Proper Lucky Labeling of Triangular Snake Graph

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Abstract The labeling is said to be lucky labeling of the graph if the vertices of the graph are labeled by natural number with satisfying the condition that sum of labels over the adjacent of the vertices in the graph are not equal and if vertices are isolated vertex then the sum of the vertex is zero. The least natural number which labelled the graph is the lucky number. The Lucky Number of graph G is denoted by $\eta(G)$. The labeling defined as proper labeling if the vertices of the graph are labeled by natural number with fulfilling the condition that label of adjacent vertices is not the same. The labeling is defined as proper lucky labeling if labeling is proper and also lucky. The proper lucky number of graph G is denoted by $\eta_p(G)$. In this paper we obtain a lucky number and proper lucky number for triangular snake graph as well as double triangular snake graph.

Keywords Triangular snake graph, Double triangular snake graph, Lucky labeling, Proper lucky labeling, Lucky edge labelling.

I.Introduction

In graph theory, graph labeling has a wide area. It has a well-built connection among numbers and the structure of the graph. It was introduced by Rosa in 1967. Labeling is a function to mark the number to a graph vertex or edge or both with the specific condition. Gallian gave a dynamic survey of graphlabeling. The proper labeling was initiated by Karonsik (2004) which mean naming adjacent vertices by a different natural number. Lucky labeling for 3 colourable graphs was studied by Ahadi (2012). Lucky labeling is compared with proper vertex coloring by Akbari (2013). The graph which fulfill the lucky labeling conditions are called lucky graph. The graph which fulfill the proper lucky labelling conditions are called proper lucky graph. Proper lucky labeling has a very close association with graph colouring. The notion of lucky edge labeling was introduced by Nellai Murugan. The main objective of this paper is to study the lucky number and proper lucky number for triangular snake graph, double triangular snake graph.

In this section, we give the basic definitions relevant to this paper. Let G(V,E) be a finite, simple and undirected graph with p vertices and q edges. A Graph labeling is the assignment of labels, traditionally represented by integers, to edges and(or) vertices of a graph to a set of labels.Let $f : V(G) \to N$ be a labeling of the vertices of a graph G by a natural number.Let S(v) indicate the sum of labels over the neighbours of the vertex v in G.If v is an isolated vertex of G we put S(v) = 0.A labeling f is Lucky if $S(u) \neq S(v)$ for an adjacent vertex u and v.A labeling f is proper lucky labeling, if u and v are adjacent in G then $f(u) \neq f(v)$and $S(u) \neq S(v)$.The proper Lucky number of G is denoted by $\eta p(G)$, is the leastnatural number k such that G has proper Lucky Labeling with {1, 2, 3, . . . , k} as theset of labels.A Triangular Snake Graph TS_n is obtained from a path $u_1, u_2, u_3, \ldots, u_n$ by connectingu_i and u_{i+1} to a new vertex v_i, for $1 \leq i \leq n$, where n is the number of edges of the path.It has $2n - 1$ vertices where $n \geq 1$, u_i have n vertices, v_i have $n - 1$ vertices and the edge set $E(TS_n) = \{u_iu_{i+1} : 1 \leq i \leq n\} \cup \{u_iv_i : 1 \leq i \leq n\} \cup \{u_{i+1}v_i : 1 \leq i \leq n\}\}$.A Double Triangular Snake Graph DTS_n is consist of $3n - 2$ vertices where $n \geq 1$,u_i have n vertices, v_i have $n - 1$ vertices, w_i have $n - 1$ vertices and the edge set $E(DTS_n) = E(TS_n) \cup \{u_iw_i : 1 \leq i \leq n\} \cup \{u_{i+1}w_i : 1 \leq i \leq n\}$. The main objective of this paper is to study the lucky number and proper lucky number for triangular snake graph, double triangular snake graph.

II. MAIN RESULTS

Theorem 2.1. The Triangular snake TSn with $n \geq 1$ admits lucky labeling with lucky number $\eta(TSn) = 2$.

Proof

Let $f : V(TSn) \to \{1, 2\}$ triangular snake graph with $2n - 1$ vertices where $n \geq 1$, u_i have n vertices, vi have $n - 1$ vertices and the edge set

$E(TS_n) = \{u_iu_{i+1} : 1 \leq i \leq n\} \cup \{u_iv_i : 1 \leq i \leq n\} \cup \{u_{i+1}v_i : 1 \leq i \leq n\}$ be defined by

$$f(u_i) = \begin{cases} 1 & \text{even } i \\ 2 & \text{odd } i \end{cases}$$

$$f(v_i) = \begin{cases} 1 & \text{even } i \\ 2 & \text{odd } i \end{cases}$$

Here we obtain the sum of neighbour vertices as,

$$S(u_i) = \begin{cases} 3 & i = 1 \\ 7 & i = \text{even } n \\ 5 & i = \text{odd } n \\ 4 & i = 6 \end{cases}$$

$$S(v_i) = \begin{cases} 4 & i = 1 \\ 3 & \text{others} \end{cases}$$

Therefore sum of adjacent vertices are not same. So, Triangular snake graph TS_n with $n \geq 1$ admits lucky labeling with lucky number $\eta(TS_n) = 2$.

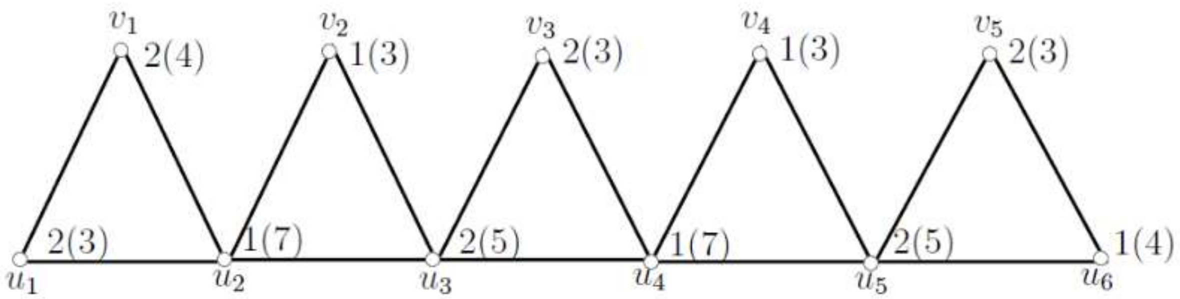

Fig 1 : Lucky Triangular Snake Graph TS_6

Theorem 2.2. The Double Triangular snake DTSn with $n \geq 1$ admits lucky labeling with lucky number $\eta(DTSn) = 2$.

Proof

Given Double Triangular Snake Graph DTSn is consist of $3n-2$ vertices where $n \geq 1$, ui have n vertices, vi have $n-1$ vertices, wi have $n-1$ vertices and the edge set

$E(DTSn) = E(TSn) \cup \{u_i w_i : 1 \leq i \leq n\} \cup \{u_{i+1} w_i : 1 \leq i \leq n\}$

Let $f : V(DTSn) \rightarrow \{1, 2\}$ for triangular snake graph be defined by

$$f(u_i) = \begin{cases} 1 & \text{even } i \\ 2 & \text{odd } i \end{cases}$$

$$f(v_i) = \begin{cases} 1 & \text{even } i \\ 2 & \text{odd } i \end{cases}$$

$$f(w_i) = \begin{cases} 2 & \text{even } i \\ 1 & \text{odd } i \end{cases}$$

Here we obtain the sum of neighbour vertices as,

$S(u_1) = 4$

$S(u_n) = 5$

$$S(u_i) = \begin{cases} 8 & i = 3, 5, \ldots \\ 10 & i = 2, 4, \ldots \end{cases}$$

$S(v_i) = 3$

$S(w_i) = 3$

Therefore sum of adjacent vertices are not same. So, Double Triangular snake graph DTSn with n ≥ 1 admits lucky labeling with lucky number η(DTSn) = 2.

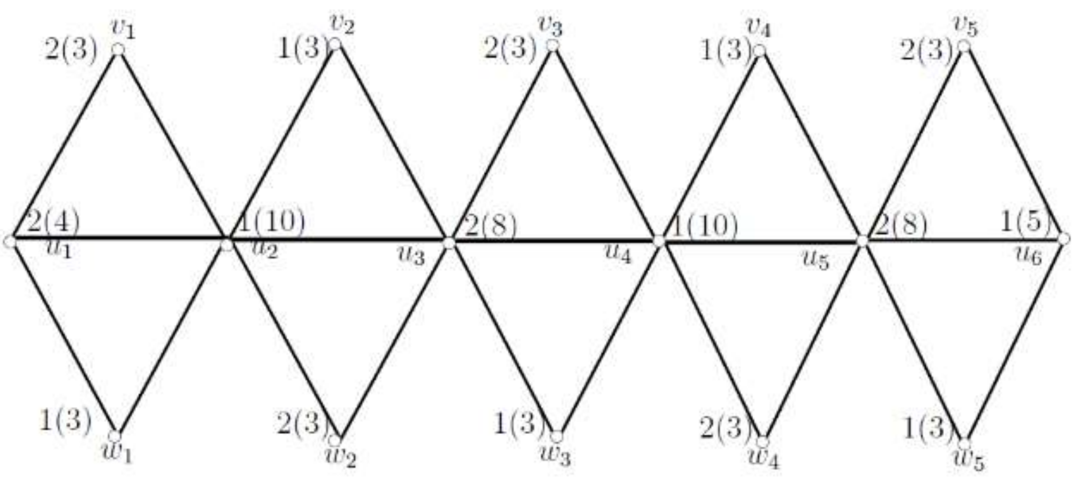

Fig 2 : Lucky double triangular snake graph DTS_6

Theorem 2.3. The Triangular snake TSn with n ≥ 1 admits proper lucky labeling with proper lucky number η_p(TSn) = 3.

Proof

Let f : V (TSn) → {1, 2, 3} for triangular snake graph with 2n − 1 vertices

where $n \geq 1$, ui have n vertices, vi have $n - 1$ vertices and the edge set
$E(TS_n) = \{u_iu_{i+1} : 1 \leq i \leq n\} \cup \{u_iv_i : 1 \leq i \leq n\} \cup \{u_{i+1}v_i : 1 \leq i \leq n\}$ be defined by

$$f(u_i) = \begin{cases} 1 & \text{when } i = 1, 4, 7, \ldots \\ 2 & \text{when } i = 2, 5, 8, \ldots \\ 3 & \text{when } i = 3, 6, 9, \ldots \end{cases}$$

$$f(v_i) = \begin{cases} 3 & \text{when } i = 1, 4, 7, \ldots \\ 1 & \text{when } i = 2, 5, 8, \ldots \\ 2 & \text{when } i = 3, 6, 9, \ldots \end{cases}$$

Here we obtain the sum of neighbour vertices as ,

$S(u_1) = 5$

$S(u_2) = S(u_5) = 5$

$S(u_3) = 6$

$S(u_4) = 10$

$S(u_6) = 3$

$$S(v_i) = \begin{cases} 3 & \text{when } i = 1, 4, 7, \ldots \\ 5 & \text{when } i = 2, 5, 8, \ldots \\ 4 & \text{when } i = 3, 6, 9, \ldots \end{cases}$$

Therefore sum of adjacent vertices and adjacent labeling are not same. So, Triangular snake graph TSn with $n \geq 1$ admits proper lucky labeling with proper lucky number $\eta p(TSn) = 3$.

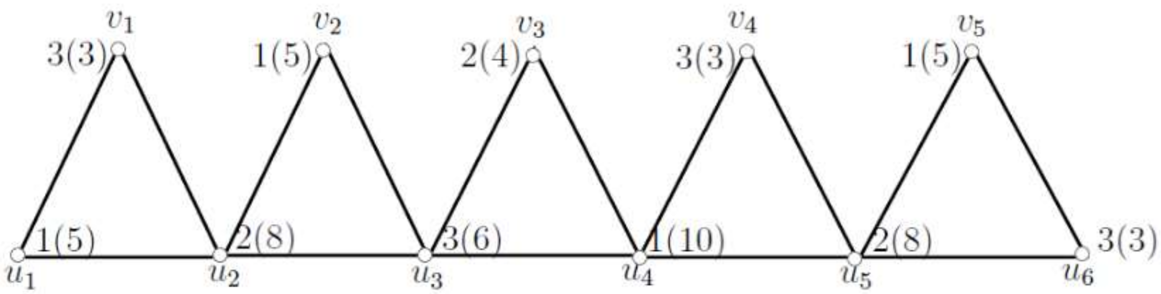

Fig 3 : Proper lucky triangular snake graph TS_n

Theorem 2.4. The Double Triangular snake DTSn with n ≥ 1 admits proper lucky labeling with proper lucky number ηp(TSn) = 3.

Proof

Given Double Triangular Snake Graph DTSn is consist of 3n−2 vertices where n ≥ 1, u_i have n vertices, v_i have n−1 vertices, w_i have n−1 vertices and the edge set

$E(DTSn) = E(TSn) \cup \{u_iw_i : 1 \leq i \leq n\} \cup \{u_{i+1}w_i : 1 \leq i \leq n\}$

Let f : V (DTS_n) → {1, 2, 3} for double triangular snake graph be defined by

$$f(u_i) = \begin{cases} 1 & \text{when } i = 1, 4, 7, \ldots \\ 2 & \text{when } i = 2, 5, 8, \ldots \\ 3 & \text{when } i = 3, 6, 9, \ldots \end{cases}$$

$$f(v_i) = \begin{cases} 3 & \text{when } i = 1, 4, 7, \ldots \\ 1 & \text{when } i = 2, 5, 8, \ldots \\ 2 & \text{when } i = 3, 6, 9, \ldots \end{cases}$$

$$f(w_i) = \begin{cases} 3 & \text{when } i = 1, 4, 7, \ldots \\ 1 & \text{when } i = 2, 5, 8, \ldots \\ 2 & \text{when } i = 3, 6, 9, \ldots \end{cases}$$

Here we obtain the sum of neighbour vertices as ,

$S(u_1) = 8$

$S(u_2) = S(u_5) = 12$

$S(u_3) = 9$

$S(u_4) = 15$

$S(u_6) = 4$

$$S(v_i) = \begin{cases} 3 & \text{when } i = 1, 4, 7, \ldots \\ 5 & \text{when } i = 2, 5, 8, \ldots \\ 4 & \text{when } i = 3, 6, 9, \ldots \end{cases}$$

$$S(w_i) = \begin{cases} 3 & \text{when } i = 1, 4, 7, \ldots \\ 5 & \text{when } i = 2, 5, 8, \ldots \\ 4 & \text{when } i = 3, 6, 9, \ldots \end{cases}$$

Therefore sum of adjacent vertices and adjacent labeling are not same. So,Double Triangular snake graph DTSn with n ≥ 1 admits proper lucky labeling with proper lucky number ηp(DTSn) = 3.

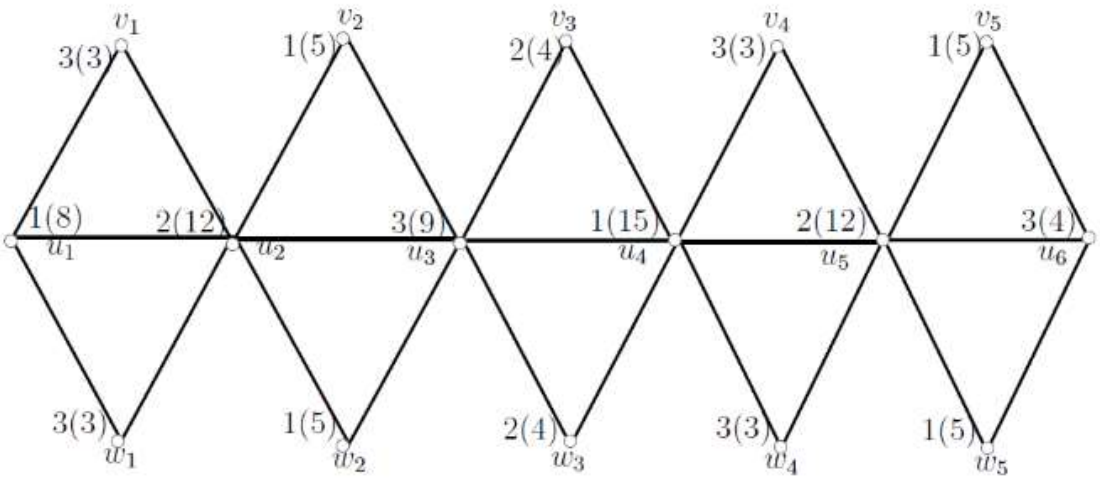

Fig 4 : Proper lucky triangular snake graph DTS_6

III. CONCLUSION

In this paper we compute the lucky number and proper lucky number of the triangular snake graph and double triangular snake graph.

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