ON ALPHA ^ GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce a new class of sets called α^g - closed sets in topological spaces and to study their properties. Further, we define and study α^g - open sets α^g - continuity.

Key Words: α^g - closed sets, α^g - open sets, α^g continuous.

1. INTRODUCTION

In 1970, Levine [9] introduced the concept of generalized closed set in the topological spaces and a class of topological spaces called T1/2 spaces. Extensive research on generalizing closedness was done in recent years by many mathematicians. In 1990, S.P. Arya and T.M. Nour [2] define generalized semi-open sets, generalized semi-closed sets. In 1993 Maki H, Devi R and Balachandran K [21] introduced generalized alpha closed (gα-closed) sets. In 2000, A. Pushpalatha[16] introduced a new class of closed sets called weakly closed(w- closed) sets .In 2007, S.S.Benchalli and R. S. Wali[3] introduced the class of set called regular w-closed(rw-closed) sets in topological spaces.

Recently Viswanathan,A.,and Ramasamy,K., (2009),introduced the concept of generalized closed sets and weakly closed sets in topological spaces, (wgα-closed sets. and wαg closed sets). In this paper, we introduce a new class of sets called alpha ^ generalized - closed sets (briefly α^g -closed sets) and we study their basic properties. We recall the following definitions, which will be used often throughout this paper.

2. PRELIMINARIES

Throughout this paper, X, Y, Z denote the topological spaces (X, τ),(Y, σ) and (Z, η) respectively, on which no separation axioms are assumed.

Definition 2.1: A subset A of a space X is called

(1) a pre-open set if $A \subseteq int(cl(A))$ and a pre-closed set if $cl(int(A)) \subseteq A$.
(2) a semi-open set if $A \subseteq cl(int(A))$ and a semi-closed set if $int(cl(A)) \subseteq A$.
(3) an α-open set if $A\subseteq int(cl(int(A)))$ and a α-closed set if $cl(int(cl(A)))\subseteq A$.
(4) a semi-preopen set (=β-open) if $A \subseteq cl(int(cl(A)))$ and a semi-pre closed set (β-closed) if $int(cl(int(A))) \subseteq A$.

The semi-closure (resp. α-closure) of a subset A of (X,τ) is denoted by scl(A) (resp. αcl(A) and spcl(A))and is the intersection of all semi-closed (resp. α-closed and semi-pre closed) sets containing **A.**

Definition 2.2: A subset A of X is called

1. a generalized closed (briefly g-closed) [9] set iff cl(A) ⊆ U whenever A ⊆ U and U is open in X.
2. Strongly generalized closed (briefly g* closed)[20] if cl(A) ⊆ U whenever A ⊆ U and U is g-open in X
3. a regular open [18] set if A = int(cl(A)) and regular closed[18] set if A = cl(int(A)) .
4. a semi generalized closed (briefly sg – closed)[4] if scl(A) ⊆ U whenever A ⊆ U and U is Semiopen in X.
5. a generalized semi closed (briefly gs – closed)[2] if scl(A) ⊆ U whenever A ⊆ U and U is open in X.
6. a generalized semi-pre closed (briefly gsp – closed)[5] if spcl(A) ⊆ U whenever A ⊆ U and U is open in X.
7. a regular generalized closed (briefly rg – closed)[15] if cl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
8. a generalized preclosed (briefly gp – closed) [10]if pcl(A) ⊆ U whenever A ⊆ U and U is open in X.
9. a generalized pre regular closed (briefly gpr – closed)[7] if pcl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
10. a weakly closed (briefly w – closed)[16] if cl(A) ⊆ U whenever A ⊆ U and U is semiopen in X.
11. a regular weakly closed (briefly rw – closed)[3] if cl(A) ⊆ U whenever A ⊆ U and U is regular semiopen in X.
12. a weakly generalized semi closed (briefly wg – closed) [13] if cl(int(A) ⊆ U whenever A ⊆ U and U is open in X.
13. a regular weakly generalized semi closed (briefly rwg – closed)[13] if cl(int(A) ⊆ U whenever A ⊆ U and U is regular open in X.
14. a regular generalized weakly semi closed (briefly rgw – closed)[17] if cl(int(A) ⊆ U whenever A ⊆ U and U is regular semi-open in X.
15. a regular^ generalized closed (r^g closed)[22] if gcl(A) ⊆U whenever A U and U is regular open.
16. gˆ - closed set [23] if cl(A)⊆U whenever A⊆U and U is semi-open in (X, τ).
17. αgˆ - closed set [24] if αcl(A)⊆U whenever A⊆U and U is gˆ -open in (X, τ).
18. αg *- closed set [25] if cl(A)⊆U whenever A⊆U and U is α-open in (X, τ).
19. sαg*- closed set [26] if αcl(A)⊆U whenever A⊆U and U is g *-open in (X, τ).
20. wgα-closed set [27] if αcl(int(A))⊆U whenever A⊆Uand U is α-open in (X, τ).
21. wαg-closed set [27] if αcl(int(A))⊆U whenever A⊆U and U is open in (X, τ).
22. ψ-closed set [28] if scl(A)⊆U whenever A⊆Uand U is sg-open in (X, τ).
23. ψg -closed set [29] if ψcl(A)⊆U whenever A⊆U and U is open in (X, τ).
24. g *ψ -closed set [30] if ψcl(A)⊆U whenever A⊆Uand U is g-open in (X, τ).
25. ψgˆ - closed set [31] if ψcl(A)⊆U whenever A⊆U and U is gˆ -open in (X, τ).
26. αψ- closed set [32] if ψcl(A)⊆U whenever A⊆U and U is α-open in (X, τ).
27. gα *- closed set [25] if cl(A)⊆U whenever A⊆U and U is α-open in (X, τ).
28. αg- closed set [33] if αcl(A)⊆U whenever A⊆U and U is open in (X, τ).
29. gα- closed set [21] if cl(A)⊆U whenever A⊆U and U is α-open in (X, τ).
30. r^g- closed set [35] if gcl(A)⊆U whenever A⊆U and U is regular open in (X, τ).

The complements of the above mentioned closed sets are their respective open sets.

Definition 2.3: A map f: X →Y is said to be
1. a continuous function[1]if f^{-1}(V) is closed in X for every closed set V in Y.
2. a pre continuous [11] if f^{-1}(V) is pre closed in X for every closed set V in Y.
3. a α-continuous function[34]if f^{-1}(V) is α-closed in X for every closed set V in Y.
5. a gs -continuous [1] if f^{-1}(V) is gs closed in X for every closed set V in Y.
6. a αg-continuous[7] if f^{-1}(V) is αg - closed in X for every closed set V in Y.
7. a rwg-continuous[13] if f^{-1}(V) is rwg- closed in X for every closed set V in Y.
8. a rgw-continuous[13] if f^{-1}(V) is rgw- closed in X for every closed set V in Y.
9. a swg –continuous[13] if f^{-1}(V) is swg- closed in X for every closed set V in Y.

3. Alpha ^ Generalized Closed Sets *(α^g* - closed sets)

Definition 3.1: A subset A of (X,τ) is called a alpha ^generalized closed **(briefly α^g closed)** if gcl(A) ⊂U, whenever A⊂ U and U is α-open in X.
We denote the family of all α^g closed sets in space X by α^GC(X).

Theorem 3.2: Every closed set of a topological space (X,τ) is α^g closed set.

Proof: Let A⊂ X be a closed set and A ⊂ U where U be α-open. Since A is closed and every closed set is g-closed, gcl(A) ⊂ cl(A) = A⊂ U. Hence A is an α^g closed set.

Remark 3.3: The converse of the above theorem need not be true as seen in the following example.

Example 3.4: Let X = {a, b, c}, τ = {X,ϕ, {a},{a, b}}. Let A = {a, c} then A is an α^g closed set but it is not a closed set.

Theorem 3.5: Every g^-closed set is α^g closed.

Proof: Let A be a g^-closed set. Let A ⊂ U where U is α-open. Since every α-open set is semi open and A is g^ closed, cl(A) ⊂ U. Every closed set is g -closed therefore gcl(A) ⊂ cl(A)⊂ U. Hence A is α^g closed.

Remark 3.6: The converse of the above theorem need not be true as seen in the following example.

Example 3.7: Let X = {a, b, c}, τ = {X,ϕ, {a},{a, b}}. Let A = {a, c} then A is α^g closed set but it is not a g^-closed set.

Theorem 3.8: Every α^g closed set αg is closed.

Proof: Let A be α^g closed. Let A ⊂ U and U be open. Since every open set is α-open set and A is α^g closed set, αcl(A) ⊂ U, Hence A is αg closed.

Remark 3.9: The converse of the above theorem need not be true as seen in the following example.

Example 3.10: Let X = {a, b, c}, τ = {X,ϕ, {a},{a, b}}. Let A = {b} then A is αg-closed set but it is not a α^g -closed set.

Theorem 3.11: Every g*closed set is α^g closed.

Proof: Let A be g*closed in (X,τ). Let A ⊂ U where U is α open. Since every α open set is g-open and A is g*closed, cl(A) ⊂ U. Every closed set is g-closed, then gcl(A) ⊂ cl(A) ⊂ U. Hence A is α^g closed.

Remark 3.12: The converse of the above theorem need not be true as seen in the following example.

Example 3.13: Let X = {a, b, c}, τ = {X, ϕ, {a}}. Let A = {a, c} then A is α^g closed set but it is not a g* -closed set.

Theorem 3.14: Every αg*closed set is α^g closed.

Let A be αg*closed in (X,τ). Let A ⊂ U where U is α open.Since A is αg*closed, cl(A) ⊂ U. Every closed set is g-closed, then gcl(A) ⊂ cl(A) ⊂ U. Hence A is α^g closed.

Remark 3.15: The converse of the above theorem need not be true as seen in the following example.

Example 3.16: Let X = {a, b, c}, τ = {X, ϕ, {a}}. Let A = {a, b} then A is α^g closed set but it is not a αg* -closed set.

Example 3.17: Let X = {a, b, c}, τ = {X,ϕ, {a},{a, b}}.Then

1.rwg closed ={X, ϕ, {a},{b}{c},{a,b}{a,c}, {b,c}}

3.rgw closed ={X, ϕ, {a},{b}{c},{a,b}{a,c}, {b,c}}

5.αg^ closed =={X, ϕ, {b}{c},{a,c}, {b,c}}

6.wαg closed = ={X, ϕ, {b}{c},{a,c}, {b,c}}

7.ψgˆ closed ={X, ϕ, {b}{c},{a,c}, {b,c}}

8. r^g closed ={X, ϕ, {a},{b}{c},{a,b}{a,c}, {b,c}}

9.α^g closed= {X, ϕ, {c},{a,c}, {b,c}}

Theorem 3.18:

1.Every α^g closed set is rwg closed.

2.Every α^g closed set is rgw closed.

3.Every α^g closed set is -αg^ closed.

4.Every α^g closed set is -wαg closed

5.Every α^g closed set is - ψgˆ closed

6.Every α^g closed set is - r^g closed

Proof: Straight forward.

Remark 3.19: The converse of the above theorem need not be true as seen in the following examples.

In Example 3.14, A = {a,b}, then A is rwg closed but not α^g closed

In Example 3.14, B = {b}, then B is rgw closed but not α^g closed set.

In Example 3.14, B = {b}, then B is αg^ closed but not α^g closed.

In Example 3.14, B = {b}, then B is wαg closed but not α^g closed.

In Example 3.14, B = {b}, then B is ψgˆ closed but not α^g closed.

In Example 3.14, B = {a}, then B is r^g closed but not α^g closed.

Remark 3.20: α^g closed sets and semi closed sets are independent to each other as seen from the following examples.

Example 3.21:

* Let X = {a, b, c,d},τ ={X, ϕ,{a},{b},{a, b}, {a,b,c}}. Let A = {b}, A is semi closed but not α^g closed and the subset {a,b,d} in X is α^g closed but not semi closed.

Remark 3.22: α^g closed sets and pre closed sets are independent to each other as seen from the following examples.

Example 3.23:

* Let X = {a, b, c,d},τ ={X, ϕ,{a},{b},{a, b}, {a,b,c}}Let A = {a, b,d}, then A is α^g closed but not preclosed.

* Let X = {a, b, c,d},τ ={X, ϕ,{a},{b},{a, b}, {a,b,c}}. Let A = {c}, then A is preclosed but not an α^g closed set.

Remark 3.24: α^g closed sets and semi-preclosed sets are independent to each other as seen from the following example.

Example 3.25:

Let X = {a, b, c,d},τ ={X, ϕ,{a},{b},{a, b}, {a,b,c}}. The subset {a} is semi- preclosed but not α^g closed and the subset {a, b,d} is α^g closed but not semi- preclosed.

Remark 3.26: α^g closed sets and gα* closed sets are independent to each other as seen from the following examples.

Example 3.27:

Let X = {a, b, c,d},τ ={X, ϕ,{a},{b},{a, b}, {a,b,c}}.. Let A = {c}, then A is gα* closed but not an α^g closed set in X and the subset{a, d} is an α^g closed set but not a gα* closed in X.

Remark 3.28: The concepts of α^g closed sets and αψ closed sets are independent of each other as seen from the following examples.

Example 3.29:

Let X = {a, b, c}, τ = {X,ϕ, {a},{a, b}}.. Let B = {b}, then B is αψ closed but it is not an α^g closed set and the subset {a, c} is an α^g closed set but not αψ closed set.

Remark 3.30: The concepts of α^g closed sets and sg closed sets are independent of each other as seen from the following example.

Example 3.31:

Let X = {a, b, c}, τ = {X,ϕ, {a},{a, b}}.. Let B = {b}, then B is sg closed but it is not an α^g closed set and the subset {a, c} is an α^g closed set but not sg closed set.

Remark 3.32: The concepts of α^g closed sets and α closed sets are independent of each other as seen from the following examples.

Example 3.33:

*Let X = {a, b, c,d},τ ={X, ϕ,{a},{b},{a, b}, {a,b,c}}Let A = {a, b,d}, then A is α^g closed but not α-closed.

* Let X = {a, b, c,d},τ ={X, ϕ,{a},{b},{a, b}, {a,b,c}}. Let A = {c}, then A is α-closed but not an α^g closed set.

Remark 3.34: α^g closed sets and sαg* closed sets are independent to each other as seen from the following examples.

Example 3.35:

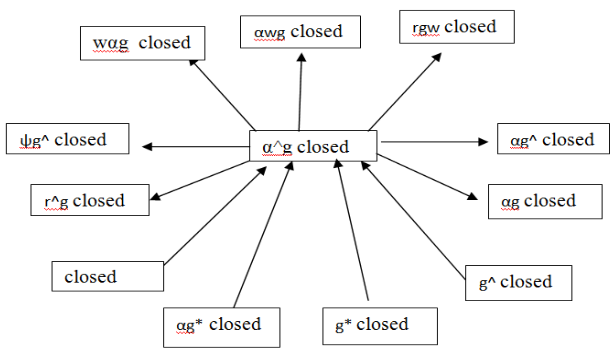
Let X = {a, b, c}, τ = {X,ϕ, {a},{a, b}}.The subset {a,b} is sαg* closed set but not an α^g closed set and the subset {b, c} is α^g closed set but not sαg* closed.

Remark 3.36: α^g closed sets and wgα closed sets are independent to each other as seen from the following examples.

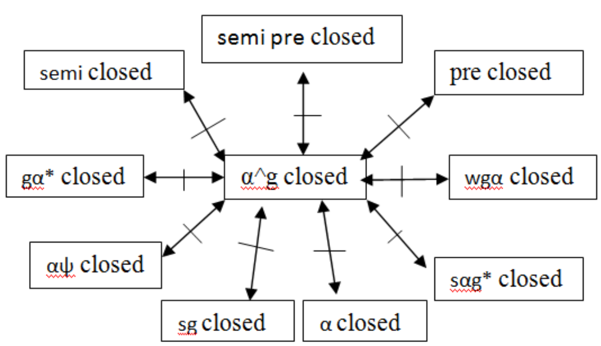
Example 3.37:

Let X = {a, b, c}, τ = {X,ϕ, {a},{a, b}}.The subset {b} is wgα closed set but not an α^g closed set and the subset {a, c} is α^g closed set but not wgα closed.

Remark 3.38: The above discussions are shown in the following diagram.



Remark 3.39: The following is the diagrammatic representation of independent concepts of the sets with α^g closed sets.



Theorem 3.40: Let A be an α^g closed set in a topological space X. Then gcl(A) – A contains no non-empty α-closed set in X.

Proof: Let F be a α- closed set such that F ⊂ gcl(A) – A. Then F⊂ X-A implies A⊂ X-F. Since A is α^g closed and X-F is α-open, then gcl(A) ⊂ X-F. That is F ⊂ X-gcl(A). Hence F⊂

gcl(A)∩(X – gcl(A)) = ϕ. Thus F =ϕ, whence gcl(A)-A does not contain nonempty α- closed set.

Remark 3.41: The converse of the above theorem need not be true , that means if gcl(A)-A contains no nonempty α-closed set , then A need not to be an α^g closed as seen in the following example.

Example 3.42:

Let X = {a, b, c,d},τ ={X, ϕ,{a},{b},{a, b}, {a,b,c}}. Let A = {b}. gcl(A) – A = {d}, it does not contain non-empty α- closed set in X. But A = {b} is not an α^g closed set.

Theorem 3.43: The finite union of two α^g closed sets are α^g closed.

Proof: Assume that A and B are α^g closed sets in X. Let A∪B ⊂ U where U is α- open. Then A⊂U and B⊂U.Since A and B are α^g closed, gcl(A) ⊂ U and gcl(B) ⊂ U. Then gcl(A∪B) = gcl(A) ∪ gcl(B) ⊂ U. Hence AUB is α^g closed.

Remark 3.44: The intersection of two α^g closed set in X need not be an α^g closed set as seen in the following example.

Example 3.45:

LetX = {a, b, c}, τ = {X,ϕ,{a}} then α^g={X, ϕ, {b},{c},{a,b}{a,c}, {b,c}} .
If A = {a, b} and B= {a, c}. Then A and B are α^g closed sets. But A∩B = {a} is not an α^g closed set.

Theorem 3.46: If A is an α^g closed subset of X such that A⊂ B⊂ gcl(A), then B is an α^g closed set.

Proof: Let B ⊂ U where U is α open. Then A ⊂ B implies A⊂U. Since A is α^g closed, gcl(A) ⊂ U. By hypothesis gcl(B) ⊂ gcl(gcl(A)) = gcl(A) ⊂ U. Hence B is α^g closed.

Remark 3.47: The converse of the above theorem need not be true as seen in the following example.

Example 3.48:

Let X = {a, b, c, d}, τ = {X,ϕ, {a},{b},{a, b}{a, b, c}}. Let A = {d} and B = {a, d}. Then A and B are α^g closed sets. But A ⊂ B is not a subset of gcl(A).

4. Regular ^ Generalized Open Set:

Definition 4.1: A set A ⊂ X is called alpha ^ generalized open (α^g open) set if and only if its compliment is alpha ^generalized closed. The collection of all α^g open sets is denoted by α^GO(X).

5.α^g Continuous and α^g Irresolute Functions:

Definition 5.1: A function f : (X,τ) →(Y,σ) is called α^g continuous if every f^{-1}(V) is α^g closed in X for every closed set V of Y.

Definition 5.2: A function f: (X,τ) →(Y,σ) is called α^g irresolute if every f^{-1}(V) is α^g closed in X for every α^g closed set V of Y.

Example 5.3: Let X = {a, b, c}, τ = {X, ϕ, {a},{a, b}} and Y = {a, b, c}, σ = {Y,ϕ,{a}}. Define f :(X,τ) →(Y,σ) by f(a)=a, f(b)=b, f(c)=c. Here the inverse image of the closed sets in Y are α^g closed sets in X. Hence f is α^g continuous.

Example 5.4: Let X={a, b, c, d}, τ = {ϕ,X,{a, b},{c},{a, b, c}} and Y = X,
σ = {Y,ϕ,{a},{c},{a, c},{a, b},{a, b, c}}.
Define f:(X,τ) →(Y,σ) by f(a)=a, f(b)=b, f(c)=c, f(d)=d. The inverse image of every α^g closed set in Y is α^g closed set in X. Hence f is α^g irresolute.

Remark 5.5: Every α^g irresolute function is α^g continuous but the converse is not true as seen in the following example.

Example 5.6: In example 5.3, f is α^g continuous but not α^g irresolute.

Remark 5.7: Every continuous function is α^g continuous. But the converse is not true as seen in the following example.

Example 5.8:
Let X= {a, b, c, d}. τ = {X,ϕ,{a},{b},{a, b},{a, b, c}} and Y = X,
σ = {Y,ϕ,{a},{c},{d},{a, c},{a, d}, {a, c, d}}. Define f :(X,τ) →(Y,σ) the identity mapping. Then f is α^g continuous but not continuous.

Theorem 5.9: Let f: (X, τ) →(Y, σ) and g:(Y, σ) →(Z, η) be any two functions. Then
(i) (gof) is α^g-continuous if g is continuous and f is α^g -continuous
(ii) (gof) is α^g -irresolute, if g is α^g –irresolute and f is α^g -irresolute.
(iii) (gof) is α^g continuous if g is α^g continuous and f is α^g -irresolute.

Proof:
(i) Let V be any closed set in (Z,η). Then f^{-1} (V) is closed in (Y,σ), since g is continuous. By hypothesisf^{-1} (g^{-1} (V)) is α^g closed in (X,τ). Hence gof is α^g continuous.
(ii) Let V be α^g closed set in (Z,η).Since g is α^g irresolute, g^{-1} (V) is α^g closed in (Y,σ). As f is α^g irresolute, f^{-1} (g^{-1} (V)) = $(gof)^{-1}$(V) is α^g closed in (X,τ).Hence gof is α^g irresolute.
(iii) Let V be closed in (Z,η).Since g is α^g continuous. g^{-1} (V) is α^g closed in (Y,σ). As f is α^g irresolute, , f^{-1} (g^{-1} (V)) =$(gof)^{-1}$(V) is α^g closed in (X,τ).Hence (gof) is α^g continuous.

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