=1}^{\infty}\frac{r_n J_\circ(rr_n)}{\lambda(P_r+r_n)J_1(r_n)}\left[\frac{-G_r}{ef} + \frac{\exp(et)\{e^3(G_r\lambda+P_r)+e^2(G_r+r_n^2)+eG_r\omega^2\lambda+G_r\omega^2\}}{e(e+f)(1+e\lambda)(e^2+\omega^2)} + \frac{\exp(-ft)\{f^3(G_r\lambda+P_r)-f^2(G_r+r_n^2)+fG_r\omega^2\lambda-G_r\omega^2\}}{f(e+f)(-1+f\lambda)(f^2+\omega^2)} + \frac{\lambda\exp(-\frac{t}{\lambda})(-P_r+\lambda r_n^2)}{(1+e\lambda)(-1+f\lambda)(1+\lambda^2\omega^2)} + \frac{\begin{Bmatrix}\{-(ef+\omega^2)(r_n^2+\lambda p_r\omega^2)-\omega^2(-e+f)(P_r-\lambda r_n^2)\}\cos\omega t \\ +\{-e\omega(r_n^2+\lambda P_r\omega^2)+f\omega(r_n^2+\lambda\omega^2-r_n^2\omega^2)+ef(P_r\omega-\lambda r_n^2)+P_r\omega^3\}\sin\omega t\end{Bmatrix}}{(e^2+\omega^2)(f^2+\omega^2)(1+\lambda^2\omega^2)}\right]. \quad (56)$$

Above Equation is the Exact Solution of shear stress for *cosine* oscillation without porous effect.

5.4.4 *Shear stress: (for* ***sine*** *oscillation.)*

The exact solution of shear stress for *sine* oscillation without porous effect is define as,

$$\tau_s(r,t) = 2\sum_{n=1}^{\infty}\frac{r_n J_\circ(rr_n)}{\lambda(P_r+r_n)J_1(r_n)}\left[\frac{G_r}{ef} + \frac{\exp(et)\{e^3G_r\lambda+e^2(G_r+P_r\omega)+e\omega(G_r\lambda\omega+r_n^2)+G_r\omega^2\}}{e(e+f)(1+e\lambda)(e^2+\omega^2)} + \frac{\exp(-ft)\{f^3G_r\lambda-f^2(G_r+P_r\omega)+f\omega(G_r\lambda\omega+r_n^2)-G_r\omega^2\}}{f(e+f)(1+e\lambda)(e^2+\omega^2)} + \frac{\exp(-\frac{t}{\lambda})(P_r-\lambda r_n^2)\lambda^2\omega}{(1+e\lambda)(-1+f\lambda)(1+\lambda^2\omega^2)} + \frac{\begin{Bmatrix}\{-ef(r_n^2+\lambda P_r\omega^2)-P_r\omega^2(e-f)-r_n^2\omega^2\}\sin\omega t \\ -\{\omega(ef+\omega^2)(P_r-\lambda r_n^2)-\omega(e+f)(r_n^2+\lambda P_r\omega^2)\}\cos\omega t\end{Bmatrix}}{(e^2+\omega^2)(f^2+\omega^2)(1+\lambda^2\omega^2)}\right]. \quad (57)$$

5.5 Newtonian without porous: ($\lambda \to 0\ \psi \to 0$)

5.5.1 *Velocity: (for* ***cosine*** *oscillation.)*

Taking Eq. (23) and assuming $\lambda \to 0\ \psi \to 0$, we get.

$$\overline{u}_H = \frac{qJ_1(r_n)}{r_n(q^2+\omega^2)} + \frac{J_1(r_n)\begin{Bmatrix}r_n^2q^2(P_rq+r_n^2)+r_n^2G_r(q^2+\omega^2) \\ -q^2(qP_r+r_n^2)(q+M+r_n^2)\end{Bmatrix}}{r_nq(q^2+\omega^2)(P_rq+r_n^2)(q+M+r_n^2)}. \quad (58)$$

By replacing $l = M + r_n^2$ and taking inverse Laplace and Hankel transformation, we get.

$$u_c(r,t) = \cos\omega t + 2\sum_{n=1}^{\infty}\frac{J_\circ(rr_n)}{r_nJ_1(r_n)}\left[\frac{G_r}{l} + \frac{\exp(\frac{-r_n^2t}{P_r})\{G_rP_r^2(r_n^4+P_r^2\omega^2)-r_n^6(P_r^2-r_n^2)\}}{P_r(lP_r-r_n^2)(r_n^4+P_r^2\omega^2)} + \frac{r_n^2\exp(-lt)\{G_r(l^2-\omega^2)+l^3(l-P_r)\}}{l(lP_r-r_n^2)(\omega^2+l^2)} + \frac{\{lP_rr_n^2\omega(r_n^2-\omega^2)+r_n^2\omega^3(P_r^2-r_n^2)\}\sin\omega t}{(l^2+\omega^2)(r_n^4+P_r^2\omega^2)} + \frac{\{-(l^2+\omega^2)(r_n^4+P_r^2\omega^2)+P_rr_n^2\omega^2(r_n^2+\omega^2)+l^2r_n^2\omega^2(P_r^2-r_n^2)\}\cos\omega t}{(l^2+\omega^2)(r_n^4+P_r^2\omega^2)}\right]. \quad (59)$$

Hence, above equation is the permanent solution of velocity field for *cosine* oscillation that shows the Newtonian's fluid properties without porous effect.

5.5.2 *Velocity: (for* ***sine*** *oscillation.)*

Similarly, the permanent solution of velocity field for *sine* oscillation that shows the Newtonian's fluid properties without porous effect, defined as.

$$u_s(r,t) = \sin\omega t + 2\sum_{n=1}^{\infty}\frac{J_\circ(rr_n)}{r_nJ_1(r_n)}\left[\frac{G_r}{l} - \frac{\exp(\frac{-r_n^2t}{P_r})G_rP_r}{+} \frac{r_n^2\exp(-lt)\{G_r(l^2+\omega^2)+l\omega(lP_r-r_n^2)\}}{l(lP_r-r_n^2)(l^2+\omega^2)} - \frac{r_n^2\cos\omega t}{(l^2+\omega^2)} + \frac{(-l+lr_n^2-\omega^2)\sin\omega t}{(l^2+\omega^2)}\right]. \quad (60)$$

5.5.3 *Shear stress: (for* ***cosine*** *oscillation.)*

Take Eq. (30), by using $\lambda \to 0\ \psi \to 0$ and taking $l = M + r_n^2$, we get.

$$\overline{\tau} = 2\sum_{n=1}^{\infty}\frac{r_n^2J_\circ(rr_n)}{J_1(r_n)}\left\{\frac{q^2(P_rq+r_n^2)+G_r(q^2+\omega^2)}{q(q^2+\omega^2)(P_rq+r_n^2)(q+l)}\right\}. \quad (61)$$

Taking Laplace inverse transformation, we get.

$$\tau_c(r,t) = 2\sum_{n=1}^{\infty}\frac{r_n^2J_\circ(rr_n)}{J_1(r_n)}\left[\frac{G_r}{hr_n^2} + \frac{\exp(\frac{-r_n^2t}{P_r})G_rP_r}{r_n^2(-lP_r+r_n^2)} + \frac{\exp(-lt)\{G_r(l^2+\omega^2)-l^2(lP_r+r_n^2)\}}{l(lP_r-r_n^2)(l^2+\omega^2)} + \frac{l\cos\omega t+\omega\sin\omega t}{(l^2+\omega^2)}\right]. \quad (62)$$

Hence, above Equation is the permanent solution of shear stress for *cosine* oscillation that shows the Newtonian's fluid properties without porous effect.

5.5.4 Shear stress: (for *sine* oscillation.)

Similarly, the permanent solution of velocity field for *sine* oscillation that shows the Newtonian's fluid properties without porous effect.

$$\tau_s(r,t) = 2\sum_{n=1}^{\infty}\frac{r_n^2 J_\circ(rr_n)}{J_1(r_n)}\left[\frac{G_r}{lr_n^2}+\frac{\exp(\frac{-r_n^2 t}{P_r})G_rP_r}{r_n^2(-lP_r+r_n^2)} + \frac{\exp(-lt)\{G_r(l^2+\omega^2)+l\omega(lP_r-r_n^2)\}}{l(lP_r-r_n^2)(l^2+\omega^2)} + \frac{h\sin\omega t-\omega\cos\omega t}{(l^2+\omega^2)}\right]. \quad (63)$$

6. Model illustration:

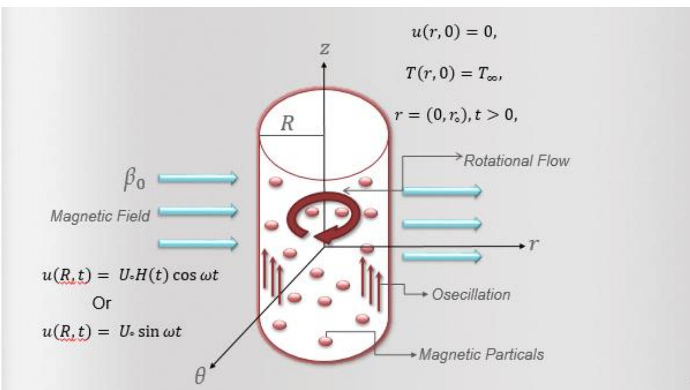


Figure 1: MHD flows of Maxwell fluid through vertical circular cylinder.

7. Result and discussion:

The effect of various fluid parameters on temperature $T(r,t)$, fluid velocity in term of *cosine* oscillation $u_c(r,t)$, fluid velocity in term of *sine* oscillation $u_s(r,t)$, shear stress in term of *cosine* oscillation $\tau_c(r,t)$ and shear stress in term of *sine* oscillation $\tau_s(r,t)$ have been drawn against r and t by using computational Mathcad software in Fig. (2 - 7).

In Figures (2a, 2b) effects of time and P_r on temperature has been drawn graphically. It is clearly seen in Fig. (a) that temperature increased by increasing time and in Fig. (b) temperature decreased by increasing prandtl number.

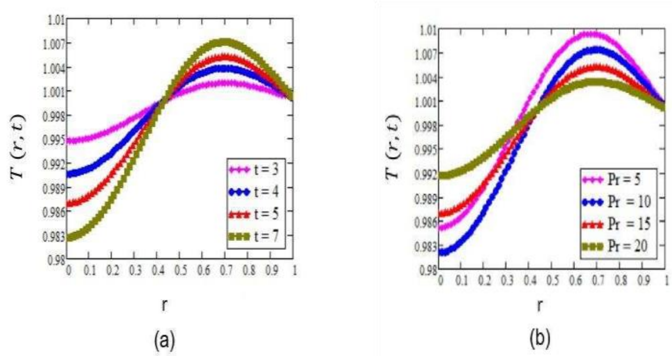


Figure 2: Temperature graph against r for different values of t and p_r.

In Figures (3a, 3b) effects of radius and prandtl number on temperature has been illustrated. Fig. (a) shows that temperature increased by increasing radius and in Fig. (b) the temperature decreased by increasing P_r number

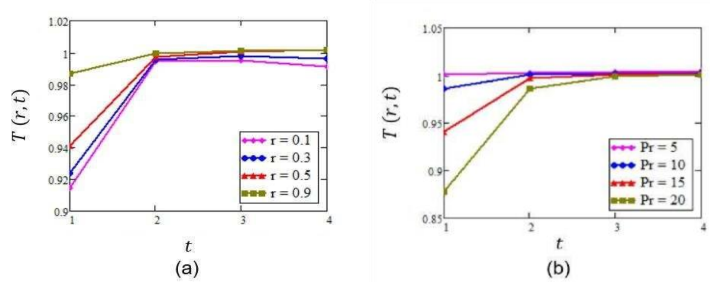
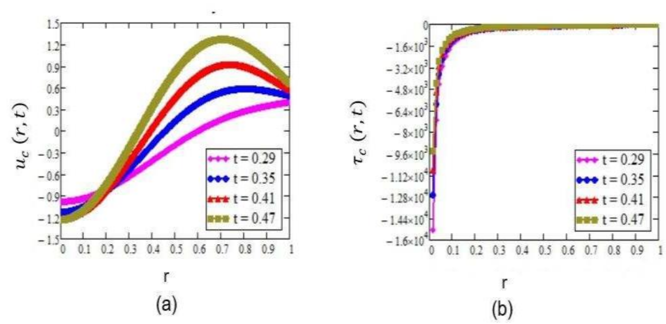


Figure 3: Temperature graph against t for different values of r and p_r

In Figures (4a - 4d) effects of time on velocity and shear stress in term of *cosine* and *sine* oscillations has been discuses. Here $G_r = 14, P_r = 20, S_c = 0.5, \lambda = 1.5, \omega = 1.5$ and $\psi = 0.1$. Fig. (4a, 4c) indicates velocity increased in the form of *cosine* and *sine* oscillation by increasing time and Fig. (4b, 4d) represents share stress increased in the form of *cosine* and *sine* oscillations by increasing time.



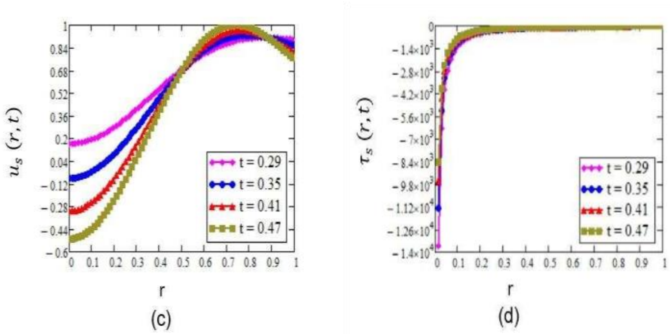


Figure 4: Velocity and shear stress graph of *cosine* and *sine* oscillation against r for different value of t.

In Figures (5a - 5d) effects of prandtl number on velocity and share stress in term of *cosine* and *sine* oscillations has been discourse. Here $G_r = 14, t = 0.3, S_c = 0.5, \lambda = 1.5, \omega = 1.5$ and $\psi = 0.1$. Fig. (5a, 5c) shows velocity decreased through boundary conditions *cosine* and *sine* oscillations and Fig. (5b, 5d) shows share stress increased through boundary conditions *cosine* and *sine* oscillations by increasing prandtl number.

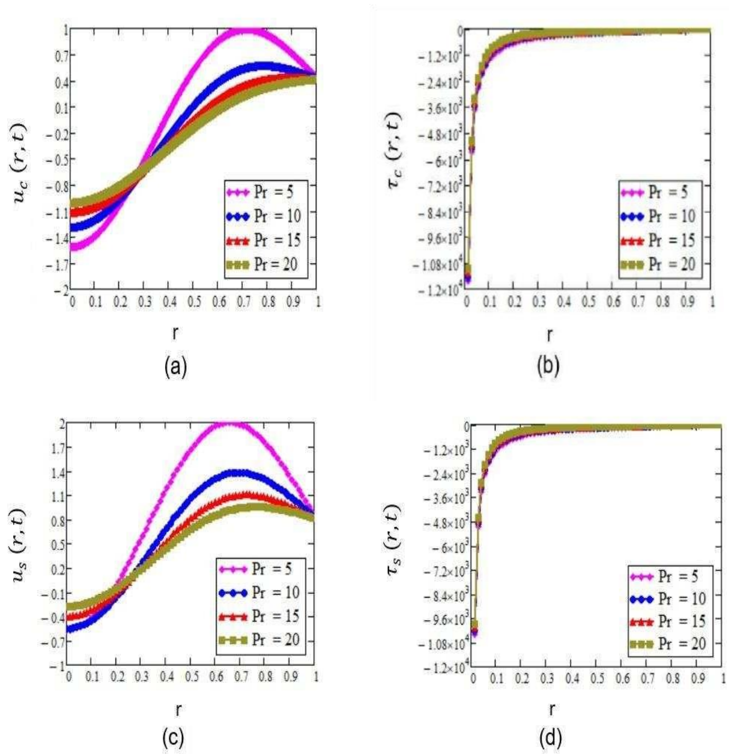


Figure 5: Velocity and shear stress graph of *cosine* and *sine* oscillations against r for different value of p_r.

In Figures (6a - 6d) effects of G_r number on velocity and share stress in term of *cosine* and *sine* oscillations has been illustrated. Here $P_r = 20, t = 0.3, S_c = 0.5, \lambda = 1.5, \omega = 1.5$ and $\psi = 0.1$. Fig. (6a, 6c) shows velocity in term of *cosine* and *sine* oscillations increased by increasing G_r number while share stress decreased by increasing G_r.

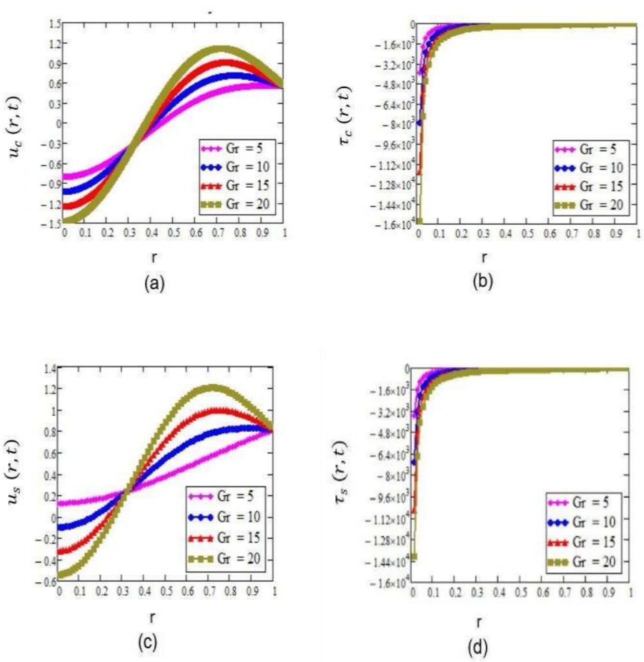


Figure 6: Velocity and shear stress graph of *cosine* and *sine* oscillations against r for different value of G_r.

In Figures (7a - 7d) effects of prandtl number on velocity and shear stress graph against t has been illustrated. Here $G_r = 14, r = 0.3, S_c = 0.5, \lambda = 1.5, \omega = 1.5$ and $\psi = 0.1$. It is clearly seen in Fig. (7a, 7c) that the velocity increased by increasing P_r number while Fig. (7b, 7d) indicates that share stress decreased by increasing P_r.

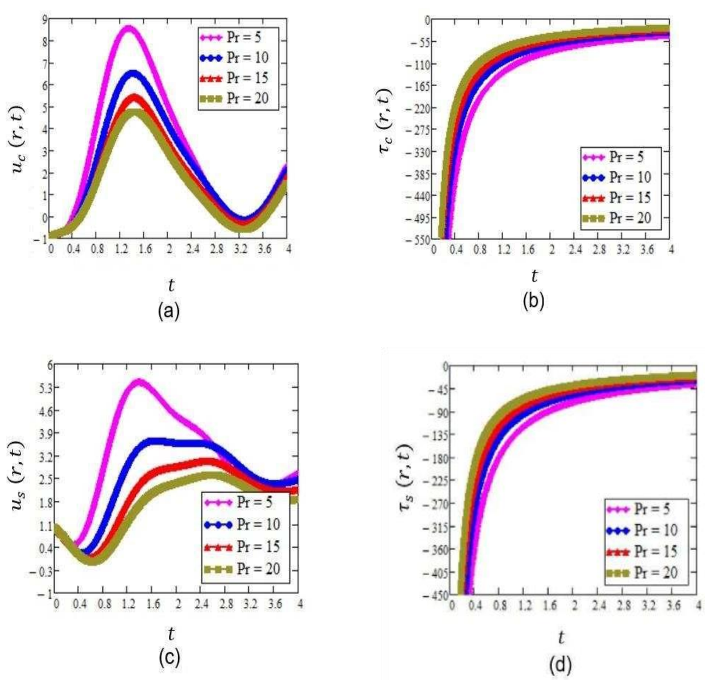


Figure 7: Velocity and shear stress graph of *sine* and *cosine* oscillation against t for different value of P_r

In Figures (8a - 8d) effects of grashuff number on velocity and shear stress graph against t has been illustrated. Here $P_r = 20, r = 0.3, S_c = 0.5, \lambda = 1.5, \omega = 1.5$ and $\psi = 0.1$. It is clearly observed in Fig. (8a, 8c) that the velocity decreased while Fig. (8b, 8d) share stress increased by increasing G_r.

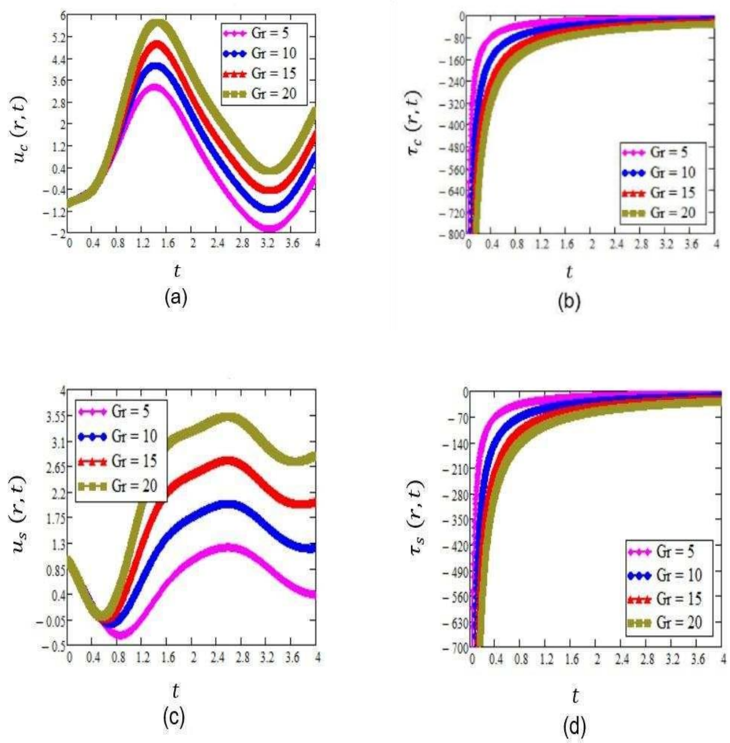


Figure 8: Velocity and shear stress graph of *sine* and *cosine* oscillations against t for different value of G_r.

8. Conclusion:

In this paper, an exact solution of heat and mass transfer of Maxwell fluid with MHD and porous effect along a vertical circular cylinder is concluded by applying combine Laplace and Hankel transformation. The motion of the fluid is longitudinal. More accurately, the objective is to determine the velocity field, shear stress and temperature to the movement inside a circular cylinder of the standardized Maxwell fluid model. Initially at $t = 0$ fluid are at rest with constant temperature T_∞. The actual movement of fluid is started by the cylinder, as the axis with velocity $cos(\omega t)$ or $sin(\omega t)$. For *sine* and *cosine* oscillation the exact transitional solution with MHD effect of velocity are $u_c(r,t)$ or $u_s(r,t)$, shear stress are $\tau_c(r,t)$ and $\tau_s(r,t)$ and temperature is $T(r,t)$. This solution satisfied all initial and boundary conditions. As special cases the corresponding solution for Maxwell fluid with MHD effect is obtained and without MHD effect is obtained by considering $M \longrightarrow 0$. The Newtonian solutions for velocity field, shear stress and temperature with MHD effect is obtained by assuming $\lambda \longrightarrow 0$ and without MHD effect is obtained by considering $\lambda \longrightarrow 0, M \longrightarrow 0$. Also Maxwell fluid without porous effect is obtained by considering $\psi \longrightarrow 0$. The Newtonian solutions for velocity field, shear stress and temperature with porous effect is obtained by assuming $\lambda \longrightarrow 0$ and without porous effect is obtained by considering $\lambda \longrightarrow 0, \psi \longrightarrow 0$.

9. Nomenclature:

g acceleration due to gravity.
k thermal conductivity of field.
ρ density.
τ shear stress.
ω frequency of oscillation.
ν kinematic viscosity.
μ coefficient of viscosity.
R Radius.
q laplace transforms parameter.
r dimension less coordinate.
λ (dimensionless) relaxation time.
ε Porosity.
σ electrical conductivity.
φ porous effect.
M magnetic effect.
$\beta_\circ$ transverse magnetic field.
β_1 radio frequency energy field applied perpendicular to $\beta_\circ$.
p_r prandtl number.
C_p acceleration due to gravity.
G_m grashof number.
G_r modified grashof number.
T_ω wall temperature.
T_∞ ambient temperature.
$H(t)$ unit step function.
$u(r,t)$ velocity field.
$\tau(r,t)$ shear stress.
$T(r,t)$ temperature distribution.

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