Center Smooth Two Restrict Complement Domination on Graphs

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***Abstract*-** Let S be a dominating set of a graph G and $RS^c \subseteq V(G)$. The set RS^c is called a center-smooth 2-RS^c set of a center smooth graph G if $|N(v)\cap RS^c| \geq 2$ for every vertex $v\in S$. The center-smooth 2-RS^c number $\gamma_{2cs}(G)$ of a graph G is the number of vertices in a center-smooth 2-RS^c set of G. In this paper, we introduce the new concept center-smooth 2-RS^Cnumber. The center-smooth 2-RS^Cnumber $\gamma_{2cs}(G)$ of G is the number of vertices in a center-smooth 2-RS^cset of G. Some results on this new parameter are established and $\gamma_{2cs}(G)$ is computed for some special graphs and also proved that $\gamma_{2cs}(G) = 6$ for Petersen graph G. A result is proved for a triangle free connected graph G with minimum degree $\delta(G) \geq 2$. The following results are also proved. (i). If a connected graph G has exactly one vertex of degree p -1, then $\gamma_{2cs}(G) = \gamma_{2cs}(\bar{G}) + \Delta(G)$ and (ii). Let G be a graph with cut edge $e = uv$ where u and v are only central vertices, $\delta(G) = 1$. If $\gamma_{2cs}(G) = p - |\{u, v\}|$, then $\gamma(G) + \gamma_{2cs}(G) = p$.

***Keywords*-** Center smooth graph, *Restrict* S^c-set, Center smooth 1^c domination number, center smooth 2-RS^c number.

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I. INTRODUCTION

We consider only finite simple undirected connected graphs. For the graph G, $V(G)$ denotes its vertex set and $E(G)$ denotes its edge set. As usual, p=|V| and q= |E| denote the number of vertices and edges of a graph G, respectively. For a connected graph $G(V, E)$and a pair u, v of vertices of G, the *distance* $d(u, v)$ between u and v is the length of a shortest u-v path in G. The *degree* of a vertex u, denoted by *deg(u)* is the number of vertices adjacent to u. A vertex u of a graph G is called a *universal vertex* if u is adjacent to all other vertices of G. A graph G is *universal graph* if every vertex in G is universal vertex. For example, the complete graph K_p is universal graph. The set of all vertices adjacent to u in a graph G, denoted by $N(u)$, is the *neighborhood* of the vertex u. The *eccentricity* $e(u)$ of a vertex u is the distance to a vertex farthest from u. Thus, $e(u) = max\{ d(u, v)/v\in V(G)\}$.

A vertex v is an *eccentric vertex* of u if $e(u)= d(u, v)$. The radius $r(G)$ is the minimum eccentricity of the vertices, whereas the diameter $diam(G)$ is the maximum eccentricity. The center of G, $C(G)=\{ v\in V(G)/e(v)= r(G)\}$.

Definition 1.1. The *S-eccentricity* $e_S(v)$ of a vertex v in $G$is $\max_{x\in S}(d(v,x))$. The *S-center* of G is $C_S(G) = \{v\in V \,|\, e_S(v) \leq e_S(x)$ for every $x\in V\}$.

Example 1.1. In figure 1, $S =\{u_1, u_3, u_6\}$ and $V - S =\{ u_2, u_4, u_5, u_7\}$. The S-eccentricity $e_S(u_1) = 3$, $e_S(u_2) = 1$, $e_S(u_3) = 3$, $e_S(u_4) = 3$, $e_S(u_5) = 3$, $e_S(u_6) = 2$, $e_S(u_7) = 3$. Then the S-center $C_S(G) = \{u_2\}$.

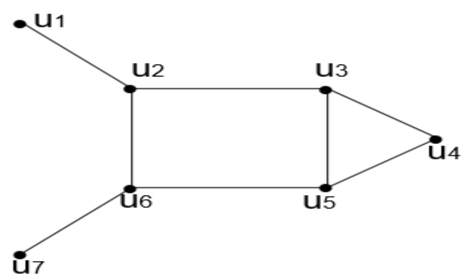

Figure 1. Center smooth graph

Definition 1.2. The S_1*-eccentricity*, $e_{S_1}(v)$ of a vertex v in S is $\max_{x\in V-S}(d(v,x))$. The S_1*center* of G is $C_{S_1}(G) = \{v\in V \,|\, e_{S_1}(v) \leq e_{S_1}(x)$for all $x\in V\}$.

Example 1.2. In figure 1, $S = \{u_1, u_3, u_6\}$ and $V- S = \{ u_2, u_4, u_5, u_7\}$. The S_1-eccentricity $e_{S_1}(u_1)$=3,$e_{S_1}(u_2)$ =1, $e_{S_1}(u_3) = 3$, $e_{S_1}(u_4) = 3$, $e_{S_1}(u_5) = 3$, $e_{S_1}(u_6) = 2$, $e_{S_1}(u_7) = 3$. Then the S_1-center, $C_{S_1}(G) = \{u_2\}$.

Definition 1.3. Let G be a graph and S be a proper set of G. G is called a *center-smooth graph* if $C_S(G) = C_{S_1}(G)$ and the set S is said to be a *center-smooth set*.

Example 1.3. In figure 1, $C_S(G) = \{u_2\} = C_{S_1}(G)$.

Definition 1.4. A set S is called 1-*dominating set* if for every vertex in V-S, there exists exactly one neighbor in S. The minimum cardinality of a 1-dominating set is denoted by $\gamma_1(G)$.

Definition 1.5. Let S be a dominating set of center smooth graph G. Then the *Restrict*-S^c(RS^c) set of a graph G is defined by RS^c= $\begin{cases} v \in RS^c; & |N(v)\cap S| = 1 \\ v \notin RS^c; & |N(v)\cap S| > 1 \end{cases}$ and the number of RS^c - set of G is denoted by $nR(G)$. If RS^c - set is independent set then the number of RS^c- set of G is denoted by $niR(G)$.

Definition 1.6. Let S be a dominating set of G and $RS^c \subseteq V(G)$. Then the set RS^c is called a *center smooth* 1^c*dominating set* of a center smooth graph G if for every vertex in S^c has at least one neighbor in S. The number of vertices in RS^c of a center smooth graph G is called *center smooth* 1^c *domination number* and it is denoted by $\gamma_1^c cs(G)$.

II. RESULTS ON CENTER-SMOOTH TWO RESTRICT COMPLEMENT DOMINATION

Definition 2.1. Let S be a dominating set of G. Then $RS^c \subseteq V(G)$ is a center-smooth 2-RS^c set of a center smooth graph G if $|N(v)\cap RS^c| \geq 2$ for every vertex $v\in S$. The center-smooth 2-RS^c number $\gamma_{2cs}(G)$ of a graph G is the number of vertices in a center-smooth 2-RS^c set of G.

Proposition 2.2. Let G be a graph with $p > 2$ vertices. If a universal vertex v of degree p - 1 then $\gamma_{2cs}(G) = p - 1$.

Proof: Let v be a universal vertex of degree p -1. Then it is clear that, v is adjacent to all other vertices in G. Since $p > 2$, then at least two or more vertices are adjacent to v in G. Thus, $|N(v)\cap RS^c| \geq 2$. Therefore, every vertex in G not in S is γ_{2cs}-set of G. Hence, $\gamma_{2cs}(G) = p - 1$.

Theorem 2.3. If there exists exactly one vertex of even degree in a tree T with $p > 2$ vertices, then $\gamma_{2cs}(T) = p - 1$ and the bounds are sharp.

Proof: Let v be a vertex of even degree and all other vertices having odd degree. Further, let U be the set of all odd degree vertices in T. Let RS^c be a γ_{2cs}-set of T and contains only the vertices in U. Suppose $v\in RS^c$, then a vertex in U is adjacent to v and $N(v) \cap RS^c = \phi$. It is a contradiction. Therefore, $v\in S$ and $N(v) \cap RS^c = RS^c \geq 2$. Hence $\gamma_{2cs}(T) = p$ -1 and the bounds are sharp for $T = P_3$ or $K_{1, p-1}$, p = 5, 7, 9…

Corollary 2.4. If there exists exactly one vertex of even degree in a tree T with p>2 vertices, then $\gamma_{2cs}(T) = \gamma_1^c cs(T)$

Proof: From the main theorem (2.3), T=P_3 or $K_{1, p-1}$, p =5, 7, 9…Clearly, $\gamma_{2cs}(T) = \gamma_1^c cs(T) = p$-1.

Theorem 2.5. If every vertex v in a tree T has an odd degree, then $\gamma_{2cs}(T) = p - k$ where k is the number of vertices which are having maximum degree in a tree T.

Proof: Assume that there exists a vertex v in a tree T has odd degree. Then an edge $xy \in E(T)$ with $N(v)= x\in RS^c$ and $y\notin RS^c$ and we can choose $x_1\in RS^c\cap N(y)$ with $x_1\neq x$. Then $N(x_1)$ has even degree, as is $|N(x_1) \cap RS^c|$. Since $y\in N(x_1)$ with $y\notin RS^c$, we can choose a vertex $x_1\neq y$ where $y_1 \in N(x_1)$ but $x_1 \notin RS^c$. Iterating this procedure, we could obtain an arbitrarily long path x, y, x_1, y_1, x_2, y_2,…in T with each $x_i\in RS^c$ and each $y_i\notin RS^c$. Thus, $\gamma_{2cs}(T) \neq p$-k. This is a contradiction. Hence each vertex v of odd degree, then $\gamma_{2cs}(T) = p$-k.

Theorem 2.6. Let G be a graph with p>2 and $\delta(G) = \Delta(G) = p$-1 then $\gamma_{2cs}(G)$=p-1 if and only if G is K_p.

Proof: Let G be any graph. Assume that, G has $\delta(G) = \Delta(G) = p$-1. Then it is clear that each vertex in G is adjacent to all vertices in G. Since p>2, every vertex v in G is adjacent to at most p-1 vertices in G. Since, RS^c-set has p-1 vertices, so that every vertex v dominates $N(v)$ and the vertices in V-$N(v)$ dominate themselves. Thus, G is K_p. Conversely, suppose G is K_p. Then any vertex $v\in V(G)$ dominate all other vertices in G. So that $\delta(G) = \Delta(G) = p$-1. Since p>2, each vertex v is adjacent to two or more vertices in G. Hence RS^c-set has p-1 vertices and so $\gamma_{2cs}(G) = p$-1.

Theorem 2.7. Let G be a graph with $\delta(G) = \Delta(G)$= 2 then $\gamma_{2cs}(G) \leq p$-1 if and only if G is C_p.

Proof: Let G be any graph and $\delta(G) = \Delta(G) = 2$.Then each vertex in G dominates 2 vertices and $p = q$. Suppose, $\gamma_{2cs}(G) \leq p$-1. Then RS^c contains atleast two vertices. Hence, G forms a cycle C_p. Conversely, suppose G is C_p. Then each vertex is adjacent to 2 vertices in G. Therefore degree of each vertex is 2. That is, $\delta(G)=\Delta(G)$=2. Let RS^c be a γ_{2cs} -set of G. If $|S|$ =1 then $|RS^c|$ =p-1. If $|S|$ >1 then $|RS^c|$ <p-1. Thus, it follows that $\gamma_{2cs}(G) \leq p$-1.

Theorem 2.8. For any graph $G = K_{1, p-1}$, $\gamma_{2cs}(G) = \gamma_1^c cs(G)$.

Proof: Let RS^c be a γ_{2cs}-set which contains only a set of end vertices in G and $S = \{v\}$ be a dominating set of G. Clearly, the vertex v is an universal vertex of G .Then $RS^c \cap N(v) = RS^c \geq 2$. Hence $\gamma_{2cs}(G) = p$ -1 and so $\gamma_1^c cs(G) = p$ - 1. Therefore, it follows that, $\gamma_{2cs}(G) = \gamma_1^c cs(G)$.

Theorem 2.9. Let G be a graph with $\delta(G)$ =1 or 2. If $\gamma_{2cs}(G) = p$-1 then $diam(G) \leq 2$.

Proof: Since $\gamma_{2cs}(G) = p$ -1, then $p > 2$. Let $S = \{v\}$ be a dominating set of G. If u and w be vertices of G such that degrees of u and w are equal to 1. Then u and w are end vertices in G and also v dominates u and w. Since RS^c-set has p-1 vertices and $diam(G)$ =2. If degree of u and w is not equal to 1, then u and w are adjacent vertices in G and also dominated by v. Then each vertex of degree is 2 and RS^c has p - 1 vertices. Clearly, $diam(G)$ =1 < 2. Hence it follows that $diam(G) \leq 2$.

Remark: The converse of the theorem (2.9) is false. For the graph C_4 or C_5, $diam(C_4$ or $C_5)$ =2 but $\gamma_{2cs}(C_4$ or $C_5) \neq p$-1 and for the graph K_2, $diam(K_2)$ =1 but $\gamma_{2cs}(K_2) \neq p$-1.

Theorem 2.10. Let G be a triangle free connected graph with minimum degree $\delta(G) \geq 2$. If $\gamma(G)$ =2, then $\gamma_{2cs}(G) \leq p$-2.

Proof: Let RS^c be a center smooth 2-dominating set of G, so that $\gamma_{2cs}(G) = |RS^c|$. Since $\gamma(G)$ =2, then there exists a pair of vertices x, $y\in G$ such that $x(y)$ is adjacent to at most one vertex in S because G is a triangle free connected graph. We show that $\gamma_{2cs}(G) \leq p$-2.We consider two cases.

Case (*i*): Each vertex in S is adjacent. Then it is clear that, every vertex in V - S is adjacent to exactly one vertex in S. Since, every vertex in RS^c is adjacent to exactly one vertex in S, then $N(v)\cap RS^c$=RS^c for all $v\in S$. Thus, $\gamma_{2cs}(G) = p$ - 2.

Case (*ii*): Each vertex in S is not adjacent. Then there exist a vertex u is adjacent to both vertex in S. Since, $\delta(G) \geq 2$, each vertex in G has atleast two neighbors, say r and s. If r is adjacent to exactly one vertex in S and u, then triangle is formed. It is a contradiction. Therefore, r is adjacent to exactly one vertex in S and s. Similarly, s is adjacent to another vertex in S and r. Clearly, $rs\in E(G)$ and G formed a cycle C_5. Therefore, we have $\gamma_{2cs}(G) < p$ - 2. In both the cases, $\gamma_{2cs}(G) \leq p$ - 2.

Remark: The converse of the theorem 2.10 is false. For the graph C_n, $n \geq 7$, $\gamma_{2cs}(C_n) \leq p$ - 2 but $\gamma(C_n) \neq 2$.

Theorem 2.11. If a connected graph G with $\gamma(G)$ =1, then $\gamma_{2cs}(G) = p - \gamma(G)$.

Proof: Since $\gamma(G)$ =1= $|\{v\}|$, then a vertex v is adjacent to all the vertices in G. Therefore, the degree of v is p -1.Clearly, $v\in S$. It implies that $|S|$ = 1 and then $|RS^c| = p -1 = p - \gamma(G)$.

Theorem 2.12. Let G be a connected graph with $diam(G) \leq 3$. If $\gamma(G)$ =2, then $\gamma_{2cs}(G) \leq p$-2.

Proof: *Case* (*i*): If $\gamma(G)$ contains an independent vertex, then atleast one vertex u is adjacent to both the vertices in $\gamma(G)$. Let RS^c be the γ_{2cs}-set of G. Since $diam(G) \leq 3$, then there exists a vertex v is adjacent to one of the vertex in $\gamma(G)$. Therefore, $\gamma_{2cs}(G) < p - 2$.

Case (*ii*): Suppose $\gamma(G)$ does not contain an independent vertex, then the two vertices in $\gamma(G)$ are adjacent. From the theorem 2.10, in case (i), $\gamma_{2cs}(G) = p - 2$. In both the cases, $\gamma_{2cs}(G) \leq p - 2$.

Remark: The converse of the theorem 2.12 is false. For the graph C_7, $\gamma_{2cs}(C_7) \leq p - 2$ but $\gamma(C_7) = 3 \neq 2$.

Theorem 2.13. If G is a connected graph with $\gamma(G) = 2$, $d(u, v) > 2$, for every $u, v \in S$ and $diam(G) > 3$, then $\gamma_{2cs}(G) = p - 2$.

Proof: Since $\gamma(G) = 2$, then there exists a pair of vertices $x, y \in V - S$ such that x is adjacent to one vertex and y is adjacent to another vertex in S because $d(u, v) > 2$. Therefore, $diam(G) = 3$. So that, from the theorem (2.12), $\gamma_{2cs}(G) \leq p - 2$. It is a contradiction to $diam(G) > 3$.Hence the result.

Theorem 2.14. The Petersen graph G has $\gamma_{2cs}(G) = 6$.

Proof: Let G be a Petersen graph with 10 vertices and 15 edges. Then G consists of two cycles C_1 and C_2 such that the cycle C_1 with vertex set $\{v_1, v_2, \ldots, v_n\}$ is nested by the another cycle C_2 with vertex set $\{u_1, u_2, \ldots, u_n\}$ and each $u_i \in C_2$ is adjacent with exactly one $v_i \in C_1$. Let RS^c be a γ_{2cs}-set of G. Further, let $u_s \in C_2$ is adjacent to more than one vertex in S. Since $d(u_i) = d(v_i) = 3$, $\forall u_i, v_i \in G$ and $\gamma(G) = 3$. Therefore $\gamma(G) = |S| = \Delta(G) = 3$. Clearly, $RS^c = V - S - \{u_s\}$. It implies that $\gamma_{2cs}(G) = p - \Delta(G) - 1 = 6$.

III. PARTICULUR VALUE FOR CENTER-SMOOTH TWO RESTRICT COMPLEMENT DOMINATION NUMBER

In this section, we identify certain graphs for which $\gamma_{2cs}(G) = p - 2$ or $p-1$. For instance $\gamma_{2cs}(B_{m,n}) = p - 2$ or $\gamma_{2cs}(K_{1,p-1}$ or $P_3) = p-1$.

Theorem 3.1. Let G be a graph with cut edge $e = uv$ where u and v are only central vertices, $\delta(G) = 1$ and $\gamma_{2cs}(G) = p - |\{u, v\}|$. Suppose RS^c is a γ_{2cs}-set of G, then a cut edge is incident to all other edges in G.

Proof: By the assumption on RS^c, $|RS^c| = p - 2$ and hence $|S| = 2$. Let $S = \{u, v\}$. Since u and v are the only central vertices of G, then u and v are in the dominating set of G. Suppose u is not adjacent to a vertex x in RS^c, then x is adjacent to v. Since, the degree of x is 1, x is not adjacent to any vertex in RS^c. Therefore RS^c has only pendant vertices. Since u and v are only the central vertices in G. Then it is clear that, e be a central edge in G. So, e dominates all other edges in G. Therefore, e is a cut edge of G and e is incident to all other edges in G.

Corollary 3.2. Let G be a graph with cut edge $e = uv$ where u and v are only central vertices, $\delta(G) = 1$. If $\gamma_{2cs}(G) = p - |\{u, v\}|$, then $\gamma(G) + \gamma_{2cs}(G) = p$.

Proof: Since u and v are the central vertices, then u and v are adjacent because $e = uv$ be a cut edge. Since, $\delta(G) = 1$, then each vertex in $V - \{u, v\}$ is a pendent vertex (by the main theorem 3.1). Clearly, $\gamma(G) = 2$. Therefore, $\gamma(G) + \gamma_{2cs}(G) = 2 + p - |\{u, v\}|$since $\gamma_{2cs}(G) = p - |\{u, v\}|$. It implies that $\gamma(G) + \gamma_{2cs}(G) = p$.

Corollary 3.3. Let G be a graph with cut edge $e = uv$ where u and v are only central vertices, $\delta(G) = 1$ and $\gamma_{2cs}(G) = p - |\{u, v\}|$, then each component of G is $K_{1,m}$ and $K_{1,n}$.

Proof: From corollary 3.2, each vertex in $V - \{u, v\}$ is a pendent vertex. Since, u and v are only the central vertices in G. Then u is adjacent to m pendant vertices and v is adjacent to n pendant vertices because $\delta(G) = 1$. From the main theorem 3.1, a cut edge e is incident to all other edges in G. Clearly, G has $m+n+2$ vertices and $m + n + 1$ edges. Therefore, G is $B_{m,n}$. Hence G has two components and each component of G is $K_{1,m}$ and $K_{1,n}$ and the common edge of G is central edge.

Theorem 3.4. Let G be a graph with $\delta(G) = 1$ and $\lfloor(G) = p-1$ where $\lfloor(G)$ is the number of end vertices in G. suppose RS^c is γ_{2cs}-set of G, then every vertex in S is adjacent to all vertices in RS^c. The bounds are sharp.

Proof: Let RS^c be γ_{2cs}-set of G. Further, let $v \in G$ be a central vertex or cut vertex in G. Therefore, v dominates all other vertices in G. Clearly, $v \in S$. Since, $V - \{v\}$ is the set of all end vertices in G. Clearly, v dominates all other end vertices in $V - \{v\}$ and $N(v) = \lfloor(G)$ and so $N(v) \cap \lfloor(G) = \lfloor(G) \geq 2$. Hence RS^c - set has p -1 vertices since $\lfloor(G) = p-1$ and v is adjacent to all vertices in RS^c. The bounds are sharp for G is $K_{1,p-1}$ or P_3.

Corollary 3.5. Let G be a graph with $\delta(G) = 1$ and $\lfloor(G) = p-1$ where $\lfloor(G)$ is the number of end vertices in G. If $\gamma_{2cs}(G) = p - 1$ then $\gamma_{2cs}(G) = \beta_0(G)$.

Proof: Since $\gamma_{2cs}(G) = p - 1 = \lfloor(G)$, it is clear that RS^c has only pendant vertices in G which are independent. Hence $\beta_0(G) = p - 1$ and so $\gamma_{2cs}(G) = \beta_0(G)$.

Corollary 3.6. Let G be a graph with $\delta(G) = 1$ and $\lfloor(G) = p - 1$ where $\lfloor(G$ is the number of end vertices in G. if $\gamma_{2cs}(G) = p-1$ then $\bar{G}$ is disconnected.

Proof: From the main theorem 3.4, G is $K_{1,p-1}$ or P_3 .We know that non adjacent vertices in G are adjacent in $\bar{G}$. Therefore, $\bar{G}$ is K_p, $(p \geq 2)$ with an isolated vertex. Hence, $\bar{G}$ is not a connected graph.

Theorem 3.7. Let G be a graph with cut edge $e = uv$ where u and v are only central vertices, $\delta(G) = 1$. If $\gamma_{2cs}(G) = p - |\{u, v\}|$, then $\gamma_{2cs}(G) = \beta_0(G)$.

Proof: From the theorem 3.1, we have RS^c has only pendent vertices which are independent. Therefore, $\beta_0(G) = p - |\{u, v\}|$. Hence, $\gamma_{2cs}(G) = \beta_0(G)$.

Theorem 3.8. Let G be a graph with $\delta(G) = 1$ and $\lfloor(G) = p-1$ where $\lfloor(G)$ is the number of end vertices in G. Then the following are equivalent:

(i) $\gamma(G) + \gamma_{2cs}(G) = p$.

(ii) $\gamma(G) = 1$ and $\gamma_{2cs}(G) = p - 1$.

(iii) G is $K_{1,p-1}$ or P_3.

Proof: (i) $\Leftrightarrow$ (ii) follows from the theorem 3.5. We prove (ii) $\Rightarrow$ (iii). Let RS^c be a γ_{2cs}-set of a graph G. Then $|RS^c| = p - 1$ and $|S| = 1$. Let $S = \{v\}$. Since, every vertex in S is adjacent to all vertices in $|RS^c|$ and $\lfloor(G) = p - 1$. Now, we claim that each vertex in RS^c is pendent vertex. If not there exists a vertex $u \in RS^c$ is adjacent to x and y where $x, y \in RS^c$. Now clearly, RS^c has less than $p - 1$ vertices. It is a contradiction to $|RS^c| = p-1$. Hence, every vertex in RS^c is pendent vertex and so G is $K_{1,p-1}$ or P_3.

We prove that (iii)$\Rightarrow$ (ii). Since G has a universal vertex v. Therefore, $\gamma(G) = 1$. Clearly, a vertex v is adjacent to all other vertices in G. Since, $V - \{v\}$ be only the end vertices and $v \in S$. Clearly, RS^c has $V - \{v\}$ vertices and so RS^c has $p - 1$ vertices. Hence, $\gamma_{2cs}(G) = p - 1$.

Theorem 3.9. If a connected graph G has exactly one vertex of degree $p - 1$, then $\gamma_{2cs}(G) = \gamma_{2cs}(\bar{G}) + \Delta(G)$.

Proof: Let G be a connected graph. Since, G has exactly one vertex of degree p-1, then G is $K_{1,p-1}$ by the theorem 3.8 and

$\gamma_{2cs}(G) = p - 1$. In $\bar{G}$, adjacent vertices in G are non-adjacent vertices in $\bar{G}$. Hence, $\bar{G}$ is disconnected. That is, $\bar{G} = K_p$ with an isolated vertex. Therefore, $\gamma_{2cs}(\bar{G})=0$. Hence, $\gamma_{2cs}(G) = \gamma_{2cs}(\bar{G}) + \Delta(G)$.

IV. CONCLUSION

In this paper, S_1 - eccentricity of a vertex and center smooth set have been defined. Also, the center smooth graph and restrict S^c set have been introduced. The center smooth 2- RS^c dominating set and center smooth 2 - RS^c domination number of some families of graphs were enumerated.

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