CONTRIBUTIONS TO THE STUDY OF SOFT CLOSEDNESS IN SOFT TOPOLOGICAL SPACES

VANI.S.V*, BALA DEEPA ARASI.K, RAMADHAN A.MOHAMMED****

*&** PG & Research Department of Mathematics,

A.P.C.Mahalaxmi college for women, Thoothukudi, TN, India

Affiliated to Manonmaniam Sundaranar University, Tirunelveli, TN, India.

**Department of Mathematics,

College of Basic Educational University of Dohuk (UoD) www.UoD.ac Dohuk, Iraq

Abstract: The concept of soft topological space was introduced by some authors. In this present paper, we offer and study a novel type of generalized soft closed sets in soft topological space, named soft Pre-semi star Generalized-closed(in brief soft-$\mathbb{P}_{\star}\mathbb{G}$- -closed) sets which is properly placed in between the class of soft pg-closed and soft gp-closed set. Relationships with each other and other weaker forms of generalized soft closed sets with counterexamples are discussed and its properties are investigated. Also we introduce and explore several characterizations and properties of this type of soft closed sets.

***Index Terms*:** Soft closed, Soft $\mathbb{P}_{\star}\mathbb{G}$-closed sets, Soft $s^{*}g$-closed sets,Soft sets, Soft Topology, sets,

I. INTRODUCTION

The concept of soft sets was initiated by Molodtsov in 1999 as a completely new approach for modelling vagueness and uncertainty. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, etc. Later Maji et.al. presented some new defintions on soft sets such as a subset, the complement of a soft set. Research works on soft sets are progressing rapidly in recent years. Muhammad Shabir and Munazza Naz introduced the soft topological spaces which are defined over an initial universe with a fixed set of parameters. The notions of soft open sets, soft closed sets, soft closure, soft interior points, soft Neighborhood of a point and soft separation axioms are also introduced and their basic properties are investigated by them. In 2012, Kannan has introduced generalized closed sets in soft topological spaces. Kannan and Raja Lakshmi have introduced soft $s^{*}g$-closed sets in soft topological spaces in 2015. Arokia rani and Albinaa paved a new path way by introducing soft generalized pre closed sets in soft topological spaces. In this present study, we define a new class of closed set called soft $\mathbb{P}^{\star}{}_{\mathbb{G}}$-closed sets in soft topological spaces and obtain its relationships with other soft closed sets. Further, we obtain the basic results and properties.

II. IDENTIFY, RESEARCH AND COLLECT IDEA

Let $\mathbb{U}$ be an initial universe set and $\mathbb{S}$ be the set of all possible parameters with respect to $\mathbb{U}$. Parameters are often attributes, characteristics or properties of the objects in $\mathbb{U}$. Let $\mathbb{P}(\mathbb{U})$ denote the power set of $\mathbb{U}$. Then a soft set over $\mathbb{U}$ is defined as follows:

2.1 Definition A pair $(\mathbb{D}, \mathbb{V})$ is called a soft set over $\mathbb{U}$ where $\mathbb{V} \subseteq \mathbb{S}$ and $\mathbb{D} : \mathbb{V} \to \mathbb{P}(\mathbb{U})$ is a set valued mapping. In other words, a soft set over $\mathbb{U}$ is a parametrized family of subsets of the universe $\mathbb{U}$. For all $\varepsilon \in \mathbb{V}$, $\mathbb{D}(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set $(\mathbb{D}, \mathbb{V})$. It is worth nothing that $\mathbb{D}(\varepsilon)$ may be arbitrary. Some of them may be empty, and some may have nonempty intersection.

2.2 Definition: A soft set $(\mathbb{D}, \mathbb{V})$ over $\mathbb{U}$ is said to be null soft set denoted by $\{\}$ if for all $\mathbb{l} \in \mathbb{V}$, $\mathbb{D}(\mathbb{l}) = \{\}$. A soft set $(\mathbb{D}, \mathbb{V})$ over $\mathbb{U}$ is said to be an absolute soft set denoted by $\tilde{\mathrm{A}}$ if for all $\mathbb{l} \in \mathbb{V}$, $\mathbb{D}(\mathbb{l}) = \mathbb{U}$.

2.3 Definition: Let $\mathbb{Y}$ be a nonempty subset of $\mathbb{U}$, then $\widetilde{\mathbb{Y}}$ denotes the soft set $(\mathrm{Y}, \mathbb{S})$ over $\mathbb{U}$ for which

$\mathbb{Y}(\mathbb{I})$=$\mathbb{Y}$, for all $\mathbb{I} \in \mathbb{S}$. In particular, $(\mathbb{U}, \mathbb{S})$ will be denoted by $\widetilde{\mathbb{U}}$.

2.4 Definition: For two soft sets $(\mathbb{D}_{1}, \mathbb{V}_{1})$ and $(\mathbb{D}_{2}, \mathbb{V}_{2})$ over $\mathbb{U}$, we say that $(\mathbb{D}_{1}, \mathbb{V}_{1})$ is a soft subset of $(\mathbb{D}_{2}, \mathbb{V}_{2})$ if $\mathbb{V}_{1} \subseteq \mathbb{V}_{2}$ and for all $\mathbb{I} \in \mathbb{V}_{1}$, $\mathbb{D}_{1}(\mathbb{I})$ and $\mathbb{D}_{2}(\mathbb{I})$ are identical approximations. We write $(\mathbb{D}_{1}, \mathbb{V}_{1}) \subseteq (\mathbb{D}_{2}, \mathbb{V}_{2})$. $(\mathbb{D}_{1}, \mathbb{V}_{1})$ is said to be soft super set of $(\mathbb{D}_{2}, \mathbb{V}_{2})$, if $(\mathbb{D}_{2}, \mathbb{V}_{2})$ is a soft subset of $(\mathbb{D}_{1}, \mathbb{V}_{1})$. We denote it by $(\mathbb{D}_{2}, \mathbb{V}_{2}) \subseteq (\mathbb{D}_{1}, \mathbb{V}_{1})$. Then $(\mathbb{D}_{1}, \mathbb{V}_{1})$ and $(\mathbb{D}_{2}, \mathbb{V}_{2})$ are said to be soft equal if $(\mathbb{D}_{1}, \mathbb{V}_{1})$ is a soft subset of $(\mathbb{D}_{2}, \mathbb{V}_{2})$ and $(\mathbb{D}_{2}, \mathbb{V}_{2})$ is a soft subset of $(\mathbb{D}_{1}, \mathbb{V}_{1})$.

2.5 Definition: The union of two soft sets of $(\mathbb{D}_{1}, \mathbb{V}_{1})$ and $(\mathbb{D}_{2}, \mathbb{V}_{2})$ over $\mathbb{U}$ is the soft set $(\mathbb{D}_{3}, \mathbb{V}_{3})$, where $\mathbb{V}_{3}$= $\mathbb{V}_{1} \cup \mathbb{V}_{2}$ and for all $\mathbb{I} \in \mathbb{V}_{3}$,

$$\mathbb{D}_{3}(\mathbb{I})=\begin{cases} \mathbb{D}_1(\mathbb{I}) & if \quad \mathbb{I} \widetilde{\in} (\mathbb{V}_1 \backslash \mathbb{V}_2) \\ \mathbb{D}_2(\mathbb{I}) & if \quad \mathbb{I} \widetilde{\in} (\mathbb{V}_2 \backslash \mathbb{V}_1) \\ [\mathbb{D}_1(\mathbb{I}) \,\widetilde{\cup}\, \mathbb{D}_2(\mathbb{I}) & if \quad \mathbb{I} \widetilde{\in} \mathbb{V}_1 \,\widetilde{\cap}\, \mathbb{V}_2] \end{cases}$$

We write $(\mathbb{D}_{1}, \mathbb{V}_{1}) \cup (\mathbb{D}_{2}, \mathbb{V}_{2}) = (\mathbb{D}_{3}, \mathbb{V}_{3})$.

The intersection $(\mathbb{D}_{3}, \mathbb{V}_{3})$ of $(\mathbb{D}_{1}, \mathbb{V}_{1})$ and $(\mathbb{D}_{2}, \mathbb{V}_{2})$ over $\mathbb{U}$, denoted $(\mathbb{D}_{1}, \mathbb{V}_{1}) \cap (\mathbb{D}_{2}, \mathbb{V}_{2})$, is defined as $\mathbb{V}_{3}$= $\mathbb{V}_{1} \cap \mathbb{V}_{2}$, and $\mathbb{D}_{3}(\mathbb{I}) = \mathbb{D}_{1}(\mathbb{I}) \cap \mathbb{D}_{2}(\mathbb{I})$ for all $\mathbb{I} \in \mathbb{V}_{3}$.

2.6 Definition: The difference $(\mathbb{D}_{3}, \mathbb{S})$ of two soft sets $(\mathbb{D}_{1}, \mathbb{S})$ and $(\mathbb{D}_{2}, \mathbb{S})$ over $\mathbb{U}$, denoted by $(\mathbb{D}_{1}, \mathbb{S}) \setminus (\mathbb{D}_{2}, \mathbb{S})$, is defined as $\mathbb{D}_{3}(\mathbb{I}) = \mathbb{D}_{1}(\mathbb{I}) \setminus \mathbb{D}_{2}(\mathbb{I})$ for all $\mathbb{I} \in \mathbb{S}$.

2.7 Definition: Let $\widetilde{\mathbb{T}}$ be the collection of soft sets over $\mathbb{U}$, then $\widetilde{\mathbb{T}}$ is said to be a soft topology on $\mathbb{U}$ if

a. $\widetilde{\{\}}$, $\widetilde{\mathbb{U}}$ are belongs to $\widetilde{\mathbb{T}}$.
b. The union of any number of soft sets in $\widetilde{\mathbb{T}}$ belongs to $\widetilde{\mathbb{T}}$.
c. The intersection of any two soft sets in $\mathbb{T}$ belongs to $\widetilde{\mathbb{T}}$.

The triplet $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$ is called a soft topological space over $\widetilde{\mathbb{U}}$ and any member of is known as soft open set in $\widetilde{\mathbb{U}}$. The complement of a soft open set is called soft closed set over $\widetilde{\mathbb{U}}$.

III.SOFT $\mathbb{P}_{\star}\mathbb{G}$-CLOSED SETS

This section is devoted to the study of soft $\mathbb{P}_{\star}\mathbb{G}$-closed sets and their properties.

3.1Definition: A soft set $(\mathbb{V}, \mathbb{S})$ in a soft topological space $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$ is said to be soft pre semi star generalizeed closed (in short soft $\mathbb{P}_{\star}\mathbb{G}$-closed) set, if soft $pcl\big(pint(\mathbb{V}, \mathbb{S})\big) \widetilde{\subseteq} (\mathbb{W}, \mathbb{S})$ whenever $(\mathbb{V}, \mathbb{S}) \widetilde{\subseteq} (\mathbb{W}, \mathbb{S})$ and $(\mathbb{W}, \mathbb{S})$is soft $s^{*}g$-open.The collection of all soft $\mathbb{P}_{\star}\mathbb{G}$-closed sets is $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$ is denoted by $C_{s\mathbb{P}_{\star}\mathbb{G}}(X)$.

3.1 Theorem :

Axiom: 1 If $(\mathbb{V}, \mathbb{S})$ is soft closed then $(\mathbb{V}, \mathbb{S})$ is soft $\mathbb{P}_{\star}\mathbb{G}$-closed.
Axiom: 2 If $(\mathbb{V}, \mathbb{S})$ is soft pre- closed then $(\mathbb{V}, \mathbb{S})$ is soft $\mathbb{P}_{\star}\mathbb{G}$-closed.
Axiom: 3 If $(\mathbb{V}, \mathbb{S})$ is soft sg- closed then $(\mathbb{V}, \mathbb{S})$ is soft $\mathbb{P}_{\star}\mathbb{G}$-closed.
Axiom: 4 If $(\mathbb{V}, \mathbb{S})$ is soft pg- closed then $(\mathbb{V}, \mathbb{S})$ is soft $\mathbb{P}_{\star}\mathbb{G}$-closed.
Axiom: 5 If $(\mathbb{V}, \mathbb{S})$ is soft $s^{*}g$- closed then $(\mathbb{V}, \mathbb{S})$ is soft $\mathbb{P}_{\star}\mathbb{G}$-closed.
Axiom: 6 If $(\mathbb{V}, \mathbb{S})$ is soft $g\alpha b$-closed then $(\mathbb{V}, \mathbb{S})$ is soft $\mathbb{P}_{\star}\mathbb{G}$-closed

3.1 Observations: The converses of the above theorems are not true in general. The following examples support our claim.

3.1 Illustration:

Axiom:1 Let us consider $\widetilde{\mathbb{U}} = \{Red(R), Green(G)\}$, $\mathbb{S} = \{b_1, b_2\}$ and $\widetilde{\mathbb{T}} = \{\widetilde{\{\}}, \widetilde{\mathbb{U}}, (\mathbb{D}_1, \mathbb{S}), (\mathbb{D}_2, \mathbb{S})\}$ be a soft topology defined on $\widetilde{\mathbb{U}}$, where $(\mathbb{D}_1, \mathbb{S}), (\mathbb{D}_2, \mathbb{S})$ are soft sets over $\widetilde{\mathbb{U}}$ defined as follows: $\mathbb{D}_1(b_1) = \{R\}$, $\mathbb{D}_1(b_2) = \widetilde{\{\}}$, $\mathbb{D}_2(b_1) = \{R, G\}$, $\mathbb{D}_2(b_2) = \widetilde{\{\}}$, then $(\mathbb{D}_3, \mathbb{S})$ is soft $\mathbb{P}_{\star}\mathbb{G}$-closed but not soft closed set in $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$,
Where $\mathbb{D}_3(b_1) = \widetilde{\{\}}$, $\mathbb{D}_3(b_2) = \{G\}$.
Axiom:2 Let us consider $\widetilde{\mathbb{U}} = \{Red(R), Green(G)\}$, $\mathbb{S} = \{b_1, b_2\}$ and $\widetilde{\mathbb{T}} = \{\widetilde{\{\}}, \widetilde{\mathbb{U}}, (\mathbb{D}_1, \mathbb{S}), (\mathbb{D}_2, \mathbb{S})\}$ be a soft topology defined on $\widetilde{\mathbb{U}}$, where $(\mathbb{D}_1, \mathbb{S}), (\mathbb{D}_2, \mathbb{S})$ are soft sets over $\widetilde{\mathbb{U}}$ defined as follows: $\mathbb{D}_1(b_1) = \{R\}$, $\mathbb{D}_1(b_2) = \widetilde{\{\}}$, $\mathbb{D}_2(b_1) = \{R, G\}$, $\mathbb{D}_2(b_2) = \widetilde{\{\}}$, then $(\mathbb{D}_3, \mathbb{S})$ is soft $\mathbb{P}_{\star}\mathbb{G}$-closed but not soft pre-closed set in $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$,
where $\mathbb{D}_3(b_1) = \{R\}$, $\mathbb{D}_4(b_2) = \{G\}$.
Axiom:3 Let us consider $\widetilde{\mathbb{U}} = \{Red(R), Green(G), Blue(B)\}$, $\mathbb{S} = \{b_1, b_2\}$ and $\widetilde{\mathbb{T}} = \{\widetilde{\{\}}, \widetilde{\mathbb{U}}, (\mathbb{D}_1, \mathbb{S}), (\mathbb{D}_2, \mathbb{S}), (\mathbb{D}_3, \mathbb{S}), (\mathbb{D}_4, \mathbb{S})\}$ be a soft topology defined on $\widetilde{\mathbb{U}}$, where $(\mathbb{D}_1, \mathbb{S}), (\mathbb{D}_2, \mathbb{S}), (\mathbb{D}_3, \mathbb{S}), (\mathbb{D}_4, \mathbb{S})$ are soft sets over $\widetilde{\mathbb{U}}$ defined as follows: $\mathbb{D}_1(b_1) = \{G\}$, $\mathbb{D}_1(b_2) = \{R\}$, $\mathbb{D}_2(b_1) = \{B\}$, $\mathbb{D}_2(b_2) = \{R, G\}$, $\mathbb{D}_3(b_1) = \{G, B\}$, $\mathbb{D}_3(b_2) = \{G, B\}$, $\mathbb{D}_4(b_1) = \{\widetilde{\mathbb{U}}\}$, $\mathbb{D}_4(b_2) = \{R, G\}$ then $(\mathbb{D}_5, \mathbb{S})$ is soft $\mathbb{P}_{\star}\mathbb{G}$-closed but not soft sg-closed set in $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$, where $\mathbb{D}_5(b_1) = \widetilde{\{\}}$, $\mathbb{D}_5(b_2) = \{R, G\}$.
Axiom:4 From the above example(1.C), the soft set $(\mathbb{D}_6, \mathbb{S})$ is soft $\mathbb{P}_{\star}\mathbb{G}$-closed but not soft pg-closed set in $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$, where $\mathbb{D}_6(b_1) = \{B\}$, $\mathbb{D}_6(b_2) = \{G\}$.

Axiom:5 Let us consider $\widetilde{\mathbb{U}} = \{Red(R), Green(G), Blue(B)\}$, $\mathbb{S} = \{b_1, b_2\}$ and $\widetilde{\mathbb{T}} = \{\widetilde{\{\}}, \widetilde{\mathbb{U}}, (\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S}), (\mathbb{D}_4,\mathbb{S}), (\mathbb{D}_5,\mathbb{S}), (\mathbb{D}_6,\mathbb{S}), (\mathbb{D}_7,\mathbb{S})\}$ be a soft topology defined on $\widetilde{\mathbb{U}}$, where $(\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S})$, $(\mathbb{D}_3,\mathbb{S}), (\mathbb{D}_4,\mathbb{S}), (\mathbb{D}_5,\mathbb{S}), (\mathbb{D}_6,\mathbb{S}), (\mathbb{D}_7,\mathbb{S})$ are soft sets over $\widetilde{\mathbb{U}}$ defined as follows: $\mathbb{D}_1(b_1) = \{\widetilde{\{\}}\}$, $\mathbb{D}_1(b_2) = \{R\}$; $\mathbb{D}_2(b_1) = \{\widetilde{\{\}}\}$, $\mathbb{D}_2(b_2) = \{G, B\}$; $\mathbb{D}_3(b_1) = \{\widetilde{\{\}}\}$, $\mathbb{D}_3(b_2) = \{\widetilde{\mathbb{U}}\}$; $\mathbb{D}_4(b_1) = \{G\}$, $\mathbb{D}_4(b_2) = \{R\}$, $\mathbb{D}_5(b_1) = \{G\}$, $\mathbb{D}_5(b_2) = \{\widetilde{\mathbb{U}}\}$; $\mathbb{D}_6(b_1) = \{R, B\}$, $\mathbb{D}_6(b_2) = \{G, B\}$;$\mathbb{D}_7(b_1) = \{R, B\}$, $\mathbb{D}_7(b_2) = \{\widetilde{\mathbb{U}}\}$ then $(\mathbb{D}_8,\mathbb{S})$ is soft $\mathbb{P}_\star\mathbb{G}$-closed but not soft s^*g-closed set in $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$, where $\mathbb{D}_8(b_1) = \{R, G\}$, $\mathbb{D}_8(b_2) = \{G\}$.

Axiom: 6 Let us consider $\widetilde{\mathbb{U}} = \{Red(R), Green(G), Blue(B)\}$, $\mathbb{S} = \{b_1, b_2\}$ and $\widetilde{\mathbb{T}} = \{\widetilde{\{\}}, \widetilde{\mathbb{U}}, (\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S}), (\mathbb{D}_4,\mathbb{S}), (\mathbb{D}_5,\mathbb{S}), (\mathbb{D}_6,\mathbb{S}), (\mathbb{D}_7,\mathbb{S})\}$ be a soft topology defined on $\widetilde{\mathbb{U}}$, where $(\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S})$, $(\mathbb{D}_3,\mathbb{S}), (\mathbb{D}_4,\mathbb{S}), (\mathbb{D}_5,\mathbb{S}), (\mathbb{D}_6,\mathbb{S}), (\mathbb{D}_7,\mathbb{S})$ are soft sets over $\widetilde{\mathbb{U}}$ defined as follows: $\mathbb{D}_1(b_1) = \{R\}$, $\mathbb{D}_1(b_2) = \{R, B\}$; $\mathbb{D}_2(b_1) = \{R, G\}$, $\mathbb{D}_2(b_2) = \{R\}$; $\mathbb{D}_3(b_1) = \{R, G\}$,$\mathbb{D}_3(b_2) = \{R, B\}$; $\mathbb{D}_4(b_1) = \{R, B\}$, $\mathbb{D}_4(b_2) = \{B\}$; $\mathbb{D}_5(b_1) = \{R, B\}$, $\mathbb{D}_5(b_2) = \{R, B\}$; $\mathbb{D}_6(b_1) = \{\widetilde{\mathbb{U}}\}$, $\mathbb{D}_6(b_2) = \{G\}$; $\mathbb{D}_7(b_1) = \{\widetilde{\mathbb{U}}\}$, $\mathbb{D}_7(b_2) = \{G, B\}$ then $(\mathbb{D}_8,\mathbb{S})$ is soft $\mathbb{P}_\star\mathbb{G}$-closed but not soft $g\alpha b$-closed set in $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$, where $\mathbb{D}_8(b_1) = \{R\}$, $\mathbb{D}_8(b_2) = \{G\}$.

3.2 Theorem :

Axiom:1 If $(\mathbb{V},\mathbb{S})$ is soft $\mathbb{P}_\star\mathbb{G}$-closed then $(\mathbb{V},\mathbb{S})$ is soft gp-closed.

Axiom:2 If $(\mathbb{V},\mathbb{S})$ is soft $\mathbb{P}_\star\mathbb{G}$-closed then $(\mathbb{V},\mathbb{S})$ is soft gpr-closed.

Axiom:3 If $(\mathbb{V},\mathbb{S})$ is soft $\mathbb{P}_\star\mathbb{G}$-closed then $(\mathbb{V},\mathbb{S})$ is soft $g\beta$-closed.

Axiom:4 If $(\mathbb{V},\mathbb{S})$ is soft $\mathbb{P}_\star\mathbb{G}$-closed then $(\mathbb{V},\mathbb{S})$ is soft rwg-closed

3.2 Observations: The reverse implication of the above theorem is not true in general. The following example supports our claim.

3.2 Illustration

Axiom:1 Let us consider $\widetilde{\mathbb{U}} = \{Red(R), Green(G), Blue(B)\}$, $\mathbb{S} = \{b_1, b_2\}$ and $\widetilde{\mathbb{T}} = \{\widetilde{\{\}}, \widetilde{\mathbb{U}}, (\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S}), (\mathbb{D}_4,\mathbb{S})\}$ be a soft topology defined on $\widetilde{\mathbb{U}}$, where $(\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S}), (\mathbb{D}_4,\mathbb{S})$ are soft sets over $\widetilde{\mathbb{U}}$ defined as follows: $\mathbb{D}_1(b_1) = \{G\}$, $\mathbb{D}_1(b_2) = \{B\}$, $\mathbb{D}_2(b_1) = \{G\}$, $\mathbb{D}_2(b_2) = \{G, B\}$, $\mathbb{D}_3(b_1) = \{R, G\}$, $\mathbb{D}_3(b_2) = \{G, B\}$, $\mathbb{D}_4(b_1) = \{R, G\}$, $\mathbb{D}_4(b_2) = \{\widetilde{\mathbb{U}}\}$ then $(\mathbb{D}_5,\mathbb{S})$ is soft gp-closed but not soft $\mathbb{P}_\star\mathbb{G}$-closed set in $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$, where $\mathbb{D}_5(b_1) = \{G, B\}$, $\mathbb{D}_5(b_2) = \{B\}$.

Axiom: 2 Let us consider $\widetilde{\mathbb{U}} = \{Red(R), Green(G)\}$, $\mathbb{S} = \{b_1, b_2\}$ and $\widetilde{\mathbb{T}} = \{\widetilde{\{\}}, \widetilde{\mathbb{U}}, (\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S})\}$ be a soft topology defined on $\widetilde{\mathbb{U}}$, where $(\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S})$ are soft sets over $\widetilde{\mathbb{U}}$ defined as follows: $\mathbb{D}_1(b_1) = \{G\}$, $\mathbb{D}_1(b_2) =\{\widetilde{\{\}}\}$, $\mathbb{D}_2(b_1) = \{R\}$, $\mathbb{D}_2(b_2) = \{G\}$, $\mathbb{D}_3(b_1) = \{R, G\}$, $\mathbb{D}_3(b_2) = \{G\}$ then $(\mathbb{D}_4,\mathbb{S})$ is soft gpr-closed but not soft $\mathbb{P}_\star\mathbb{G}$ - closed set in $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$, where $\mathbb{D}_4(b_1) = \{R, G\}$, $\mathbb{D}_4(b_2) = \{\widetilde{\{\}}\}$

Axiom: 3 In the above example(II.B), the soft set $(\mathbb{D}_5,\mathbb{S})$ is soft $g\beta$-closed but not soft $\mathbb{P}_\star\mathbb{G}$ - closed set in $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$, where $\mathbb{D}_5(b_1) = \{G\}$, $\mathbb{D}_5(b_2) = \{\widetilde{\{\}}\}$

Axiom: 4 Let us consider $\widetilde{\mathbb{U}} = \{Yellow(Y), Red(R), Green(G), Blue(B)\}$, $\mathbb{S} = \{b_1\}$ and $\widetilde{\mathbb{T}} = \{\widetilde{\{\}}, \widetilde{\mathbb{U}}, (\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S})\}$ be a soft topology defined on $\widetilde{\mathbb{U}}$, where $(\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S})$ are soft sets over $\widetilde{\mathbb{U}}$ defined as follows: $\mathbb{D}_1(b_1) = \{R\}$, $\mathbb{D}_2(b_1) = \{Y, G\}$, $\mathbb{D}_3(b_1) = \{Y, R, G\}$,then $(\mathbb{D}_4,\mathbb{S})$ is soft rwg-closed but not soft $\mathbb{P}_\star\mathbb{G}$-closed set in $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$, where $\mathbb{D}_4(b_1) = \{R, G\}$.

4. INDEPENDENCY OF SOFT $\mathbb{P}_\star\mathbb{G}$-CLOSED SET WITH OTHER SOFT CLOSED SETS

The following examples shows that SOFT $\mathbb{P}_\star\mathbb{G}$-CLOSED SET is independent of soft g-closed set (4.A), soft β-closed(4.B), soft gs-closed set(4.C), soft Q-set(4.D)

A. Let us consider $\widetilde{\mathbb{U}} = \{Red(R), Green(G), Blue(B)\}$, $\mathbb{S} = \{b_1, b_2\}$ and $\widetilde{\mathbb{T}} = \{\widetilde{\{\}}, \widetilde{\mathbb{U}}, (\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S}), (\mathbb{D}_4,\mathbb{S}), (\mathbb{D}_5,\mathbb{S}), (\mathbb{D}_6,\mathbb{S}), (\mathbb{D}_7,\mathbb{S})\}$ be a soft topology defined on $\widetilde{\mathbb{U}}$, where $(\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S})$, $(\mathbb{D}_3,\mathbb{S}), (\mathbb{D}_4,\mathbb{S}), (\mathbb{D}_5,\mathbb{S}), (\mathbb{D}_6,\mathbb{S}), (\mathbb{D}_7,\mathbb{S})$ are soft sets over $\widetilde{\mathbb{U}}$ defined as follows: $\mathbb{D}_1(b_1) = \{G\}$, $\mathbb{D}_1(b_2) = \{R\}$, $\mathbb{D}_2(b_1) = \{G\}$, $\mathbb{D}_2(b_2) = \{R, B\}$, $\mathbb{D}_3(b_1) = \{R, G\}$, $\mathbb{D}_3(b_2) = \{R, G\}$, $\mathbb{D}_4(b_1) = \{R, G\}$, $\mathbb{D}_4(b_2) = \{\widetilde{\mathbb{U}}\}$, $\mathbb{D}_5(b_1) = \{G, B\}$, $\mathbb{D}_5(b_2) = \{R\}$, $\mathbb{D}_6(b_1) = \{G, B\}$, $\mathbb{D}_6(b_2) = \{R, B\}$, $\mathbb{D}_7(b_1) = \{\widetilde{\mathbb{U}}\}$, $\mathbb{D}_7(b_2) = \{R, G\}$ then $(\mathbb{D}_8,\mathbb{S})$ is soft g-closed but not soft $\mathbb{P}_\star\mathbb{G}$-closed set in $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$, where $\mathbb{D}_8(b_1) = \{R, G\}$, $\mathbb{D}_8(b_2) = \{\widetilde{\mathbb{U}}\}$. since the only soft open set containing $(\mathbb{D}_8,\mathbb{S})$ is $\widetilde{\mathbb{U}}$, and $\left(pcl(pint(\mathbb{D}_8,\mathbb{S}))\right) = cl(\mathbb{D}_8,\mathbb{S}) = \widetilde{\mathbb{U}}$.

➢ Let us consider $\widetilde{\mathbb{U}} = \{Red(R), Green(G), Blue(B)\}$, $\mathbb{S} = \{b_1, b_2\}$ and $\widetilde{\mathbb{T}} = \{\widetilde{\{\}}, \widetilde{\mathbb{U}}, (\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S}), (\mathbb{D}_4,\mathbb{S}), (\mathbb{D}_5,\mathbb{S}), (\mathbb{D}_6,\mathbb{S}), (\mathbb{D}_7,\mathbb{S})\}$ be a soft topology defined on $\widetilde{\mathbb{U}}$, where $(\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S})$, $(\mathbb{D}_3,\mathbb{S}), (\mathbb{D}_4,\mathbb{S}), (\mathbb{D}_5,\mathbb{S}), (\mathbb{D}_6,\mathbb{S}), (\mathbb{D}_7,\mathbb{S})$ are soft sets over $\widetilde{\mathbb{U}}$ defined as follows: $\mathbb{D}_1(b_1) = \{B\}$, $\mathbb{D}_1(b_2) = \widetilde{\{\}}$, $\mathbb{D}_2(b_1) = \widetilde{\{\}}$, $\mathbb{D}_2(b_2) = \{R\}$, $\mathbb{D}_3(b_1) = \{G\}$, $\mathbb{D}_3(b_2) = \{G\}$, $\mathbb{D}_4(b_1) = \{G\}$, $\mathbb{D}_4(b_2) = \{R,G\}$, $\mathbb{D}_5(b_1) = \{B\}$, $\mathbb{D}_5(b_2) = \{R\}$, $\mathbb{D}_6(b_1) = \{G,B\}$, $\mathbb{D}_6(b_2) = \{G\}$, $\mathbb{D}_7(b_1) = \{G,B\}$, $\mathbb{D}_7(b_2) = \{R,G\}$. Here $(\mathbb{D}_8,\mathbb{S})$ is soft g-closed but not Soft $\mathbb{P}_\star\mathbb{G}$-closed set in $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$, where $\mathbb{D}_8(b_1) = \{B\}$, $\mathbb{D}_8(b_2) = \widetilde{\{\}}$.

Since $(\mathbb{D}_8,\mathbb{S})$ is soft open also soft s^*g-open and $\left(pcl(pint(\mathbb{D}_8,\mathbb{S}))\right)$=$(\mathbb{D}_8,\mathbb{S})$, $cl(\mathbb{D}_8,\mathbb{S}) = \{(b_1,\{G,B\}), (b_{12},\{B\})\} \widetilde{\nsubseteq} (\mathbb{D}_8,\mathbb{S})$

Hence soft $\mathbb{P}_\star\mathbb{G}$-closedness is independent of soft g-closedness.

B. Let us consider $\widetilde{\mathbb{U}} = \{Red(R), Green(G)\}$, $\mathbb{S} = \{b_1, b_2\}$ and $\widetilde{\mathbb{T}} = \{\widetilde{\{\}}, \widetilde{\mathbb{U}}, (\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S}), (\mathbb{D}_4,\mathbb{S}),, (\mathbb{D}_5,\mathbb{S})\}$ be a soft topology defined on $\widetilde{\mathbb{U}}$, where $(\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S}), (\mathbb{D}_4,\mathbb{S}),, (\mathbb{D}_5,\mathbb{S})$ are soft sets over $\widetilde{\mathbb{U}}$ defined as follows: $\mathbb{D}_1(b_1) = \{R\}$, $\mathbb{D}_1(b_2) =\widetilde{\{\}}$, $\mathbb{D}_2(b_1) = \widetilde{\{\}}$, $\mathbb{D}_2(b_2) = \{G\}$, $\mathbb{D}_3(b_1) = \widetilde{\{\}}$, $\mathbb{D}_3(b_2) = \{R,G\}$, $\mathbb{D}_4(b_1) = \{R\}$, $\mathbb{D}_4(b_2) = \{R\}$,$\mathbb{D}_5(b_1) = \{R\}$, $\mathbb{D}_5(b_2) =\{\widetilde{\mathbb{U}}\}$, here $(\mathbb{D}_6,\mathbb{S})$ is soft $\mathbb{P}_\star\mathbb{G}$ - closed but not soft β-closed set in $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$, where $\mathbb{D}_6(b_1) = \{G\}$, $\mathbb{D}_6(b_2) = \{R\}$. Since $\beta cl(\mathbb{D}_6,\mathbb{S}) = \{(b_1,\{R,G\}), (b_2,\{R\})\} \widetilde{\nsubseteq} (\mathbb{D}_7,\mathbb{S})$. Where $(\mathbb{D}_7,\mathbb{S})$= $\{(b_1,\{G\}), (b_2,\{\widetilde{\mathbb{U}}\})\}$

➢ Let us consider $\widetilde{\mathbb{U}} = \{Red(R), Green(G)\}$, $\mathbb{S} = \{b_1, b_2\}$ and $\widetilde{\mathbb{T}} = \{\widetilde{\{\}}, \widetilde{\mathbb{U}}, (\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S})\}$ be a soft topology defined on $\widetilde{\mathbb{U}}$, where $(\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S})$ are soft sets over $\widetilde{\mathbb{U}}$ defined as follows: $\mathbb{D}_1(b_1) = \{R\}$, $\mathbb{D}_1(b_2) =\widetilde{\{\}}$, $\mathbb{D}_2(b_1) = \{G\}$, $\mathbb{D}_2(b_2) = \widetilde{\{\}}$, $\mathbb{D}_3(b_1) = \{R,G\}$, $\mathbb{D}_3(b_2) = \widetilde{\{\}}$, here $(\mathbb{D}_4,\mathbb{S})$ is soft β - closed but not soft $\mathbb{P}_\star\mathbb{G}$ -closed set in $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$, where $\mathbb{D}_4(b_1) = \{R\}$, $\mathbb{D}_4(b_2) = \widetilde{\{\}}$. Since $\left(pcl\left(pint((\mathbb{D}_4,\mathbb{S}))\right)\right) \widetilde{\subseteq} \{(b_1,\{R\}), (b_2,\{R,G\})\} \widetilde{\nsubseteq} (\mathbb{D}_4,\mathbb{S})$.

Hence soft $\mathbb{P}_\star\mathbb{G}$ –closedness is independent of soft β – closedness.

C. Let us consider $\widetilde{\mathbb{U}} = \{Red(R), Green(G), Blue(B)\}$, $\mathbb{S} = \{b_1, b_2\}$ and $\widetilde{\mathbb{T}} = \{\widetilde{\{\}}, \widetilde{\mathbb{U}}, (\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S}), (\mathbb{D}_4,\mathbb{S})\}$ be a soft topology defined on $\widetilde{\mathbb{U}}$, where $(\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S}), (\mathbb{D}_4,\mathbb{S})$ are soft sets over $\widetilde{\mathbb{U}}$ defined as follows: $\mathbb{D}_1(b_1) = \{G\}$, $\mathbb{D}_1(b_2) = \{R\}$, $\mathbb{D}_2(b_1) = \{B\}$, $\mathbb{D}_2(b_2) = \{R,G\}$, $\mathbb{D}_3(b_1) = \{G,B\}$, $\mathbb{D}_3(b_2) = \{G,B\}$, $\mathbb{D}_4(b_1) = \{\widetilde{\mathbb{U}}\}$, $\mathbb{D}_4(b_2) = \{R,G\}$. Here $(\mathbb{D}_5,\mathbb{S})$ is soft $\mathbb{P}_\star\mathbb{G}$-closed but not soft gs-closed set in $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$, where $\mathbb{D}_5(b_1) = \widetilde{\{\}}$, $\mathbb{D}_5(b_2) = \{R,G\}$. Since $scl(\mathbb{D}_5,\mathbb{S}) = \{(b_1,\{\widetilde{\mathbb{U}}\}), (b_2,\{\widetilde{\mathbb{U}}\})\} \widetilde{\nsubseteq} (\mathbb{D}_1,\mathbb{S})$.

Then it is soft $\mathbb{P}_\star\mathbb{G}$ closed but not soft gs-closed in $(\widetilde{\mathbb{U}},\widetilde{\mathbb{T}}, \mathbb{V})$. Hence soft $\mathbb{P}_\star\mathbb{G}$-closedness is independent of soft gs-closedness.

D. Let us consider $\widetilde{\mathbb{U}} = \{Red(R), Green(G), Blue(B), Yellow(Y)\}$, $\mathbb{S} = \{b_1, b_2\}$ and $\widetilde{\mathbb{T}} = \{\widetilde{\{\}}, \widetilde{\mathbb{U}}, (\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S})\}$ be a soft topology defined on $\widetilde{\mathbb{U}}$, where $(\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S})$are soft sets over $\widetilde{\mathbb{U}}$ defined as follows: $\mathbb{D}_1(b_1) = \{R\}$, $\mathbb{D}_1(b_2) = \{G\}$, $\mathbb{D}_2(b_1) = \{R,G\}$, $\mathbb{D}_2(b_2) = \{R,G,B\}$, $\mathbb{D}_3(b_1) = \{R,G,Y\}$, $\mathbb{D}_3(b_2) = \{\widetilde{\mathbb{U}}\}$, $\mathbb{D}_4(b_1) = \{\widetilde{\mathbb{U}}\}$. Here $(\mathbb{D}_5,\mathbb{S})$ is soft $\mathbb{P}_\star\mathbb{G}$-closed but not soft Q- set in $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$, where $\mathbb{D}_5(b_1) = \{R\}$, $\mathbb{D}_5(b_2) = \{Y\}$. Since $soft\left(cl(int(\mathbb{D}_5,\mathbb{S}))\right) \neq soft\left(int(cl(\mathbb{D}_5,\mathbb{S}))\right)$.

➢ Let us consider $\widetilde{\mathbb{U}} = \{Red(R), Green(G)\}$, $\mathbb{S} = \{b_1, b_2\}$ and $\widetilde{\mathbb{T}} = \{\widetilde{\{\}}, \widetilde{\mathbb{U}}, (\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S}), (\mathbb{D}_4,\mathbb{S}), (\mathbb{D}_5,\mathbb{S})\}$ be a soft topology defined on $\widetilde{\mathbb{U}}$, where $(\mathbb{D}_1,\mathbb{S}), (\mathbb{D}_2,\mathbb{S}), (\mathbb{D}_3,\mathbb{S}), (\mathbb{D}_4,\mathbb{S}), (\mathbb{D}_5,\mathbb{S})$ are soft sets over $\widetilde{\mathbb{U}}$ defined as follows: $\mathbb{D}_1(b_1) = \{R\}$, $\mathbb{D}_1(b_2) =\widetilde{\{\}}$, $\mathbb{D}_2(b_1) = \widetilde{\{\}}$, $\mathbb{D}_2(b_2) = \{G\}$, $\mathbb{D}_3(b_1) = \widetilde{\{\}}$, $\mathbb{D}_3(b_2) = \{R,G\}$, $\mathbb{D}_4(b_1) = \{R\}$, $\mathbb{D}_4(b_2) = \{R\}$, $\mathbb{D}_5(b_1) = \{R\}$, $\mathbb{D}_5(b_2) = \{R,G\}$. Here $(\mathbb{D}_3,\mathbb{S})$ is soft Q-set but not soft $\mathbb{P}_\star\mathbb{G}$ -closed set in $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$, Since $\left(pcl\left(pint((\mathbb{D}_3,\mathbb{S}))\right)\right) = \widetilde{\mathbb{U}} \widetilde{\nsubseteq} (\mathbb{D}_3,\mathbb{S})$.

Hence soft $\mathbb{P}_\star\mathbb{G}$-closedness is independent of soft Q-set closedness.

5. CHARACTERIZATION OF SOFT $\mathbb{P}_\star\mathbb{G}$--CLOSED SET

5.1 Theorem: The Union of two soft $\mathbb{P}_\star\mathbb{G}$ −closed Subsets of $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$ is soft $\mathbb{P}_\star\mathbb{G}$ −closed Subsets of $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$

5.1 Corollary: The intersection of two soft $\mathbb{P}_\star\mathbb{G}$ −closed Subsets of $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$ is soft $\mathbb{P}_\star\mathbb{G}$ −closed Subsets of $(\widetilde{\mathbb{U}}, \widetilde{\mathbb{T}}, \mathbb{V})$

5.2 Theorem: A soft set $(\mathbb{V},\mathbb{S})$ is a soft $\mathbb{P}_\star\mathbb{G}$ −closed if and only if soft $\left(pcl(pint(\mathbb{V},\mathbb{S}))\right) - (\mathbb{V},\mathbb{S})$ does not contain any non-empty soft s^*g -closed sets.

5.3 Theorem: If $(\mathbb{V},\mathbb{S})$ is a soft $\mathbb{P}_{\star}\mathbb{G}$ −closed set in $(\widetilde{\mathbb{U}},\widetilde{\mathbb{T}},\mathbb{V})$, then soft $\big(pcl(pint(\mathbb{V},\mathbb{S}))\big)-(\mathbb{V},\mathbb{S})$ contains only null soft closed set.

5.4 Theorem: If $(\mathbb{V}_1,\mathbb{S})$ is a soft $\mathbb{P}_{\star}\mathbb{G}$ −closed set in $(\widetilde{\mathbb{U}},\widetilde{\mathbb{T}},\mathbb{V})$ and $(\mathbb{V}_2,\mathbb{S})$ is a soft s^*g-closed set in $(\widetilde{\mathbb{U}},\widetilde{\mathbb{T}},\mathbb{V})$, then $(\mathbb{V}_1,\mathbb{S})\,\widetilde{\cap}\,(\mathbb{V}_2,\mathbb{S})$ is soft $\mathbb{P}_{\star}\mathbb{G}$ −closed.

5.5 Theorem: If $(\mathbb{V}_1,\mathbb{S})$ and $(\mathbb{V}_2,\mathbb{S})$ are two soft sets in a soft topological space

$(\widetilde{\mathbb{U}},\widetilde{\mathbb{T}},\mathbb{V})$ and $(\mathbb{V}_2,\mathbb{S})\,\widetilde{\subseteq}\,(\mathbb{V}_1,\mathbb{S})$, $(\mathbb{V}_2,\mathbb{S})$ is soft $\mathbb{P}_{\star}\mathbb{G}$ −closed set relative to $(\mathbb{V}_1,\mathbb{S})$ and $(\mathbb{V}_1,\mathbb{S})$ is soft $\mathbb{P}_{\star}\mathbb{G}$ −closed set in $(\widetilde{\mathbb{U}},\widetilde{\mathbb{T}},\mathbb{V})$, then $(\mathbb{V}_2,\mathbb{S})$ is soft $\mathbb{P}_{\star}\mathbb{G}$ −closed set relative to $(\widetilde{\mathbb{U}},\widetilde{\mathbb{T}},\mathbb{V})$.

5.6 Theorem: If $(\mathbb{V}_1,\mathbb{S})$ is soft $\mathbb{P}_{\star}\mathbb{G}$ −closed set in a soft topological space $(\widetilde{\mathbb{U}},\widetilde{\mathbb{T}},\mathbb{V})$ and $(\mathbb{V}_1,\mathbb{S})\,\widetilde{\subseteq}\,(\mathbb{V}_2,\mathbb{S})\,\widetilde{\subseteq}\,\big(pcl(pint(\mathbb{V}_1,\mathbb{S}))\big)$, then $(\mathbb{V}_2,\mathbb{S})$ is soft $\mathbb{P}_{\star}\mathbb{G}$ −closed set.

5.7 Theorem: Let $(\mathbb{V},\mathbb{S})$ is soft set in a soft topological space $(\widetilde{\mathbb{U}},\widetilde{\mathbb{T}},\mathbb{V})$.

If $\Big(cl\Big(int\big(cl(\mathbb{V},\mathbb{S})\big)\Big)\Big)\,\widetilde{\subseteq}\,(\mathbb{W},\mathbb{S})$, whenever $(\mathbb{V},\mathbb{S})\,\widetilde{\subseteq}\,(\mathbb{W},\mathbb{S})$ and $(\mathbb{W},\mathbb{S})$ be a soft s^*g-open set, then $(\mathbb{V},\mathbb{S})$ is soft $\mathbb{P}_{\star}\mathbb{G}$ −closed set.

5.8 Theorem: If a Soft subset $(\mathbb{V},\mathbb{S})$ of $\widetilde{\mathbb{U}}$ is soft $\mathbb{P}_{\star}\mathbb{G}$ −closed Subsets of $\widetilde{\mathbb{U}}$, Then $\big(pcl(pint(\mathbb{V},\mathbb{S}))\big)-(\mathbb{V},\mathbb{S})$ does not contain any non-empty soft s^*g-open set in $\widetilde{\mathbb{U}}$.

Proof: Suppose that $(\mathbb{V},\mathbb{S})$is soft $\mathbb{P}_{\star}\mathbb{G}$ −closed subsets of $\widetilde{\mathbb{U}}$. We prove the result by contradiction.

Let $(\mathbb{W},\mathbb{S})$ be soft s^*g-open such that $\big(pcl(pint(\mathbb{V},\mathbb{S}))\big)-(\mathbb{V},\mathbb{S})\,\widetilde{\supseteq}\,(\mathbb{W},\mathbb{S})$ and

$(\mathbb{W},\mathbb{S})\neq\emptyset$

Now,
$(\mathbb{W},\mathbb{S})\,\widetilde{\subseteq}\,\big(pcl(pint(\mathbb{V},\mathbb{S}))\big)\,(\mathbb{V},\mathbb{S})$Therefore $(\mathbb{W},\mathbb{S})\,\widetilde{\subseteq}\,\widetilde{\mathbb{U}}$-$(\mathbb{W},\mathbb{S})$, Since $(\mathbb{V},\mathbb{S})$ is soft $\mathbb{P}_{\star}\mathbb{G}$ −closed subsets of $\widetilde{\mathbb{U}}$.

By definition of soft $\mathbb{P}_{\star}\mathbb{G}$ −closed subsets of $\widetilde{\mathbb{U}}$, $\big(pcl(pint(\mathbb{V},\mathbb{S}))\big)-(\mathbb{V},\mathbb{S})\,\widetilde{\subseteq}\,\widetilde{\mathbb{U}}$-$(\mathbb{W},\mathbb{S})$, so $(\mathbb{W},\mathbb{S})\,\widetilde{\subseteq}\,\Big\{\widetilde{\mathbb{U}}-\big(pcl(pint(\mathbb{V},\mathbb{S}))\big)\Big\}$. Also $(\mathbb{W},\mathbb{S})\,\widetilde{\subseteq}\,\big(pcl(pint(\mathbb{V},\mathbb{S}))\big)$

Therefore $(\mathbb{W},\mathbb{S})\,\widetilde{\subseteq}\,\Big[\big(pcl(pint(\mathbb{V},\mathbb{S}))\big)\,\widetilde{\cap}\,\Big\{\widetilde{\mathbb{U}}-\big(pcl(pint(\mathbb{V},\mathbb{S}))\big)\Big\}\Big]$

That is, $(\mathbb{W},\mathbb{S})$ $=\{\widetilde{\{\}}\}$. This is contradicts to $(\mathbb{W},\mathbb{S})\neq\{\widetilde{\{\}}\}$. Hence $\big(pcl(pint(\mathbb{V},\mathbb{S}))\big)-(\mathbb{V},\mathbb{S})$ does not contain any non-empty soft s^*g-open set in $\widetilde{\mathbb{U}}$. It does not contain any non-empty soft s^*g-open set in $\widetilde{\mathbb{U}}$. But $(\mathbb{W},\mathbb{S})$ is not soft $\mathbb{P}_{\star}\mathbb{G}$ −closed of $\widetilde{\mathbb{U}}$.

5.1 Observation: The Converse of the above theorem need not be true as seen from the following Example

5.1 Illustration: Let us consider $\widetilde{\mathbb{U}}=\{Red(R),Green(G),Blue(B)\}$, $\mathbb{S}=\{b_1,b_2\}$

and $\widetilde{\mathbb{T}}=\{\widetilde{\{\}},\widetilde{\mathbb{U}},(\mathbb{D}_1,\mathbb{S}),(\mathbb{D}_2,\mathbb{S}),(\mathbb{D}_3,\mathbb{S}),(\mathbb{D}_4,\mathbb{S}),(\mathbb{D}_5,\mathbb{S}),(\mathbb{D}_6,\mathbb{S}),(\mathbb{D}_7,\mathbb{S})\}$ be a soft topology defined on $\widetilde{\mathbb{U}}$, where $(\mathbb{D}_1,\mathbb{S}),(\mathbb{D}_2,\mathbb{S})$, $(\mathbb{D}_3,\mathbb{S}),(\mathbb{D}_4,\mathbb{S}),(\mathbb{D}_5,\mathbb{S}),(\mathbb{D}_6,\mathbb{S}),(\mathbb{D}_7,\mathbb{S})$ are soft sets over $\widetilde{\mathbb{U}}$ defined as follows: $\mathbb{D}_1(b_1)=\{G\}$, $\mathbb{D}_1(b_2)=\{\widetilde{\{\}}\}$, $\mathbb{D}_2(b_1)=\{\widetilde{\{\}}\}$, $\mathbb{D}_2(b_2)=\{R\}$, $\mathbb{D}_3(b_1)=\{R\}$, $\mathbb{D}_3(b_2)=\{R\}$, $\mathbb{D}_4(b_1)=\{G\}$, $\mathbb{D}_4(b_2)=\{R\}$, $\mathbb{D}_5(b_1)=\{R,G\}$, $\mathbb{D}_5(b_2)=\{R\}$, $\mathbb{D}_6(b_1)=\{G,B\}$, $\mathbb{D}_6(b_2)=\{G,B\}$, $\mathbb{D}_7(b_1)=\{G,B\}$, $\mathbb{D}_7(b_2)=\{\widetilde{\mathbb{U}}\}$.

Solution:

We take the soft subset

$(\mathbb{D}_8,\mathbb{S})=\{b_1,\{R,G\}\}\{b_2,\{\widetilde{\{\}}\}\}$; $\big(pcl(pint(\mathbb{D}_8,\mathbb{S}))\big)=\{b_1,\{R,G\}\}\{b_2,\{R,G\}\}=(\mathbb{D}_6,\mathbb{S})$. Therefore $\big(pcl(pint(\mathbb{D}_8,\mathbb{S}))\big)-(\mathbb{D}_8,\mathbb{S})$ $=\{b_1,\{G,B\}\}\{b_2,\{R,G\}\}=(\mathbb{D}_9,\mathbb{S})$ Here $(\mathbb{D}_9,\mathbb{S})$ is not soft s^*g-open set. Therefore $\big(pcl(pint(\mathbb{D}_8,\mathbb{S}))\big)-(\mathbb{D}_8,\mathbb{S})$ does not contain any non-empty soft s^*g-open set in $\widetilde{\mathbb{U}}$. But $(\mathbb{D}_8,\mathbb{S})$ is not soft $\mathbb{P}_{\star}\mathbb{G}$ −closed of $\widetilde{\mathbb{U}}$.

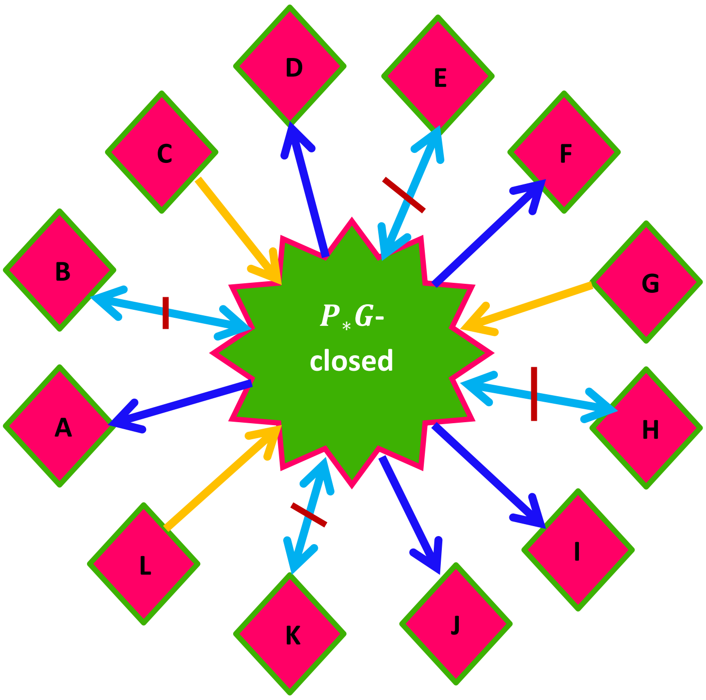
5.9 Theorem: If a Soft subset $(\mathbb{V},\mathbb{S})$ of $\widetilde{\mathbb{U}}$ is soft $\mathbb{P}_{\star}\mathbb{G}$ −closed and soft s^*g-open set in $\widetilde{\mathbb{U}}$, Then $(\mathbb{V},\mathbb{S})$ is soft pre-closed.

5.10 Theorem: Let $(\widetilde{\mathbb{U}},\widetilde{\mathbb{T}},\mathbb{V})$ be a soft topological space over $\widetilde{\mathbb{U}}$ and $(\mathbb{V},\mathbb{S})$ be soft $\mathbb{P}_{\star}\mathbb{G}-$ closed in $\widetilde{\mathbb{U}}$. $(\mathbb{V},\mathbb{S})$ is soft pre closed if and only if $\big(pcl(pint(\mathbb{V},\mathbb{S}))\big)-(\mathbb{V},\mathbb{S})$ is soft s^*g closed.

5.11 Theorem: If a Soft subset $(\mathbb{V},\mathbb{S})$ of $\widetilde{\mathbb{U}}$ is soft $\mathbb{P}_{\star}\mathbb{G}-$ closed if and only if soft

$\left(pcl\left(pint(\mathbb{V},\mathbb{S})\right)\right) - (\mathbb{V},\mathbb{S})$ contains only null soft $s^{*}g$-closed set in $\widetilde{\mathbb{U}}$.

We depict the above discussions in the following diagram:



A. Soft $g\beta$-closed
B. Soft g-closed
C. Soft pre-closed
D. Soft gp-closed
E. Soft β-closed
F. Soft gpr-closed
G. Soft pg-closed
H. Soft Q-SET
I. Soft $s^{*}g$-closed
J. Soft rwg-closed
K. Soft gs-set
L. Soft $g\alpha b$-closed

where $A \longrightarrow B$ represents A implies B but not conversely and $A \nLeftrightarrow B$ represents A and B are independent.

Conclusion

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied the soft set theory, which is initiated by Molodtsov and easily applied to many problems having uncertainties from social life. In the present work, we have continued to study the properties of soft topological spaces. In our future work, we will go on studying the properties of soft $\mathbb{P}_{\star}\mathbb{G}$- open sets and soft $\mathbb{P}_{\star}\mathbb{G}$ -closed sets such as hereditary of them. And will discuss some theorems on the equivalence of soft $\mathbb{P}_{\star}\mathbb{G}$ -separate spaces. We hope that the findings in this paper will help researcher enhance and promote the further study on soft topology to carry out a general framework for their applications in practical life.

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Authors

First Author-VANI.S.V, M.Sc., M.Phil., Registration No : 19112012092003(Full-Time), Research Scholar Of Mathematics, PG & Research Department of Mathematics, A.P.C.Mahalaxmi college for women, Thoothukudi, TN, India, Affiliated to Manonmaniam Sundaranar University, Tirunelveli, TN, India.
E-Mail: vanikathir16@gmail.com
Second Author- BALA DEEPA ARASI.K, M.Sc., M.Phil., Ph.D., Assistant Professor Of Mathematics, PG & Research Department of Mathematics, A.P.C.Mahalaxmi college for women, Thoothukudi, TN, India, Affiliated to Manonmaniam Sundaranar University, Tirunelveli, TN, India.
E-Mail: baladeepa@apcmcollege.ac.in
Third Author-RAMADHAN A.MOHAMMED, Ph.D., Department of Mathematics-College of Basic Educational University of Dohuk (UoD) www.UoD.ac, Dohuk, Iraq. E-Mail: ramadhan.hajani@uod.ac
Correspondence Author- VANI.S.V, E-Mail: vanikathir16@gmail.com & kvrithiksundhararaj@gmail.com
Mobile: 9487482412