Sum Labeled Annihilator Domination

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Subject classification: 05Cxx.

Abstract: Let $G(V,E)$ be a simple (p,q) graph. A function f^* is called a sum labeled Annihilator dominating function if $f : E(G) \to \{1,2,...,q\}$ such that the induced map f^* defined by $f^*(v_i) = \begin{cases} 0 & if \sum f(e_i) \equiv 0 (\text{mod } 2) \\ 1 & else \end{cases}$

where e_i is an edge incident with v_i. And the set $\{v_i / f^*(v_i) = 1\}$ is a minimal sum labeled annihilator dominating set.

Index Terms: Sum labeled annihilator domination, Triangular ladder, $Z - P_m$ graph, sum labeled annihilator edge domination.

I.INTRODUCTION

Domination and labeling are two different concepts in graph theory. We make an attempt to combine them both. In this paper we introduce a new type of graph domination called sum labeled annihilator domination . We label the vertices(edges) of the graph G by imposing some conditions on the label of the edges(vertices) thereby determining the minimal annihilator(edge) dominating set.

II. DEFINITIONS AND THEOREMS

Definition: A dominating set [3] D of a graph G is said to be an annihilator dominating set, if its induced subgraph $\langle V - D \rangle$ is a graph containing only isolated vertices. The annihilator domination number $\gamma_a(G)$ is the minimum cardinality of an annihilator dominating set.

Definition: An edge dominating set F of a graph G is said to be an annihilator edge dominating set, if its induced subgraph $\langle E - F \rangle$ is a graph containing only isolated edges. The annihilator edge domination number $\gamma_a'(G)$ is the minimum cardinality of an annihilator edge dominating set.

Definition: The graph obtained from the graph G by adding a pendant edge to each vertex of G is denoted by G^+.

Definition: $Z - P_m$ is obtained from the pair of path $P_m^{'}$ and $P_m^{''}$ by joining i^{th} vertex of $P_m^{'}$ with $(i+1)^{th}$ vertex of $P_m^{''}$.

Definition: The graph join $G_1 + G_2$ is obtained from G_1 and G_2 by joining every vertex of G_1 to all the vertices of G_2.

Definition: The triangular ladder (TL_n) is obtained from a ladder by including the edges $v_i u_{i+1}$ for $i = 1,2,...,n-1$ with $2n$ vertices and $4n-3$ edges.

Theorem: Path graph $P_m; m = 3, 4x \ and \ 4x+1, x \geq 1$ admits sum labeled annihilator domination.

Proof: Let G be a path graph P_m. For P_3, Define $f: E(G) \to \{1,2\}$ by $f(e_1) = 1, f(e_2) = 2$.
For P_4, Define $f: E(G) \to \{1,2,3\}$ by $f(e_1) = 1, f(e_2) = 3, f(e_3) = 2$.
For P_5, Define $f: E(G) \to \{1,2,3,4\}$ by $f(e_1) = 1, f(e_2) = 3, f(e_3) = 2$.
For both $m = 4x, 4x+1; x \geq 1$ Define $f: E(G) \to \{1,2,...,m\}$ by

$$f(e_1) = 2, f(e_2) = 3, f(e_3) = 1 \ for all \ P_m,$$

$$f(e_i) = \begin{cases} i \quad for \ i = 4k \ \ and \ 4k+3 \ , \ 1 \leq x \leq x-1 \\ i+1 \ \ for \ \ i = 4k+1 \ , 1 \leq k \leq x-1 \\ i-1 \ \ for \ i = 4k+2 \ , 1 \leq k \leq x-1. \end{cases}$$

For $m = 4x+1, f(e_i) = i \ \ for \ i = 4k; 1 \leq k \leq x, \ i = 4k+3; 1 \leq k \leq x-1.$

The induced vertex labeling are $f^*(v_i) = \begin{cases} 0 \ \ if \ \sum f(e_j) \equiv 0 (\text{mod } 2) \\ 1 \ \ else \end{cases}$

where e_j is an edge incident with v_i. **Case i)** When $m = 4x, D = \{v_2, v_4, ..., v_m\}$ is the minimal annihilator dominating set [2], since any annihilator dominating set with fewer elements than D is neither annihilating nor dominating. Also this set satisfies $\{v_i / f^*(v_i) = 1\}$. Hence D is a minimal sum labeled annihilator dominating set**.** **Case ii)** When $m = 4x+1, D = \{v_2, v_4, ..., v_{m-1}\}$ is the minimal sum labeled annihilator dominating set as in the above case.

Theorem: Cycle graph C_m admits sum labeled annihilator domination iff $m = 4x \ or \ 4x-1$.

Proof: Let G be a cycle graph C_m. Let $V(G) = \{v_i / 1 \leq i \leq m\}$ and $E(G) = \{v_i v_{i+1} : 1 \leq i \leq m-1\} \cup \{v_m v_1\}$. **Case i)** When $m = 4x-1$ Define $f: E(G) \to \{1,2,...,m\}$ by $f(e_{2i-1}) = i : 1 \leq i \leq \frac{m+1}{2}, \ f(e_{2i}) = \frac{m+1}{2} + i : 1 \leq i \leq \frac{m-1}{2}$. The induced vertex labeling are

$$f^*(v_i)=\begin{cases}0 & if \sum f(e_i)\equiv 0(\mathrm{mod}\ 2)\\ 1 & else\end{cases}$$

where e_i is an edge incident with v_i and the set $D=\{v_2,v_4,...,v_{m-1},v_m\}$ is a minimal annihilator dominating set and this set satisfies $\{v_i / f^*(v_i)=1\}$. Therefore D is a minimal sum labeled annihilator dominating set. **Case ii)** when $m=4x$, Define $f:E(G)\to\{1,2,...,m\}$ by $f(e_{2i-1})=i{:}1\le i\le\frac{m+1}{2}$, $f(e_{2i})=\frac{m+1}{2}+i:1\le i\le\frac{m-1}{2}$. The induced vertex labeling are

$$f^*(v_i)=\begin{cases}0 & if \sum f(e_j)\equiv 0(\mathrm{mod}\ 2)\\ 1 & else\end{cases}$$

where e_j is an edge incident with v_i and the set $D=\{v_2,v_4,...,v_{m-1},v_m\}$ is a minimal annihilator dominating set also D satisfies $\{v_i / f^*(v_i)=1\}$. Therefore D is a sum minimal annihilator dominating set. Case iii) Conversely assume cycle graph C_m admits sum labeled annihilator domination. To prove $m=4x\ or\ 4x-1$. Suppose $m\ne 4x,4x-1$. When we label the edges continually with odd numbers for the $\left\lceil\frac{m}{2}\right\rceil$ vertices and even numbers for the rest of the vertices then we get only two vertices with odd label which cannot form an annihilator dominating set. Here the only possible way to label in the order odd number, odd number, even number, even number, odd number, odd number and so on with two odd numbers and two even numbers alternately. Even then we cannot get a minimal annihilator dominating set.

Theorem: Comb graph P_{2m}^+ concedes sum labeled annihilator domination.

Proof: Let G be a comb graph P_{2m}^+. Let $\{v_1,v_2,...,v_m,u_1,u_2,...,u_m\}$ be the vertices of P_{2m}^+. Let $e_i=v_iv_{i+1}:1\le i\le m-1,\ f_i=v_iu_i:1\le i\le m.$ Define $f:E(G)\to\{1,2,...,m\}$ by $f(e_i)=i{:}1\le i\le m,\ f(f_i)=m-1+i:1\le i\le m.$ The induced vertex labeling are

$$f^*(v_i)=\begin{cases}0 & if \sum f(e_j)\equiv 0(\mathrm{mod}\ 2)\\ 1 & else\end{cases}$$

where e_j is an edge incident with v_i. The set that satisfies $\{v_i / f^*(v_i)=1\}$ is a minimal sum annihilator dominating set $D.$ Here $D=\{v_1,u_2,v_3,u_4,...,v_{m-1},u_m\}.$

Theorem: Crown graph C_{2m}^+ concedes sum labeled annihilator domination.

Proof: Let G be a Crown graph C_{2m}^+. Let $\{v_1,v_2,...,v_m,u_1,u_2,...,u_m\}$ be the vertices of P_{2m}^+. Let $e_i=v_iv_{i+1}:1\le i\le m-1, e_m=v_mv_1,\ f_i=v_iu_i:1\le i\le m.$ Define $f:E(G)\to\{1,2,...,m\}$ by $f(e_i)=i{:}1\le i\le m,\ f(f_i)=m+i:1\le i\le m.$ The induced vertex labeling are

$$f^*(v_i)=\begin{cases}0 & if \sum f(e_j)\equiv 0(\mathrm{mod}\ 2)\\ 1 & else\end{cases}.$$

where e_j is an edge incident with v_i. The set that satisfies $\{v_i / f^*(v_i)=1\}$ is a minimal sum labeled annihilator dominating set D. Here $D=\{v_1,u_2,v_3,u_4,...,v_{m-1},u_m\}$.

Result: Star graph does not admit sum labeled annihilator domination.

Theorem: Wheel graph $W_m, m=4x+1,4x+2$ admits sum labeled annihilator domination.

Proof: Let G be the wheel graph W_m. The vertex set of W_m is $\{v,v_1,v_2,...,v_m\}$ where v is the apex vertex. Let $\{e_i, f_i\}$ be the edges where

$e_i = v_iv_{i+1} : 1\le i\le m-1, e_m = v_mv_1, f_i = vv_i : 1\le i\le m.$ Define $f: E(G)\to\{1,2,...,m\}$ by

$f(e_i)=m+(i+1) : 1\le i\le m-1, f(e_m)=m+1, f(f_i)=i : 1\le i\le m.$ The induced vertex labeling

$$f^*(v_i)=\begin{cases}0 & if \sum f(e_j)\equiv 0(\text{mod } 2)\\ 1 & else\end{cases}$$

where e_j is an edge incident with v_i . The set that satisfies $\{v_i / f^*(v_i)=1\}$ is a minimal sum labeled annihilator dominating set D. Here $D=\{v,v_2,v_4,...,v_m\}$.

Theorem: $K_1+K_{1,2n}$ satisfies sum labeled annihilator domination.

Proof: Let $K_1+K_{1,2n}$ with $V(G)=\{x,y\}\cup\{w_j : 1\le i\le m\}$ and

$E(G)=\{e, f_j, g_j, h_j, j_j : j=1 \ to \ n\}$ where

$e=xy, f_j = xu_j, g_j = yu_j, h_j = xv_j, j_j = yv_j : 1\le j\le n.$ Define $f: E(G)\to\{1,2,...,m\}$ by

$f(e)=1, f(f_j)=4j-2, f(g_j)=4j, f(h_i)=4j-1, f(j_j)=4j+1; j=1,2,...,n.$ The induced

vertex labeling are $f^*(v_i)=\begin{cases}0 & if \sum f(e_j)\equiv 0(\text{mod } 2)\\ 1 & else\end{cases}$

where e_j is an edge incident with v_i. The set that satisfies $\{v_i / f^*(v_i)=1\}$ is a minimal sum labeled annihilator dominating set D. Here $D=\{x,y\}$.

III. SUM LABELED ANNIHILATOR EDGE DOMINATION

Let $G(V,E)$ be a simple (p,q) graph. A function f^* is called a sum labeled Annihilator edge dominating function if $f: V(G)\to\{1,2,..., p\}$ such that the induced map f^* defined by

$$f^*(e_i)=\begin{cases}0 & if \ f(v_j)+f(v_k)\equiv 0(\text{mod } 2)\\ 1 & else\end{cases}$$

where v_j, v_k are the end vertices of e_i and the set $\{e_i / f^*(e_i)=1\}$ is called a minimal sum labeled annihilator edge dominating set.

Theorem: Path graph $P_m; m=4x-1,4x \ and \ 4x+1, x\ge 1$ admits sum labeled annihilator edge domination.

Proof: Let G be a path graph P_mP. Let $V(G)=\{v_i\}_{i=1}^m$ and $E(G)=\{v_iv_{i+1}\}_{i=1}^{m-1}$.

Case i) when $m = 4x.$ Define $f : V(G) \to \{1,2,...,m\}$ by
$f(v_i) = \frac{i+1}{2}, i = 1,3,5,...,m$; $f(v_j) = \frac{(m+1)+j}{2}, j = 2,4,6,...,m-1.$ The induced edge labeling are

$$f^*(e_i) = \begin{cases} 0 & if \ f(v_i) + f(v_j) \equiv 0(\text{mod } 2) \\ 1 & else \end{cases}$$

where v_i, v_j are the end vertices of e_i. The set that satisfies $\{e_i / f^*(e_i) = 1\}$ is a minimal sum labeled annihilator edge dominating set F. Here $F = \{e_2, e_4, ..., e_{m-1}\}.$

Case ii) when $m = 4x.$ Define $f : V(G) \to \{1,2,...,m\}$ by
$f(v_i) = \frac{i+1}{2}, i = 1,3,5,...,m-1$; $f(v_j) = \frac{m+j}{2}, j = 2,4,6,...,m.$ The induced edge labeling are

$$f^*(e_i) = \begin{cases} 0 & if \ f(v_i) + f(v_j) \equiv 0(\text{mod } 2) \\ 1 & else \end{cases}$$

where v_i, v_j are the end vertices of e_i. The set that satisfies $\{e_i / f^*(e_i) = 1\}$ is a minimal sum labeled annihilator edge dominating set F. Here $F = \{e_2, e_4, ..., e_{m-2}\}.$

Case iii) when $m = 4x+1.$. Define $f : V(G) \to \{1,2,...,m\}$ by
$f(v_i) = \frac{i+1}{2}, i = 1,3,5,...,m$; $f(v_j) = \frac{2m-j+2}{2}, j = 2,4,6,...,m-1.$ The induced edge labeling are $f^*(e_i) = \begin{cases} 0 & if \ f(v_j) + f(v_k) \equiv 0(\text{mod } 2) \\ 1 & else \end{cases}$

where v_j, v_k are the end vertices of e_i. The set that satisfies $\{e_i / f^*(e_i) = 1\}$ is a minimal sum annihilator edge dominating set F. Here $F = \{e_2, e_4, ..., e_{m-1}\}$. Any annihilator edge dominating set with fewer elements than F is neither annihilating nor edge dominating.

Theorem: Comb graph P_m^+ admits sum labeled annihilator edge domination.

Proof: Let G be a comb graph P_m^+. Let $\{v_1, v_2, ..., v_m, u_1, u_2, ..., u_m\}$ be the vertices of P_{2m}^+. Let $e_i = v_i v_{i+1} : 1 \le i \le m-1, f_i = v_i u_i : 1 \le i \le m.$ Define $f : V(G) \to \{1,2,...,m\}$ by
$f(v_i) = i : 1 \le i \le m, \ f(u_j) = m + j : 1 \le j \le m.$. The induced edge labeling are

$$f^*(e_i) = \begin{cases} 0 & if \ f(v_j) + f(v_k) \equiv 0(\text{mod } 2) \\ 1 & else \end{cases}$$

where v_j, v_k are the end vertices of e_i. $F = \{e_2, e_4, ..., e_{m-1}\}$ is minimal annihilator edge dominating set. Also this set satisfies $\{e_i / f^*(e_i) = 1\}$. Therefore, F is the minimal sum labeled annihilator edge dominating set.

Theorem: Crown graph C_{2m}^+ concedes sum labeled annihilator edge domination.

Proof: Let G be a Crown graph C_{2m}^+. Let $\{v_1, v_2, ..., v_m, u_1, u_2, ..., u_m\}$ be the vertices of P_{2m}^+. Let $e_i = v_i v_{i+1} : 1 \le i \le m-1, e_m = v_m v_1, f_i = v_i u_i : 1 \le i \le m.$ Define $f : V(G) \to \{1,2,...,m\}$ by

$f(v_i)=i:1\le i\le m,\ f(u_i)=m+i:1\le i\le m+i.$ The induced edge labeling are

$$f^*(e_i)=\begin{cases}0 & if\ f(v_j)+f(v_k)\equiv 0(\text{mod } 2)\\ 1 & else\end{cases}$$

where v_j, v_k are the end vertices of e_i. $F=\{e_2,e_4,...,e_m\}$ is minimal annihilator edge dominating set. Also this set satisfies $\{e_i / f^*(e_i)=1\}$. Therefore, F is the minimal sum labeled annihilator edge dominating set.

Theorem: Cycle graph $C_m, m=4x,4x-1$ concedes sum labeled annihilator edge domination.

Proof: Let G be a cycle graph C_m. Let $V(G)=\{v_i / 1\le i\le m\}$ and $E(G)=\{v_iv_{i+1}:1\le i\le m-1\}\cup\{v_mv_1\}$. **Case i)** When $m=4x-1$ Define $f:V(G)\to\{1,2,...,m\}$ by

$$f(v_i)=\begin{cases}i & for\ i=4k-3\ and\ 4k-2,\ 1\le k\le x\\ i+1 & for\ i=4k-1, 1\le k\le x-1\\ i-1 & for\ i=4k, 1\le k\le x-1.\end{cases}$$

The induced edge labeling are $f^*(e_i)=\begin{cases}0 & if\ f(v_j)+f(v_k)\equiv 0(\text{mod } 2)\\ 1 & else\end{cases}$

where v_j, v_k are the end vertices of e_i. The set that satisfies $\{e_i / f^*(e_i)=1\}$ is a minimal sum labeled annihilator edge dominating set F. Here $F=\{e_2,e_4,...,e_{m-1},e_m\}$.

Case ii) When $m=4x-1$. Define $f:V(G)\to\{1,2,...,m\}$ by

$$f(v_i)=\begin{cases}\dfrac{i+1}{2} & for\ i=1,3,5,...,m-1\\ \dfrac{m+i}{2} & for\ i=2,4,6,...,m\end{cases}$$

The induced edge labeling are $f^*(e_i)=\begin{cases}0 & if\ f(v_j)+f(v_k)\equiv 0(\text{mod } 2)\\ 1 & else\end{cases}$

where v_j, v_k are the end vertices of e_i. The set that satisfies $\{e_i / f^*(e_i)=1\}$ is a minimal sum annihilator edge dominating set F. Here $F=\{e_2,e_4,...,e_m\}$.

Theorem: $Z-P_m$ [1] admits sum labeled annihilator edge domination.

Proof: Let G be a $Z-P_m$ graph. Let $V(G)=\{u_iv_i\}_{i=1}^m, E(G)=\{e_i, f_i, g_i\}$ where $e_i=v_iv_{i+1}, f_i=u_iu_{i+1}, g_i=u_i\,v_{i+1}:1\le i\le m-1.$ **Case i)** When m is odd. Define $f:V(G)\to\{1,2,...,m\}$ by $f(v_i)=i:1\le i\le m,\ f(u_i)=m+i:1\le i\le m$

The induced edge labeling are $f^*(e_i)=\begin{cases}0 & if\ f(v_j)+f(v_k)\equiv 0(\text{mod } 2)\\ 1 & else\end{cases}$

where v_j, v_k are the end vertices of e_i. **Case ii)** When m is even. Define $f:V(G)\to\{1,2,...,m\}$ by $f(v_i)=i:1\le i\le m,\ f(u_i)=2m-i+1:1\le i\le m.$

The induced edge labeling are $f^*(e_i)=\begin{cases}0 & if\ f(v_j)+f(v_k)\equiv 0(\text{mod } 2)\\ 1 & else\end{cases}$

where v_j, v_k are the end vertices of e_i. In both the cases, The set that satisfies $\{e_i / f^*(e_i)=1\}$ is a minimal sum labeled annihilator edge dominating set F. Here

$F=\{e_1, e_2, ..., e_{m-1}, f_1, f_2, ..., f_{m-1}\}$.

Theorem: Triangular Ladder TL_n concedes sum labeled annihilator edge domination.

Proof: Let TL_n be a triangular ladder[1] then we define vertex and edge set as

$V(G)=\{u_i v_i\}_{i=1}^{m}, E(G)=\{e_i, f_i, g_i, h_i\}$ where

$e_i=u_i v_i : 1\le i\le m, f_i=u_i u_{i+1}, g_i=v_i v_{i+1}, h_i=u_{i-1} v_i : 1\le i\le m-1.$

Case i) When m is odd. Define $f: V(G)\to\{1,2,...,m\}$ by

$f(v_i)=i : 1\le i\le m,\ f(u_i)=m+i : 1\le i\le m$

The induced edge labeling are $f^*(e_i)=\begin{cases}0 & if\ f(v_j)+f(v_k)\equiv 0(\text{mod } 2)\\ 1 & else\end{cases}$

where v_j, v_k are the end vertices of e_i.

Case ii) When m is even. Define $f: V(G)\to\{1,2,...,m\}$ by

$f(v_i)=i : 1\le i\le m,\ f(u_i)=2m-i+1 : 1\le i\le m$

The induced edge labeling are $f^*(e_i)=\begin{cases}0 & if\ f(v_j)+f(v_k)\equiv 0(\text{mod } 2)\\ 1 & else\end{cases}$

where v_j, v_k are the end vertices of e_i. Then for both cases the set that satisfies $\{e_i / f^*(e_i)=1\}$ is a minimal sum labeled annihilator edge dominating set F. Here

$F=\{e_1, e_2, ..., e_{m-1}, f_1, f_2, ..., f_{m-1}, g_1, g_2, ..., g_{m-1}\}$.

IV. CONCLUSION

In this article we have define sum labeled annihilator domination and sum labeled annihilator edge domination and shown that some graph concedes it.

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