

## CONTRA NANO $\alpha^*_{AS}$ CONTINUOUS FUNCTION IN NANO TOPOLOGICAL SPACE

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### ABSTRACT

The purpose of this study is to introduce a new class of contra continuous function called Contra nano  $\alpha^*_{AS}$ -continuous function in Nano topological spaces. Some of its properties are analysed. The equivalent condition for a function to be contra  $N\alpha^*_{AS}$  -continuous function is established. Further Contra  $N\alpha^*_{AS}$  -irresolute function is defined and few of its properties are discussed.

**Keywords:** Nano topological space, Nano contra continuous function,  $N\alpha^*_{AS}$  -closed set,  $N\alpha^*_{AS}$  -continuous function,  $N\alpha^*_{AS}$  -irresolute function.

### I.INTRODUCTION

The concept of topology was first developed in 17<sup>th</sup> century by Gottfried Leibniz. The concept of nano topology was introduced by Lellis Thivagar et al. We introduced  $N\alpha^*_{AS}$  - closed set in nano topological space.  $N\alpha^*_{AS}$  - closed map, open map, continuous and homeomorphism was also discussed in the previous papers and their properties were analysed. In this paper we introduce contra  $N\alpha^*_{AS}$  - continuous and contra  $N\alpha^*_{AS}$  - irresolute function, also some of their properties were discussed.

### II.PRELIMINARIES

The following are the necessary concepts and definitions that are used in this work.

**Definition 2.1:[1]** Let  $U$  be a non-empty finite set of objects called the universe and  $R$  be an equivalence relation on  $U$  named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair  $(U, R)$  is said to be the approximation space. Let  $X \subseteq U$ . Then,

1.The lower approximation of  $X$  with respect to  $R$  is the set of all objects, which can be for certain classified as  $X$  with respect to  $R$  and it is denoted by  $L_R(X)$ .

$L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$ , where  $R(x)$  denotes the equivalence class determined by  $x \in U$ .

2.The upper approximation of  $X$  with respect to  $R$  is the set of all objects, which can be possibly classified as  $X$  with respect to  $R$  and it is denoted by  $U_R(X)$ .

$U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$ .

3.The boundary region of  $X$  with respect to  $R$  is the set of all objects, which can be classified neither as  $X$  nor as not -  $X$  with respect to  $R$  and it is denoted by  $B_R(X)$ .

$B_R(X) = U_R(X) - L_R(X)$ .

**Definition 2.2:[1]** Let  $U$  be the universe,  $R$  be an equivalence relation on  $U$  and  $\tau_R(X) = \{U, \emptyset, U_R(X), L_R(X), B_R(X)\}$  where  $X \subseteq U$ .  $R(X)$  satisfies the following axioms:

1.  $U$  and  $\emptyset \in \tau_R(X)$ ,

2. The union of the elements of any sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$ ,

3. The intersection of the elements of any finite sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$ .

That is,  $\tau_R(X)$  is a topology on  $U$  called the nano topology on  $U$  with respect to  $X$ . We call  $(U, \tau_R(X))$  as the nano topological space. The elements of  $\tau_R(X)$  are called as nano open sets. The complement of nano open sets is called nano closed sets.

**Remark 2.3:[1]** If  $\tau_R(X)$  is the nano topology on  $U$  with respect to  $X$ , then the set  $B = \{U, \emptyset, L_R(X), B_R(X)\}$  is the basis for  $\tau_R(X)$ .

**Definition 2.4:[1]** A subset  $A$  of a nano topological space  $(U, \tau_R(X))$  is called nano  $\alpha^*_{AS}$  (briefly  $N\alpha^*_{AS}$ ) closed sets if  $N\alpha cl(A) \subseteq Nint(V)$  whenever  $A \subseteq V$  and  $V$  is nano open.

**Definition 2.5:[2]** Let  $(U, \tau_R(X))$  and  $(V, \sigma_R(Y))$  be a nano topological spaces. Then the function  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  is said to be nano continuous on  $U$  if the inverse image of every nano open set in  $V$  is nano open in  $U$ .

**Definition 2.6:[2]** A function  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  is called Nano  $\alpha^*_{AS}$ -continuous (briefly  $N\alpha^*_{AS}$ -continuous) if the inverse image of every Nano closed set in  $(V, \sigma_R(Y))$  is  $N\alpha^*_{AS}$ -closed in  $(U, \tau_R(X))$ .

**Definition 2.7:[4]** A function  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  is called nano contra continuous if the inverse image of every nano open set in  $(V, \sigma_R(Y))$  is nano closed set in  $(U, \tau_R(X))$ .

**Definition 2.8:[5]** A function  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  is called nano contra  $g$ -continuous if the inverse image of every nano open set in  $(V, \sigma_R(Y))$  is nano  $g$ -closed set in  $(U, \tau_R(X))$ .

### III.CONTRA $N\alpha^*_{AS}$ – CONTINUOUS

In this section, the notion of Contra  $N\alpha^*_{AS}$  - continuous is introduced and its properties are investigated.

**Definition 3.1:** Let  $(U, \tau_R(X))$  and  $(V, \sigma_R(Y))$  be two nano topological spaces. A function  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  is called contra  $N\alpha^*_{AS}$  - continuous if the inverse image of every nano open set in  $(V, \sigma_R(Y))$  is  $N\alpha^*_{AS}$  closed set in  $(U, \tau_R(X))$ .

**Example 3.2:** Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ . Let  $X = \{a, b\} \subseteq U$  and  $\tau_R(X) = \{U, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ . Then  $N\alpha^*_{AS}$  closed sets are  $= \{U, \emptyset, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}$ . Let  $V = \{a, b, c, d\}$  with  $V/R = \{\{a, b\}, \{c, d\}\}$ . Let  $Y = \{a, b\} \subseteq V$  and  $\sigma_R(Y) = \{U, \emptyset, \{a, b\}\}$ . Let  $f: U \rightarrow V$  defined by  $f(a) = d, f(b) = c, f(c) = a, f(d) = b$  then  $f^{-1}(a) = c, f^{-1}(b) = d, f^{-1}(c) = b, f^{-1}(d) = a$ . Thus, the inverse image  $\{a, b\}$  in  $V$  i.e.  $f^{-1}(a, b) = \{c, d\}$  which is a  $N\alpha^*_{AS}$  closed set in  $U$ . Thus,  $f$  is contra  $N\alpha^*_{AS}$  – continuous.

**Theorem 3.3:** Let  $U$  and  $V$  are any two Nano Topological spaces. Let  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$ ,  $f$  is contra  $N\alpha^*_{AS}$  – continuous function if and only if inverse image of every Nano closed set in  $V$  is  $N\alpha^*_{AS}$  open in  $U$ .

**Proof:** Let  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  and  $S$  be a Nano closed set in  $V$ . Since  $f$  is contra  $N\alpha^*_{AS}$  – continuous function  $f^{-1}(V - S) = U - f^{-1}(S)$  is  $N\alpha^*_{AS}$  closed in  $U$ . Hence  $f^{-1}(S)$  is  $N\alpha^*_{AS}$  open set in  $V$ . Conversely, let  $S$  be a Nano open set in  $V$ . By assumption  $f^{-1}(V - S)$  is  $N\alpha^*_{AS}$  open set.  $f^{-1}(V - S) = U - f^{-1}(S)$ ,  $f^{-1}(S)$  is  $N\alpha^*_{AS}$  closed set in  $U$ . Hence  $f$  is Contra  $N\alpha^*_{AS}$  - continuous function.

**Theorem 3.4:** Every Nano contra continuous function is contra  $N\alpha^*_{AS}$  - continuous function.

**Proof:** Let  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  be a nano contra continuous function and  $S$  be a

nano open set in  $V$ . Since  $f$  is nano contra continuous function  $f^{-1}(S)$  is closed set in  $U$ . Since every nano closed set is  $N\alpha^*_{AS}$  closed  $f^{-1}(S)$  is  $N\alpha^*_{AS}$  closed set in  $U$ . Hence  $f$  is contra  $N\alpha^*_{AS}$  - continuous function.

**Remark 3.5:** The converse of the above theorem need not be true as shown in the following example.

**Example 3.6:** Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ . Let  $X = \{a, b\} \subseteq U$  and  $\tau_R(X) = \{U, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ . Then  $N\alpha^*_{AS}$  closed sets are  $= \{U, \emptyset, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}$ . Let  $V = \{a, b, c, d\}$  with  $V/R = \{\{a, b\}, \{c, d\}\}$ . Let  $Y = \{a, b\} \subseteq V$  and  $\sigma_R(Y) = \{U, \emptyset, \{a, b\}\}$ . Let  $f: U \rightarrow V$  defined by  $f(a) = d, f(b) = c, f(c) = a, f(d) = b$  then  $f^{-1}(a) = c, f^{-1}(b) = d, f^{-1}(c) = b, f^{-1}(d) = a$ . Thus, the inverse image  $\{a, b\}$  in  $V$  i.e.  $f^{-1}(a, b) = \{c, d\}$  which is a  $N\alpha^*_{AS}$  closed set in  $U$ . Thus,  $f$  is contra  $N\alpha^*_{AS}$  - continuous. But  $\{c, d\}$  is not a nano closed set in  $U$ . Thus,  $f$  is not nano contra continuous function.

**Theorem 3.7:** Every nano contra  $\alpha$  - continuous function is contra  $N\alpha^*_{AS}$  - continuous function.

**Proof:** Let  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  be a nano contra  $\alpha$  - continuous function and  $S$  be a nano open set in  $V$ . Then the inverse image of  $S$  under the map  $f$  is nano  $\alpha$  - closed set in  $U$ . Since every nano  $\alpha$  - closed set is  $N\alpha^*_{AS}$  closed,  $f^{-1}(S)$  is  $N\alpha^*_{AS}$  closed set in  $U$ . Hence  $f$  is contra  $N\alpha^*_{AS}$  - continuous function.

**Remark 3.8:** The converse of the above theorem need not be true as shown in the following example.

**Example 3.9:** Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ . Let  $X = \{a, b\} \subseteq U$  and  $\tau_R(X) = \{U, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ . Then  $N\alpha^*_{AS}$  closed sets are  $= \{U, \emptyset, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}$ . Let

$V = \{a, b, c, d\}$  with  $V/R = \{\{a, b\}, \{c, d\}\}$ . Let  $Y = \{a, b\} \subseteq V$  and  $\sigma_R(Y) = \{U, \emptyset, \{a, b\}\}$ . Let  $f: U \rightarrow V$  defined by  $f(a) = d, f(b) = c, f(c) = a, f(d) = b$  then  $f^{-1}(a) = c, f^{-1}(b) = d, f^{-1}(c) = b, f^{-1}(d) = a$ . Thus, the inverse image  $\{a, b\}$  in  $V$  i.e.  $f^{-1}(a, b) = \{c, d\}$  which is a  $N\alpha^*_{AS}$  closed set in  $U$ . Thus,  $f$  is contra  $N\alpha^*_{AS}$  - continuous. But  $\{c, d\}$  is not a nano  $\alpha$ - closed set in  $U$ . Thus,  $f$  is not nano contra  $\alpha$ - continuous function.

**Theorem 3.10:** Let  $U$  and  $V$  are any two Nano Topological spaces. Let  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  be nano contra continuous function.  $f$  is nano contra  $g$  - continuous iff it is contra  $N\alpha^*_{AS}$  - continuous function.

**Proof: Necessity:** Let  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  be a nano contra  $g$  - continuous function and  $S$  be a nano open set in  $V$ . Then the inverse image of  $S$  under the map  $f$  is nano  $g$  - closed set in  $U$ . We know that a set is nano  $g$  - closed set iff it is  $N\alpha^*_{AS}$  closed,  $f^{-1}(S)$  is  $N\alpha^*_{AS}$  closed set in  $U$ . Hence  $f$  is contra  $N\alpha^*_{AS}$  - continuous function.

**Sufficient:** Assume  $f$  is contra  $N\alpha^*_{AS}$  - continuous. Let  $S$  be any Nano open set in  $(V, \sigma_R(Y))$ . Then  $f^{-1}(S)$  is  $N\alpha^*_{AS}$  - closed in  $(U, \tau_R(X))$ . Since [6] A set is Nano  $g$ -closed set iff it is  $N\alpha^*_{AS}$ -closed. Then,  $f^{-1}(S)$  is  $Ng$ -closed in  $(U, \tau_R(X))$ . Therefore,  $f$  is contra  $Ng$ -continuous.

**Theorem 3.11:** Every nano contra  $g\alpha$  - continuous function is contra  $N\alpha^*_{AS}$  - continuous function.

**Proof:** Let  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  be a nano contra  $g\alpha$  - continuous function and  $S$  be a nano open set in  $V$ . Then the inverse image of  $S$  under the map  $f$  is nano  $g\alpha$  - closed set in  $U$ . Since every nano  $g\alpha$  - closed set is  $N\alpha^*_{AS}$  closed,  $f^{-1}(S)$  is  $N\alpha^*_{AS}$  closed set in  $U$ . Hence  $f$  is contra  $N\alpha^*_{AS}$  - continuous function.

**Remark 3.12:** The converse of the above theorem need not be true as shown in the following example.

**Example 3.13:** Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a, b\}, \{c, d\}\}$ . Let  $X = \{a, b\} \subseteq U$  and  $\tau_R(X) = \{U, \emptyset, \{a, b\}\}$ . Let  $V = \{a, b, c, d\}$  with  $V/R = \{\{b\}, \{c\}, \{a, d\}\}$ . Let  $Y = \{a, c\} \subseteq V$  and  $\sigma_R(Y) = \{U, \emptyset, \{c\}, \{a, d\}, \{a, c, d\}\}$ . Let  $f: U \rightarrow V$  defined by  $f(a) = b, f(b) = a, f(c) = d, f(d) = c$  then  $f^{-1}(a) = b, f^{-1}(b) = a, f^{-1}(c) = d, f^{-1}(d) = c$ . Then  $f$  is contra  $N\alpha^*_{AS}$ -continuous but not contra nano  $g\alpha$ -continuous.

**Theorem 3.14:** Let  $U$  and  $V$  are any two Nano Topological spaces. Let  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  be nano contra continuous function.  $f$  is nano contra  $\alpha g$ -continuous iff it is contra  $N\alpha^*_{AS}$ -continuous function.

**Proof: Necessity:** Let  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  be a nano contra  $\alpha g$ -continuous function and  $S$  be a nano open set in  $V$ . Then the inverse image of  $S$  under the map  $f$  is nano  $\alpha g$ -closed set in  $U$ . We know that a set is nano  $\alpha g$ -closed set iff it is  $N\alpha^*_{AS}$  closed,  $f^{-1}(S)$  is  $N\alpha^*_{AS}$  closed set in  $U$ . Hence  $f$  is contra  $N\alpha^*_{AS}$ -continuous function.

**Sufficient:** Assume  $f$  is contra  $N\alpha^*_{AS}$ -continuous. Let  $S$  be any Nano open set in  $(V, \sigma_R(Y))$ . Then  $f^{-1}(S)$  is  $N\alpha^*_{AS}$ -closed in  $(U, \tau_R(X))$ . Since, [6] A set is Nano  $\alpha g$ -closed set iff it is  $N\alpha^*_{AS}$ -closed. Then,  $f^{-1}(S)$  is  $N\alpha g$ -closed in  $(U, \tau_R(X))$ . Therefore,  $f$  is contra  $N\alpha g$ -continuous.

**Theorem 3.15:** Every nano contra  $g^*$ -continuous function is contra  $N\alpha^*_{AS}$ -continuous function.

**Proof:** Let  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  be a nano contra  $g^*$ -continuous function and  $S$  be a nano open set in  $V$ . Then the inverse image of  $S$  under the map  $f$  is nano  $g^*$ -closed set in  $U$ . Since every nano  $g^*$ -closed set is  $N\alpha^*_{AS}$  closed,  $f^{-1}(S)$  is  $N\alpha^*_{AS}$

closed set in  $U$ . Hence  $f$  is contra  $N\alpha^*_{AS}$ -continuous function.

**Remark 3.16:** The converse of the above theorem need not be true as shown in the following example.

**Example 3.17:** Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a, b\}, \{c, d\}\}$ . Let  $X = \{a, b\} \subseteq U$  and  $\tau_R(X) = \{U, \emptyset, \{a, b\}\}$ . Let  $V = \{a, b, c, d\}$  with  $V/R = \{\{b\}, \{c\}, \{a, d\}\}$ . Let  $Y = \{a, c\} \subseteq V$  and  $\sigma_R(Y) = \{U, \emptyset, \{c\}, \{a, d\}, \{a, c, d\}\}$ . Let  $f: U \rightarrow V$  defined by  $f(a) = b, f(b) = a, f(c) = d, f(d) = c$  then  $f^{-1}(a) = b, f^{-1}(b) = a, f^{-1}(c) = d, f^{-1}(d) = c$ . Then  $f$  is contra  $N\alpha^*_{AS}$ -continuous but not contra nano  $g^*$ -continuous.

**Remark 3.18:** The composition of two contra  $N\alpha^*_{AS}$ -continuous need not always be a contra  $N\alpha^*_{AS}$ -continuous as seen from the following example.

**Example 3.19:** Let  $(U, \tau_R(X)), (V, \sigma_R(Y))$  and  $(W, \lambda_R(Z))$  be three nano topological spaces where  $U=V=W=\{a, b, c, d\}$  then the nano open sets are  $\tau_R(X) = \{U, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ ,  $\sigma_R(Y) = \{V, \emptyset, \{a, b\}\}$  and  $\lambda_R(Z) = \{W, \emptyset, \{c\}, \{a, d\}, \{a, c, d\}\}$ . Define two functions  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  and  $g: (V, \sigma_R(Y)) \rightarrow (W, \lambda_R(Z))$ . Clearly these two functions are contra  $N\alpha^*_{AS}$ -continuous. But their composition  $g \circ f: (U, \tau_R(X)) \rightarrow (W, \lambda_R(Z))$  is not contra  $N\alpha^*_{AS}$ -continuous because for the nano open set  $\{c\}$  in  $(W, \lambda_R(Z))$ ,  $(g \circ f)^{-1} = f^{-1}(g^{-1}(c)) = f^{-1}(d) = \{a\}$  is not  $N\alpha^*_{AS}$ -closed in  $(U, \tau_R(X))$ . Hence  $g \circ f: (U, \tau_R(X)) \rightarrow (W, \lambda_R(Z))$  is not contra  $N\alpha^*_{AS}$ -continuous. Thus, the composition of two contra  $N\alpha^*_{AS}$ -continuous need not always be a contra  $N\alpha^*_{AS}$ -continuous.

**Remark 3.20:** Contra  $N\alpha^*_{AS}$ -continuous and  $N\alpha^*_{AS}$ -continuous are independent.

**Example 3.21:** Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a, b\}, \{c, d\}\}$ . Let  $X = \{a, b\} \subseteq U$  and  $\tau_R(X) = \{U, \emptyset, \{a, b\}\}$ . Let  $V = \{a, b, c,$

$d\}$  with  $V/R = \{\{b, \{c\}, \{a, d\}\}\}$ . Let  $Y = \{a, c\} \subseteq V$  and  $\sigma_R(Y) = \{U, \emptyset, \{c\}, \{a, d\}, \{a, c, d\}\}$ . Then nano closed set  $= \{U, \emptyset, \{c, d\}\}$ . Let  $f: U \rightarrow V$  defined by  $f(a) = c, f(b) = d, f(c) = b, f(d) = a$  then  $f^{-1}(a) = d, f^{-1}(b) = c, f^{-1}(c) = a, f^{-1}(d) = b$ . Then  $f$  is  $N\alpha^*_{AS}$ -continuous. Since  $f^{-1}\{c\} = \{a\}$  is not  $N\alpha^*_{AS}$ -closed set in  $U$ ,  $f$  is not contra  $N\alpha^*_{AS}$ -continuous function.

**Example 3.22:** Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ . Let  $X = \{a, b\} \subseteq U$  and  $\tau_R(X) = \{U, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ . Then  $N\alpha^*_{AS}$  closed sets are  $= \{U, \emptyset, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$ . Let  $V = \{a, b, c, d\}$  with  $V/R = \{\{a, b\}, \{c, d\}\}$ . Let  $Y = \{a, b\} \subseteq V$  and  $\sigma_R(Y) = \{U, \emptyset, \{a, b\}\}$ . Then nano closed sets are  $= \{U, \emptyset, \{c, d\}\}$ . Let  $f: U \rightarrow V$  defined by  $f(a) = d, f(b) = c, f(c) = a, f(d) = b$  then  $f^{-1}(a) = c, f^{-1}(b) = d, f^{-1}(c) = b, f^{-1}(d) = a$ . Thus, the inverse image  $\{a, b\}$  in  $V$  i.e.  $f^{-1}\{a, b\} = \{c, d\}$  which is a  $N\alpha^*_{AS}$  closed set in  $U$ . Thus,  $f$  is contra  $N\alpha^*_{AS}$ -continuous. But  $f^{-1}\{c, d\} = \{a, b\}$  is not a  $N\alpha^*_{AS}$  closed set in  $U$ . Thus,  $f$  is not  $N\alpha^*_{AS}$ -continuous.

#### IV. CONTRA $N\alpha^*_{AS}$ -IRRESOLUTE FUNCTION

In this section we introduce Contra  $N\alpha^*_{AS}$ -irresolute function and discuss some of its properties.

**Definition 4.1:** Let  $(U, \tau_R(X))$  and  $(V, \sigma_R(Y))$  be two nano topological spaces. A function  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  is called contra  $N\alpha^*_{AS}$ -irresolute if the inverse image of every  $N\alpha^*_{AS}$  closed(open) set in  $(V, \sigma_R(Y))$  is  $N\alpha^*_{AS}$  closed(open) set in  $(U, \tau_R(X))$ .

**Example 4.2:** Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a, \{c\}, \{b, d\}\}\}$ . Let  $X = \{a, b\} \subseteq U$  and  $\tau_R(X) = \{U, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$ . Then  $N\alpha^*_{AS}$ -closed  $= \{U, \emptyset, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$ . Let  $V = \{a, b, c, d\}$  with  $V/R = \{\{b\}, \{c\}, \{a, d\}\}$ . Let  $Y = \{a, c\} \subseteq V$  and  $\sigma_R(Y) = \{U, \emptyset, \{c\},$

$\{a, d\}, \{a, c, d\}\}$ . Then  $N\alpha^*_{AS}$  closed set  $= \{U, \emptyset, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$ . Let  $f: U \rightarrow V$  defined by  $f(a) = a, f(b) = d, f(c) = b, f(d) = c$  then  $f^{-1}(b) = c, f^{-1}\{a, b\} = \{a, c\}, f^{-1}\{b, c\} = \{c, d\}, f^{-1}\{b, d\} = \{b, c\}, f^{-1}\{a, b, c\} = \{a, c, d\}, f^{-1}\{a, b, d\} = \{a, b, c\}, f^{-1}\{b, c, d\} = \{b, c, d\}$ . The inverse image of every Nano  $N\alpha^*_{AS}$ -closed set in  $(V, \sigma_R(Y))$  is  $N\alpha^*_{AS}$ -closed in  $(U, \tau_R(X))$ . Then  $f$  is contra  $N\alpha^*_{AS}$ -irresolute.

**Theorem 4.3:** In a nano topological space, the composition of two contra  $N\alpha^*_{AS}$ -irresolute functions is also a contra  $N\alpha^*_{AS}$ -irresolute function.

**Proof:** Consider the contra  $N\alpha^*_{AS}$ -irresolute functions  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  and  $g: (V, \sigma_R(Y)) \rightarrow (W, \lambda_R(Z))$ . Claim,  $g \circ f: (U, \tau_R(X)) \rightarrow (W, \lambda_R(Z))$  is a contra  $N\alpha^*_{AS}$ -irresolute function. As  $g$  is considered to be a contra  $N\alpha^*_{AS}$ -irresolute function, by definition for every  $N\alpha^*_{AS}$ -closed set  $S$  of  $(W, \lambda_R(Z))$ ,  $g^{-1}(S)$  is a  $N\alpha^*_{AS}$ -closed set in  $(V, \sigma_R(Y))$ . Since  $f$  is  $N\alpha^*_{AS}$ -irresolute,  $f^{-1}(g^{-1}(S))$  is  $N\alpha^*_{AS}$ -closed in  $(U, \tau_R(X))$ . Hence  $(g \circ f)$  is a contra  $N\alpha^*_{AS}$ -irresolute.

**Theorem 4.4:** In a nano topological space, the composition of two contra  $N\alpha^*_{AS}$ -irresolute function is  $N\alpha^*_{AS}$ -irresolute function.

**Proof:** Consider the contra  $N\alpha^*_{AS}$ -irresolute functions  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  and  $g: (V, \sigma_R(Y)) \rightarrow (W, \lambda_R(Z))$ . Claim,  $g \circ f: (U, \tau_R(X)) \rightarrow (W, \lambda_R(Z))$  is a  $N\alpha^*_{AS}$ -irresolute function. Let  $S$  be a  $N\alpha^*_{AS}$ -open set in  $W$ .  $g^{-1}(S)$  is a  $N\alpha^*_{AS}$ -closed set in  $V$ , since  $g$  is contra  $N\alpha^*_{AS}$ -irresolute function. Since  $f$  is also contra  $N\alpha^*_{AS}$ -irresolute,  $f^{-1}(g^{-1}(S))$  is  $N\alpha^*_{AS}$ -closed in  $U$ . Hence  $(g \circ f)$  is a  $N\alpha^*_{AS}$ -irresolute function.

**Theorem 4.5:** Every contra  $N\alpha^*_{AS}$ -irresolute function is contra  $N\alpha^*_{AS}$ -continuous function.

**Proof:** Let  $f: (U, \tau_R(X)) \rightarrow (V, \sigma_R(Y))$  be a Contra  $N\alpha^*_{AS}$ -irresolute function. Let  $S$  a nano closed set in  $V$ .  $S$  is  $N\alpha^*_{AS}$ -closed in  $V$ , since every nano closed set is  $N\alpha^*_{AS}$ -closed. Hence  $f^{-1}(S)$  is  $N\alpha^*_{AS}$ -open in  $U$ . Hence  $f$  is contra  $N\alpha^*_{AS}$ -continuous function.

**Remark 4.6:** Every contra  $N\alpha^*_{AS}$ -continuous function need not be contra  $N\alpha^*_{AS}$ -irresolute function as shown in the following example.

**Example 4.7:** Let  $U = \{a, b, c, d\}$  with  $U/R = \{\{a, b\}, \{c, d\}\}$ . Let  $X = \{a, b\} \subseteq U$  and  $\tau_R(X) = \{U, \emptyset, \{a, b\}\}$ . Let  $V = \{a, b, c, d\}$  with  $V/R = \{\{b\}, \{c\}, \{a, d\}\}$ . Let  $Y = \{a, c\} \subseteq V$  and  $\sigma_R(Y) = \{U, \emptyset, \{c\}, \{a, d\}, \{a, c, d\}\}$ . Then nano closed set =  $\{U, \emptyset, \{c, d\}\}$ . Let  $f: U \rightarrow V$  defined by  $f(a) = c, f(b) = d, f(c) = b, f(d) = a$  then  $f^{-1}(a) = d, f^{-1}(b) = c, f^{-1}(c) = a, f^{-1}(d) = b$ . Then  $f$  is contra  $N\alpha^*_{AS}$ -continuous but not contra  $N\alpha^*_{AS}$ -irresolute.

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